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**TESIS**

# PRONOSTICO DEL TIPO DE CAMBIO MEDIANTE MODELOS ECONOMÉTRICOS, OPERADORES DE AGREGACIÓN HEAVY Y LA METODOLOGÍA DE LOS EFECTOS OLVIDADOS. MÉXICO 1994-2017

**PRESENTA**
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# CAPÍTULO I. Introducción

## 1.1 Introducción

Los resultados de la crisis en México de 1994, expresan Cartens & Werner (2000), Martínez & Werner (2002) y Ramos & Torres (2005), forzaron a abandonar el régimen de flotación predeterminado y adoptar el de libre flotación ante las divisas del mundo. Los responsables de las oscilaciones del precio afirman Popov & Stutzmann (2003) son los cambios en la oferta y la demanda de los mercados financieros, influidos por numerosos factores externos y el régimen cambiario, lo cual genera un tipo de cambio inesperado, indeterminado y volátil que pone en riesgo los flujos de efectivo futuros de la empresa. (e.g., Adler & Dumas, 1984; Flood & Lessard, 1986; Lewent & Kearney, 1990; Bartram,2008)

El efecto de adoptar el régimen de libre flotación en tipo de cambio ha generado una incertidumbre considerable en cuestión del valor de las divisas en México, principalmente en el tipo de cambio USD/MXN el cual del año 1994 al 2014 se ha enfrentado a una alta variación de precio y volatilidad (Ver Figura 1-2).

La exposición al riesgo cambiario, expresan Chaney (2016) y Chatterjee *et al.,* (2013), se encuentra vinculada a las operaciones diarias de la compañía, entre las que incluyen importación, exportación, inversión y prestamos en moneda extranjera, de tal manera, que el entorno en donde operan las organizaciones es volátil e incierto (Al-Momani & Gharaibeh, 2008). Por lo que, ante estos escenarios, señala Wu *et al.,* (2012), es necesario el diseño de estrategias que permitan reducir el riesgo a niveles aceptables, por lo que pronosticar el tipo de cambio futuro se vuelve una necesidad para las organizaciones.

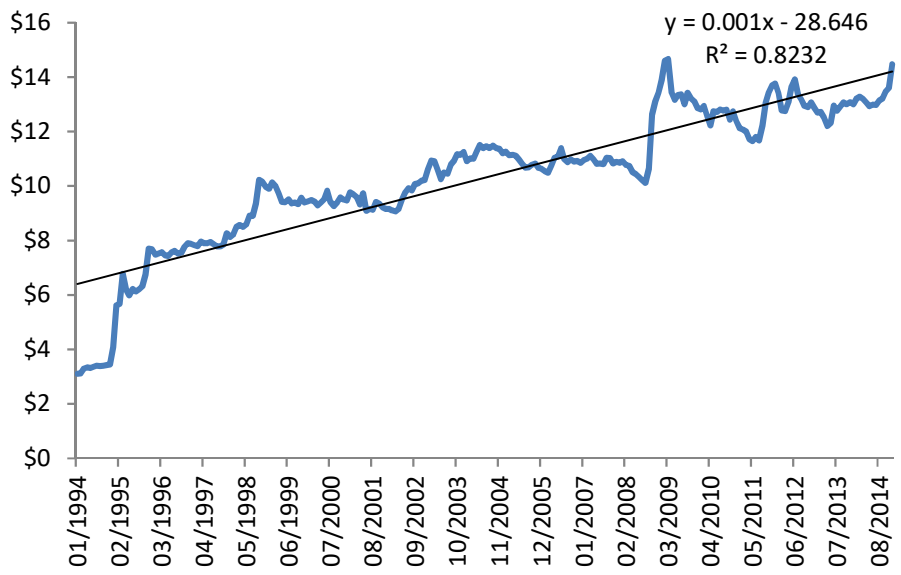



**Figura 1. Tipo de cambio spot USD/MXN 1994-2014[1]**

Fuente: Elaboración propia con datos de Banxico (2015)

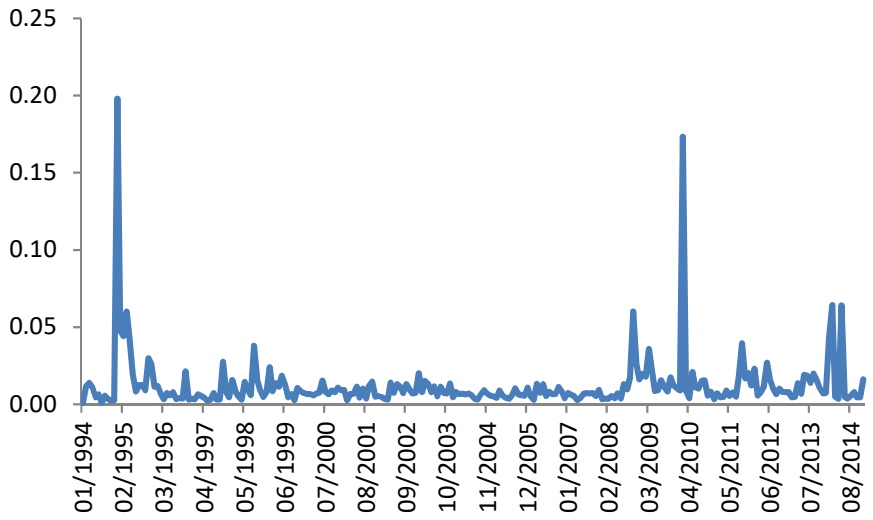



**Figura 2. Volatilidad del tipo de cambio USD/MXN 1994-2014**

Fuente: Elaboración propia con datos de Banxico (2015)

[1] El coeficiente de determinación o $r^2$ es la proporción de la varianza explicada por el modelo de regresión, lo que permite medir el éxito de predecir la variable dependiente con la variable independiente. Mientras mas cercana sea a 1 tendrá mayor poder de predicción (Nagelkerke, 1991).

Bajo estas condiciones de incertidumbre, expresan Kaufmann & Gil Aluja (1990) se requieren herramientas que auxilien en la toma de decisiones y en la definición de estrategias, ya que el riesgo cambiario, comenta Kavaliova (2007) lleva a los decisores a la necesidad de medir con precisión y racionalidad la exposición y diseñar estrategias que permitan reducirlo a niveles aceptables, por lo que determinar el tipo de cambio futuro se torna necesario para poder reconocer, controlar y minimizar el riesgo.

Por lo comentado, la presente tesis se focaliza en analizar la eficiencia del uso de nuevas herramientas de pronóstico del tipo de cambio en contraste con los modelos tradicionales. Lo anterior se ha realizado a través de la publicación de diferentes artículos en diversos congresos, libros y revistas indizadas.

Finalmente se presenta una nueva extensión del operador *Ordered Weighted Average* (OWA), denominado *Heavy Ordered Weighted Moving Average* (HOWMA) el cual permitió incluir dentro del pronóstico de las variables exógenas que integran a los modelos econométricos información histórica, expectativas futuras y conocimiento de los expertos del mercado financiero; proporcionando una herramienta que contribuye al desarrollo de escenarios futuros y apoya la toma de decisiones dentro de la organización.

## 1.2 Planteamiento del problema

Una de las interrogantes centrales en la literatura sobre gestión del riesgo cambiario está centrado en reducir el nivel de exposición económica, a partir de minimizar el error de pronóstico del tipo de cambio forward (Meese & Rogoff, 1983; Faust *et al.,* 2003; Engel *et al,* 2015). El tipo de cambio es el precio relativo más importante en economías emergentes; su volatilidad creciente, obliga a los tomadores de decisiones a atenuar el grado de exposición, mediante la estructuración de modelos teóricos y herramientas metodológicas que viabilicen minimizar el error del pronóstico y, en consecuencia, el riesgo cambiario (Jorion & Sweeney, 1996; Zhang & Hu,1988; Bodyanskiy & Popov, 2006; Majhi, *et al.,* 2009).

En la literatura, se compendian modelos cuyas variables explicativas tradicionales son la inflación, tasa de interés y flujos monetarios de balanza de pagos. Sin embargo, estos no han generado resultados satisfactorios, siendo superador por modelos basados en caminos aleatorios[2] a corto plazo (Meese & Rogoff, 1983; Taylor, 1995; Cheung *et al.,* 2002; Obstfeld & Rogoff, 2001).

Por otro lado, se identifican los modelos no estructurados entre los que sobresalen los de regresión, Markov, redes neuronales y algoritmos genéticos (Huang *et al.,* 2005; Preminger & Franck, 2007; Nikolsko-Rzhevskyy & Prodan, 2012).

Recientemente, se introducen los modelos basados en el agente, que integran el conocimiento e interpretación de expertos, relajando la hipótesis que postula la presencia de información homogénea y de participantes perfectamente informados, por lo que el grado de aversión al riesgo, habilidades, experiencia y conocimiento

[2] El pronóstico del camino aleatorio es aquel en donde el nivel (log) del tipo de cambio nominal se predice que se mantendrá dentro del mismo nivel (log) (Engel *et al.,* 2015).

del entorno global, desempeñan un rol sobresaliente en la toma de decisiones (Menkhoff & Taylor, 2007; Chen, 2011; Pacelli *et al.,* 2011; Dick *et al.,* 2015).

Las limitaciones que presentan los modelos con bases econométricas, señala Phillips (2003), que aun cuando el modelo este correctamente formulado, su funcionamiento depende de las estimaciones de las variables exógenas, las cuales de forma generalizada se hacen a través de series de tiempo (e.g. Fischer & Planas, 2000). Sin embargo, esta técnica solamente contempla la información histórica, sin considerar expectativas y conocimiento de los expertos dentro del pronóstico, de tal forma que como alternativa a los modelos tradicionales Goldberg & Tenorio (1997), Lyons & Evans (2002), Wu & Goo (2005) señalan que la opinión que tienen los participantes en el mercado de divisas, al interpretar tanto información pública y privada, permiten asumir las expectativas de equilibrio del tipo de cambio.

Por su parte, Pastor (2000) y Krugman (2000) señalan que la ineficiencia de los modelos se debe a que son una simplificación de la realidad. Por lo que aun cuando el fenómeno estudiado está basado en un grupo de variables teóricamente aceptadas, algunas son olvidadas, por lo que su impacto e influencia son ignorados (Gil-Aluja, 1999)..

Ante esta problemática, se plantea el uso de operadores de agregación de información que permitan generar nuevos escenarios. Dentro de los que han sido ampliamente estudiado y utilizado se encuentra el *Ordered Weighted Average* (OWA) desarrollado por Yager (1988), el cual ha sido aplicado y trabajado en diversas áreas (Merigó & Gil-Lafuente (2010); Zhou & Chen (2013); Emrouznejad & Marra (2014)).

En el área financiera el operador OWA ha sido utilizado en la evaluación de portafolios de inversión (Singh *et al.,* 2010), monitoreo inteligente de mercados financieros (Fonooni & Moghadam, 2008), evaluación del desempeño de las empresas (Cheng *et al.,* 2012), pronostico del precio de acciones (Cheng *et al.,* 2013), entre otros.

Dentro de las extensiones del operador OWA, se encuentran los operadores Heavy OWA (HOWA), los cuales, mediante el uso de un vector de peso heavy, no limitado a una suma igual a uno, permiten subestimar o sobrestimar los resultados de acuerdo a la información disponible y las expectativas de los tomadores de decisión (Yager, 2002).

Un modelo para detectar los olvidos en el proceso de toma de decisiones, el cual vincula causas y efectos a través de matrices de incidencia para obtener las relaciones indirectas a través de información de expertos es la metodología de los efectos olvidados, desarrollada por Kaufmann & Gil-Aluja (1988), las cuales una vez identificadas pueden incluirse dentro de los modelos tradicionales para mejorar la eficiencia de los mismos.

Por lo tanto, afirman De los Cobos *et al.* (2010) y Álvarez (2014), que los modelos borrosos llegan a ser más versátiles que los modelos estadísticos convencionales, ya que integran relaciones funcionales cuando las variables no presentan valores únicos, sino que trabaja por medio de intervalos de confianza en posibilidad y no en probabilidad.

En la presente investigación se plantea que el uso de operadores de agregación de información heavy y la metodología de los efectos olvidados, permiten generar nuevos pronósticos sobre el tipo de cambio futuro, los cuales ayudaran a gestionar de una forma más eficiente el riesgo cambiario dentro de las organizaciones.

### 1.3 Formulación del problema de investigación

Los modelos basados en fundamentales económicos[3] para la determinación del tipo de cambio futuro, se han visto limitados, obteniendo resultados eficientes al largo plazo, pero ineficientes al corto plazo (Cheung *at al.,* 2005; Engel *et al.,* 2007). Al respecto, se identifica que no se contemplan todos los elementos del fenómeno, por lo que los resultados llegan a diferir ampliamente de lo estimado (Kaufmann & Gil-Aluja, (1986); Linares-Mustarós, *et al.* (2015)).

Ante la ineficiencia de los modelos tradicionales de pronóstico, es necesario explorar nuevas alternativas que incluyan variables representativas olvidadas, al igual que expectativas y conocimiento de los expertos del mercado financiero, planteándose como pregunta central de investigación:

¿Cuál es la efectividad de los modelos econométricos basados en fundamentales económicas para pronosticar la evolución futura del tipo de cambio, a través del desempeño esperado de las variables exógenas, apoyado de operadores de agregación heavy y la metodología de los Efectos Olvidados?

Adicionalmente se establecen las siguientes interrogantes complementarias:

[3] Los fundamentales económicos se refieren a las teóricas clásicas que indican que el tipo de cambio se encuentra relacionado con los movimientos en ciertas variables macroeconómicas como son la inflación, tasas de interés y balanza de pago (Groen, 2001; Engel, Mark & West, 2007; Rogoff & Stravrakeva, 2008; Molodtsova & Papell, 2008).

¿Cuál es la efectividad de pronóstico del tipo de cambio a través del desempeño de las variables exógenas mediante operadores *Heavy Ordered Weighted Moving Average* (HOWMA)?

¿Cuáles son las variables que permiten anticipar la volatilidad y delinear la trayectoria del tipo de cambio en el corto, mediano y largo plazo?

### 1.4. Justificación

El uso de fundamentales para pronosticar el tipo de cambio no siempre ha demostrado un resultado eficiente, usualmente generando mejores resultados con modelos basados en caminos aleatorios (Meese & Rogoff, 1983; Rossi, 2013). Dentro de los factores que debilitan los modelos que utilizan variables macroeconómicas se encuentra que los agentes económicos no reaccionan igual ante los cambios de los fundamentales, por lo que es necesario incluir sus expectativas dentro de los modelos de pronóstico (Bacchetta & van Wincoop, 2013; De Grauwe & De Grimaldi, 2005). A su vez, Bartolini & Giorgianni (2001), Engel & West (2005) y Balke *et al.* (2013) identifican que existen factores no observados como la prima de riesgo, cambios en la demanda monetaria y otros determinantes no fundamentales que afectan el tipo de cambio de forma considerable.

Ante esta situación, la generación de modelos que contemplen la información proporcionada por los expertos del mercado financiero y variables que no han sido observadas dentro de los modelos fundamentales, permitirán responder a los cambios rápidos del entorno, construyendo una metodología adaptable y flexible a las necesidades del entorno financiero (Gil-Lafuente, 2005; Chen, 2011; Dick *et al.,* 2015).

## 1.5 Objetivos e hipótesis

*Objetivo general*

Identificar la efectividad de los modelos econométricos basados en fundamentales económicos para pronosticar la evolución futura del tipo de cambio, a través del desempeño esperado de las variables exógenas, apoyado de operadores de agregación heavy y la metodología de los Efectos Olvidados

*Objetivo especifico*

Analizar la efectividad de pronóstico de los instrumentos de la matemática de la incertidumbre a través de operadores de agregación de información heavy.

Determinar las variables que permitan anticipar la volatilidad y delinear la trayectoria del tipo de cambio futuro.

*Hipótesis*

Bajo las condiciones inciertas y mutables de los mercados globalizados, afirma González (2004) es necesario encontrar modelos que permitan mejorar la toma de decisiones de los empresarios, Kauffman & Gil Aluja (1986) señalan que el uso de la lógica borrosa y la metodología de los efectos olvidados permite encontrar herramientas adecuadas bajo estos escenarios de incertidumbre.

Los modelos macroeconómicos para determinar el tipo de cambio, indican Evans & Lyons (2008) tienen un pobre rendimiento para pronosticar al corto plazo, por lo que es necesario encontrar las relaciones de causalidad que se obviaron dentro de las variables iniciales (Kaufmann & Gil Aluja, 1988), lo que perfeccionará al modelo y permitirá generar mejores resultados.

De tal forma, la inclusión de la opinión de los expertos, mediante el uso de operadores de agregación de información, como es el operador OWA (Yager, 1988) y la metodología de los efectos olvidados (Kaufmann & Gil-Aluja, 1988) permiten generar nuevos modelos adecuados a la incertidumbre de los mercados, lo que permitirá generar nuevas estrategias para tomar decisiones. Ante esto, el uso de dichas técnicas dentro de modelos econométricos para pronosticar el futuro del tipo de cambio permite plantear la siguiente hipótesis:

«La inclusión de la metodología de los efectos olvidados y operadores de agregación heavy dentro de los modelos econométricos basados en fundamentales económicos disminuye de forma significativa el error de pronóstico, en contraste de los modelos tradicionales que pronostican el valor esperado de sus variables exógenas a través de la técnica de series de tiempo».

## 1.7 Estructura y contenido

La presente tesis doctoral se encuentra estructurada en seis capítulos, los cuales desarrollan de forma complementaria el tema objeto de investigación. A continuación, se describen brevemente cada uno de ellos.

En el capítulo 1, se lleva a cabo la Introducción, en la cual se encuentra el planteamiento del problema y se justifica la elección del tema. Adicionalmente se fijan los objetivos e hipótesis.

En el capítulo 2, se estudia el tipo de cambio. Riesgo y modelos fundamentales, en el cual se repasan los conceptos existentes sobre el tema. Para esto primero se presenta la gestión del riesgo cambiario. Posteriormente, se presenta la evolución

del tipo de cambio en México y los modelos fundamentales de basados en variables macroeconómicas.

En el capítulo 3, se presentan la lógica borrosa y los operadores de agregación de información. Aquí se desarrolla una reseña de las generalidades de la lógica borrosa y se presentan los operadores de agregación de información, específicamente el operador *OWA* y sus extensiones.

En el capítulo 4, se plantea la metodología utilizada en la presente investigación, la cual utiliza diversas técnicas como modelos econométricos, series de tiempo y operadores de agregación de información. Específicamente, la presente investigación plantea el uso del experton, la metodología de los efectos olvidados y los operadores *Heavy Ordered Weighted Moving Average* (HOWMA) como técnicas auxiliares al pronóstico del tipo de cambio tradicional.

En el capítulo 5, se muestran los resultados que se obtuvieron de utilizar los modelos de agregación de información y la metodología de los efectos olvidados para determinar las variables exógenas que integraron cada uno de los modelos econométricos basados en fundamentales econométricos.

En el capítulo 6, se presentan las conclusiones alcanzadas tras el desarrollo de los capítulos anteriores y también se muestran las futuras líneas de investigación que son fruto del trabajo realizado.

Finalmente, se muestran las referencias consultada para la realización de la presente tesis doctoral y los apéndices de la misma.

De forma esquemática, el presente trabajo de investigación se estructura de la siguiente forma

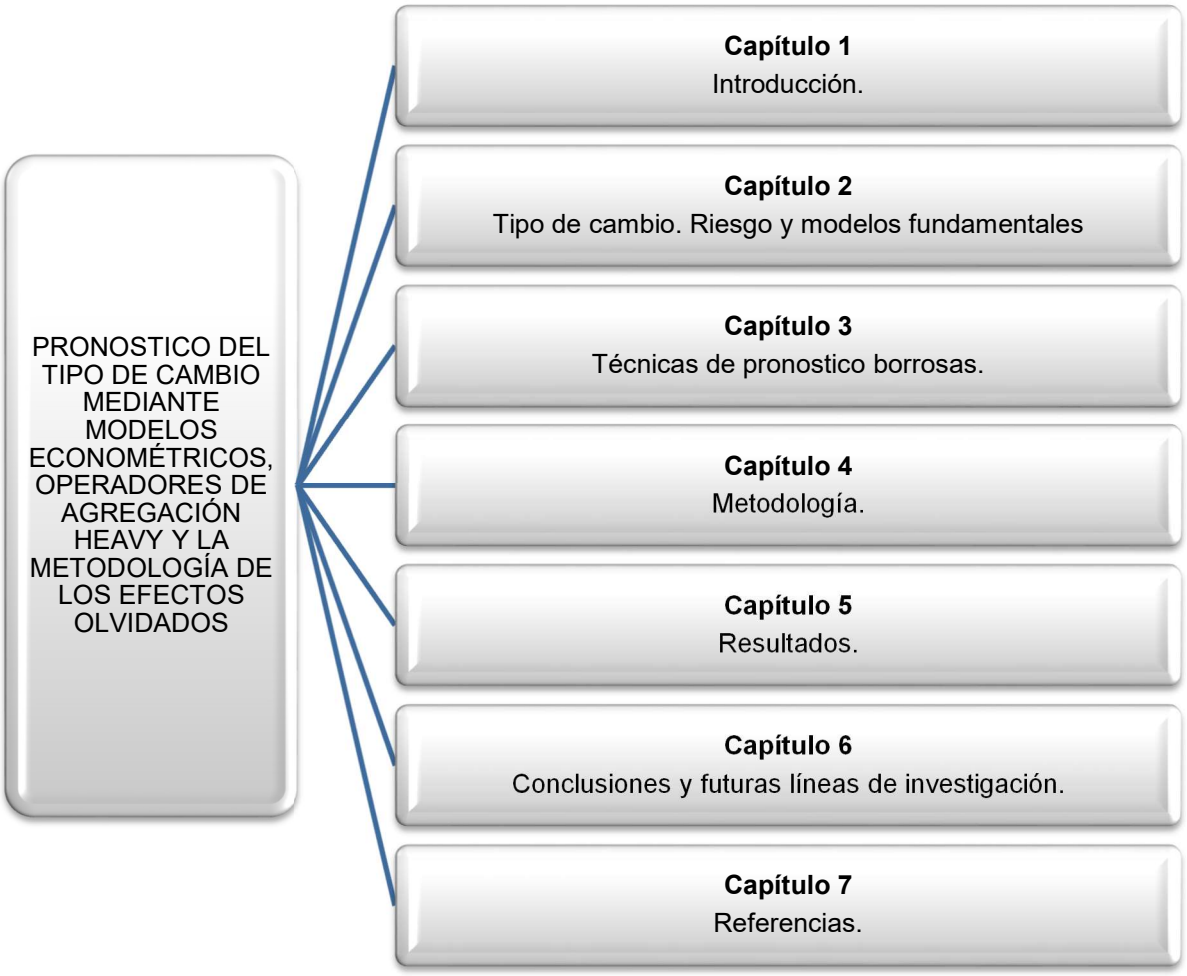



**Figura 3. Esquema de estructura del trabajo de investigación**

Fuente: Elaboración propia

# CAPÍTULO II. Tipo de cambio. Riesgo y modelos fundamentales

## 2.1 La gestión del riesgo cambiario

Los resultados de la crisis en México de 1994, expresan Ramos & Torres (2005) forzaron a abandonar el régimen de flotación predeterminado y adoptar un régimen de libre flotación de peso ante las diversas divisas del mundo. Los responsables de las oscilaciones del precio de las monedas afirman Popov & Stutzmann (2003) se deben a movimientos en la oferta y demanda de los mercados financieros, regímenes cambiarios y diversos factores externos.

Los riesgos no son siempre explícitos, visibles o percibidos de manera inmediata, existen algunos que no se identifican o se encuentran ocultos, los cuales no son otra cosa que efectos de efectos, de una acumulación de causas. Al respecto Gil-Lafuente (2008) afirma que los eventos y hechos que nos rodean son parte de un sistema o subsistema, por lo que existen incidencias que permanecen ocultas que evitan la visualización del problema.

Las innovaciones de los mercados financieros y la volatilidad del precio de los insumos y productos finales, lleva a los decisores a desarrollar estrategias que permitan reducir los riesgos dentro del mercado. El riesgo cambiario se asocia con la depreciación o apreciación no anticipada de la paridad, la cual de acuerdo a Bernanke & Frank (2007) se encuentra determinada por las leyes de mercado, lo que significa que su cambio es inesperado, indeterminado y volátil.

Ante las fluctuaciones imprevistas del tipo de cambio, las organizaciones ven modificados sus flujos de efectivo esperados por lo que deben administrar el riesgo, el cual se define de acuerdo a Griffin & Ebert (2005) como el poder de conservar los ingresos y activos de una empresa al reducir la amenaza de pérdidas debido a eventos incontrolables.

Las definiciones de exposición al riesgo cambiario son similares, Flood & Lessard (1986), Priestley & Odeggard (2002) y Rupeika-Apoga (2005) concuerdan que es la medida en que los flujos futuros de la compañía se vean alterados por la fluctuación no anticipada del tipo de cambio

Dentro de los tipos de exposición al riesgo cambiario, identifican Dhanani (2004), Díez de Castro & Mascareñas (1994) y Peyrard (1986) las siguientes:

A) Riesgo de translación. Es un riesgo contable; representa las ganancias o pérdidas adquiridas por la reestructuración de los estados financieros de las empresas foráneas subsidiarias en la moneda local para realizar los estados consolidados.

B) Riesgo de transacción. Es el riesgo de los flujos de efectivo, el cual se materializa cuando la compañía busca convertir los flujos de efectivo en moneda extranjera en la moneda local.

C) Riesgo operacional o económica. Es el riesgo de los movimientos a largo plazo de los flujos de efectivo, de tal manera que el tipo de cambio aquí puede tener efectos profundos en los flujos de efectivo de las organizaciones debido a que estos todavía no existen.

La exposición del riesgo cambiario, expresan Díez de Castro & Mascareñas (1998) se encuentra vinculada con las operaciones cotidianas de la empresa, dentro de las cuales destacan las importaciones, exportaciones, inversiones y prestamos en moneda extranjera. En este sentido, se intensifica la importancia de la

administración del riesgo cambiario, ya que se encuentra asociada a las ventas, costos y rendimientos de la organización.

En México desde 1954 la determinación del tipo de cambio ha pasado por distintos regímenes, comenzando por un régimen fijo hasta llegar al tipo de cambio de libre flotación (Ver tabla 2), lo que ha generado una creciente incertidumbre acerca del valor futuro de las paridades. Ante esto, señalan Popov y Stuzmann (2003) la gestión de la exposición del riesgo cambiario necesita una visión clara del fenómeno.

Tabla 1. Regímenes cambiarios en México desde 1954

| Fecha | Régimen | Tipo de cambio | Cotizaciones | |
|---|---|---|---|---|
| | | | Inicio | Fin |
| 19 de abril de 1954 - 31 de agosto de 1976 | Paridad Fija | Fijo | $ 12.50 | $ 12.50 |
| 1 de septiembre de 1976 - 5 de agosto de 1982 | Sistema de flotación controlada | Operaciones en billetes/documentos | $ 20.50 | $ 48.79 |
| 6 de agosto de 1982 - 31 de agosto de 1982 | Sistema cambiario múltiple | General | $ 75.33 | $ 104.00 |
| | | Preferencial | $ 49.13 | $ 49.81 |
| | | Mex-dólar | $ 69.50 | $ 69.50 |
| 1 de septiembre de 1982 19 de diciembre de 1982 | Control generalizado de cambios | Preferencial | $ 50.00 | $ 70.00 |
| | | Ordinario | $ 70.00 | $ 70.00 |
| 20 de diciembre de 1982 - 4 de agosto de 1985 | Control de cambios | Controlado | $ 95.05 | $ 281.34 |
| | | Especial | $ 70.00 | $ 281.51 |
| | | Libre | $ 149.25 | $ 344.50 |
| 5 de agosto de 1985 - 10 de noviembre de 1991 | Flotación regulada | Controlado | $ 282.30 | $ 3,073.00 |
| | | Libre | $ 344.50 | $ 3,068.90 |
| 11 de noviembre de 1991 - 21 de diciembre de 1994 | Bandas cambiarias con desliz controlado | FIX | $ 3,074.03 | N $ 3.9970 |
| 22 de diciembre de 1994 – presente | Libre flotación | FIX | N $ 4.8875 | - |

Fuente. Banco de México (2009)

Frente a la incertidumbre de la paridad del tipo de cambio y la preocupación por prever y compensar esos movimientos existen teorías que pretenden proporcionar una base para posibles predicciones, así como explicar la razón por la cual diferentes divisas tienen diferentes precios. Díez de Castro & Mascareñas (1998) explican que la principal razón de este fenómeno son las variaciones entre la oferta y demanda de las divisas, dentro de las causas se encuentran el comercio internacional de bienes, la inversión, la especulación y el arbitraje.

## 2.2 Modelos fundamentales de pronóstico del tipo de cambio

### *2.2.1 Teoría de la paridad del poder de compra*

La teoría de la paridad del poder de compra (PPP por sus siglas en inglés *purchasing power parity),* asienta Dornbusch (1985), se basa en que el tipo de cambio entre dos monedas es determinado por el cambio del nivel de precios de los dos países. De tal modo que el tipo de cambio se ajusta a los diferenciales de la inflación, a lo que argumenta Solnik (1993) y Taylor (2004) que si los precios de los bienes de un país se incrementan en comparación de otro, el tipo de cambio del país deberá depreciarse para mantener un precio real similar en los dos países

En este sentido la fórmula de la paridad de poder de compra para el caso del tipo de cambio USD/MXN es:

$$\frac{S^1}{S^o} = \frac{1+I_F}{1+I_D} \tag{2}$$

donde: $S^0$ es el tipo de cambio de contado al inicio del periodo, $S^1$ es el tipo de cambio de contado al final del periodo, $I_F$ es la tasa de inflación del periodo en Estados Unidos y $I_D$ es la tasa de inflación del periodo en México

*2.2.2 Teoría de la paridad de las tasas de interés*

La teoría de la paridad de las tasas de interés, señalan Solnik (1993) y Avilés (2014) que, en mercados de dinero libre, el diferencial de las tasas de interés debe ser igual al descuento o prima a futuro, por lo que existe paridad si la diferencia entre las tasas de interés compensa la prima a futuro de la moneda más fuerte.

Por lo que la relación matemática para el tipo de cambio USD/MXN seria:

$$\frac{F}{S^O} = \frac{1+r_F}{1+r_D} \tag{3}$$

En donde: F= Tipo de cambio futuro, $S^0$=Tipo de cambio de contado, $r_F$= Tasa de interés en Estados Unidos y $r_D$= Tasa de interés en México

*2.2.3 Enfoque monetario del tipo de cambio*

El desarrollo de los modelos monetarios para la determinación del tipo de cambio de acuerdo a Wilson (2009) se ven influenciados por tres teorías: la teoría de la cantidad del dinero, el enfoque de equilibrio de Cambridge y la teoría monetaria Keynesiana.

En la teoría de la cantidad del dinero, Fisher (1911) establece una ecuación de cambio que señala

$$MV = PT \tag{4}$$

En donde M es el dinero, V es la velocidad, P es el nivel de precios y T se refiere al número total de transacciones usando dinero. Por lo tanto, explica que el dinero y la velocidad en que se ve reflejado en la economía es igual al nivel de precios por el número total de transacciones.

El enfoque de equilibrio de Cambridge, es una extensión del modelo de Fisher, el cual de acuerdo a Humphrey (2004), integra la demanda de depósitos, es decir la cantidad de dinero necesario para cubrir las necesidades presentes y futuras se

encuentra en dependencia de los ingresos y las tasas de interés. Por lo que la formula queda expresada

$$M/P = kY \tag{5}$$

En donde M es el dinero, P es el nivel de precios, Y es el ingreso real nacional y k es una constante relacionada con el deseo de mantener el dinero.

Dentro de la teoría monetaria de Keynes, comenta Bertocco (2006) existe una relación entre la demanda del dinero y las tasas de interés, en el sentido de que, a mayor tasa de rendimiento de activos de renta fija, más probable que se invierta dinero.

Con las bases anteriores se encuentra que la oferta de dinero debería ser igual que la demanda de dinero, por lo que:

$$M_d = M_s \tag{6}$$

El modelo de Cambridge sugiere que la demanda real del dinero está relacionada con el ingreso real, por lo que retomando su formula

$$M_s = kPY \tag{7}$$

En donde $M_s$ corresponde a la oferta de dinero, k es una constante, P es el nivel de precios e Y es el ingreso real nacional.

Si se considera la ley del precio único, que de acuerdo a Haskel & Holger (2001) es que los precios de bienes idénticos deben ser iguales en dos o más mercados, por lo que

$$E = \frac{P}{P_F} \tag{8}$$

En donde E es el tipo de cambio nominal, $P_D$ son los precios domésticos y $P_F$ son los precios en el extranjero. Dentro de la ecuación encontramos que un cambio en el nivel de precios domésticos requiere un cambio proporcional en el precio

extranjero para mantener el tipo de cambio estable, de lo contrario el tipo de cambio absorberá estas diferencias.

Similar a la fórmula para la determinación de la oferta del dinero doméstico, se considera

$$M_{SF} = k_F P_F Y_F \tag{9}$$

En donde $M_{SF}$ es la oferta de dinero en el extranjero, $k_F$ es la constante en el extranjero, $P_F$ es el nivel de precios en el extranjero y $Y_F$ es el ingreso real en el extranjero, por lo que si sustituimos la ecuación (7) y (8) en la ecuación (9)

$$E = \frac{M_S k_F Y_F}{M_{SF} kY} \tag{10}$$

Utilizando logaritmo dentro de la ecuación (10) encontramos que

$$e = (m - m_F) - (y - y_F) - (logk - logk_F) \tag{11}$$

En donde *e* es el log del tipo de cambio, *m* es el log de la oferta de dinero doméstica, $m_F$ es el log de la oferta de dinero extranjero, *y* es el log del ingreso real, $y_F$ es el log del ingreso real extranjero.

Retomando la propuesta de Keynes, la cual señala que k no es una constante, sino que es la tasa de interés, la cual afecta negativamente la oferta de dinero y positivamente con el ingreso, por lo que la demanda para la balanza es

$$m - p = a + by - ci \tag{12}$$

$$m_F - p_F = a + by_F - ci_F \tag{13}$$

En donde *i* es la tasa de interés nominal; *a,b* y *c* son constantes. Por lo que si se utiliza el logaritmo de la ley del precio único

$$e = p - p_F \tag{14}$$

Por lo que utilizando la formula (12), (13) y (14) se obtiene

$$e = (m - m_F) - b(y - y_F) + c(i - i_F) \tag{15}$$

En este sentido la teoría monetaria propone que el tipo de cambio es un fenómeno que se ve afectado por la oferta del dinero, nivel de ingreso y tasas de interés.

El modelo de los precios flexibles (Frenkel, 1976) incorpora los elementos de la teoría monetaria del dinero al tipo de cambio, considerando que los precios se ajustan inmediatamente al mercado de dinero, por lo que se asume que los capitales domésticos y extranjeros son perfectamente sustitutos. Si consideramos que los coeficientes para la demanda de dinero doméstica y extranjera son diferentes, encontraríamos que

$$m_S - p = \alpha_0 + \alpha_1 y - \alpha_2 i \tag{16}$$

$$m_{SF} - p_F = \beta_0 + \beta_1 y_F - \beta_2 i_F \tag{17}$$

En donde $\alpha$ y $\beta$ son coeficientes constantes. Por lo que utilizando la ecuación (13), (15) y (16) se obtiene

$$e_t = \gamma_0 + \gamma_1 m_t + \gamma_2 m_{Ft} + \gamma_3 y_t + \gamma_4 y_{Ft} + \gamma_5 i_t + \gamma_6 i_{Ft} + u_t \tag{18}$$

En donde $\gamma$ son coeficientes constantes y u es el termino de error. Algunos efectos dentro de las variables que afectan el tipo de cambio se encuentran:

a) El incremento de la oferta monetaria nacional, generaría un exceso de oferta por lo que los precios nacionales aumentarían en proporción y por lo tanto se depreciaría el tipo de cambio en dicha proporción.

b) Un aumento en el ingreso real nacional crearía un exceso de demanda, siempre que la oferta esta fija, se produciría una disminución de los precios nacionales en la misma proporción y por lo tanto llevaría a una apreciación del tipo de cambio.

c) Un aumento de las tasas de interés nominal reflejarían expectativas de inflación, por lo que reducirían las demandas de saldos reales, aumentarían los precios y depreciarían la moneda.

d) Tomando en cuenta las variaciones extranjeras se encuentran efectos similares, en donde un incremento de la oferta monetaria extranjera, disminución del ingreso

real extranjero y aumento en la tasa de interés nominal extranjera generaría una apreciación del tipo de cambio.

*2.2.4 Teoría del tipo de cambio basada en la balanza de pagos*

Dornbush (1979) plantea que el tipo de cambio se ajusta a al equilibrio de las entradas y salidas provenientes de las transacciones internacionales de bienes, servicios y activos, de tal forma que la cuenta corriente se ve afecta por el tipo de cambio debido al cambio relativo del precio y por lo tanto la competitividad, la cuenta capital por otra parte se ve afectada por las expectativas de los inversionistas y las tasas de interés. La fórmula que explica este fenómeno es

$$BoP = 0 = C\left(\frac{EP_F}{P}, Y, Y_F\right) + K(i, i_F, s) \tag{19}$$

En donde BoP es la balanza de pagos, C es la cuenta corriente, E es el tipo de cambio, P es el nivel de precio doméstico, $P_F$ es el nivel de precios en el extranjero, Y es el ingreso real domestico, $Y_F$ es el ingreso real extranjero, K es la tasa de entrada de capital, i es la tasa de interés doméstico, $i_F$ es la tasa de interés en el extranjero y s es la especulación.

Dentro de los efectos que generarían cambios dentro de las variables en el tipo de cambio, se encuentran

a) Un incremento en las tasas de interés domestica genera una apreciación debido a que esto llevaría a un incremento en las entradas de flujo de capital o una reducción de la salida de los flujos de capital, por lo que la balanza de pagos se encontraría en superávit y por lo tanto el tipo de cambio deberá apreciarse para volver al equilibrio.

b) Una disminución en el nivel de precios en el extranjero generaría un desequilibrio de la cuenta corriente, lo que generaría un incremento en las importaciones o disminución de exportaciones y por lo tanto sería necesario que el tipo de cambio se apreciara para volver al equilibrio de la balanza de pagos.

### *2.2.5 Limitaciones de los modelos tradicionales de pronóstico del tipo de cambio*

Los modelos de tipo de cambio han presentado limitaciones para pronosticar de forma eficiente. En cuestión de la Teoría del Poder de Compra señala Haidar (2011) que dentro de las principales razones por la falta de eficiencia de este modelo son

a) La canasta básica de bienes y servicios puede ser diferente entre los países comparados

b) Las políticas de proteccionismo puedes ser aplicadas, como son impuestos que prevengan el libre comercio de bienes y servicios

c) Las economías de forma general no pueden ser comparadas en cuestión de su nivel de ingreso, diferencia entre precios de los sectores y la elasticidad de precios de los bienes.

En cuestión a la teoría de la paridad en las tasas de interés Kohler (2008) indica que dentro de las principales razones de variación son

a) La conducta irracional de los inversionistas

b) Los datos son inapropiados para el calculo

c) Cambios en los regímenes de tipo de cambio y la lenta adaptación de los inversionistas ante estos cambios

d) La prima de riesgo que tiene como función compensar el riesgo adquirido

El enfoque monetario del tipo de cambio analizado a través del modelo de precios flexibles de Frankel tiene como limitaciones que no identifica la influencia de la política fiscal, a su vez se basa en las suposiciones de la paridad del poder de compra y paridad de tasas de interés que de acuerdo a Yong & Ling (2000) no cuenta con la suficiente evidencia empírica para ser considerados fuertes

predictores, adicionalmente este modelo supone que el tipo de cambio se adapta en el instante que se generan los cambio de las variables macroeconómicas.

# CAPÍTULO III. Lógica borrosa y operadores de agregación de información

## 3.1 Generalidades de la lógica borrosa

La idea de la lógica borrosa tiene sus antecedentes en el principio del tercio excluso que fue desarrollado por Aristóteles (382-322 a.C.) el cual establece que una proposición es verdadera o falsa, pero no ambas a la vez. Al tratamiento de la lógica planteada por Aristóteles se incorporan las ideas de Boole (1916) acerca de la lógica binaria. Adicionalmente, Lukasiewicz (1910) propone el principio de valencia; de tal modo que se les asignaron valores a las variables en tres sentidos: verdadero (1), falso (0) e indeterminado (0.5).

Einstein señalaba que en la medida en que las leyes de las matemáticas se refieren a la realidad, no son ciertas y en medidas en que son ciertas, no se refieren a la realidad. Dentro del pensamiento, señala Gil Lafuente (2001) que no todo es binario (0,1) sino que dentro del pensamiento existen matices, lo que resulta en una lógica con frecuencia imprecisa, vaga y borrosa, lo que lleva a trasladarse de la lógica binaria a la lógica multivalente.

La lógica multivalente, comenta Klimova (2009) sirve como antecedente para la lógica borrosa y la metodología de los efectos olvidados, se basa en la idea que existe un espacio entre la verdad completa y la falsedad completa, caracterizada por una función de pertenencia para la descripción del fenómeno. La lógica borrosa, indica Barcellos de Paula (2010) incorpora a la lógica clásica una serie de posibilidades, caracterizados por la imprecisión, debido a que el mundo está basado en una escala de grises.

La teoría de los subconjuntos borrosos[4] constituye una teoría matemática dentro de la lógica multivalente. El origen se encuentra dentro de los trabajos de Zadeh (1965)

[4] Los tipos y operaciones básicas que son aplicables a los conjuntos borrosos se encuentran en el Apéndice 1.

en donde lo define como: un conjunto borroso (clase) A en X se encuentra caracterizado por una función de pertenencia $fA\ (x)$, la cual se asocia a cada punto de X en un intervalo de números $[0,1]$ con el valor de $fA\ (x)$ en *x* representa el grado de pertenencia de *x* en A. Si el conjunto es binario, la función de pertenencia solo puede tomar valores de 0 y 1, lo que indica si pertenece o no a A.

De acuerdo a Kaufmann & Aluja (1986) los subconjuntos borrosos mejoran el reflejo de la realidad, de forma que la función característica de pertenencia $fA\ (x)$ pueda asumir no solo valores binarios $[0,1]$, sino una gama de opciones que se adecuen a las realidades de percepción de los objetos de los seres humanos, por lo que dicha función debe ser flexible.

La teoría de los subconjuntos borrosos, señalan Kaufmann & Gil Aluja (1986) es una parte de las matemáticas que se adapta a la subjetividad como de lo incierto. El nuevo tratamiento de la incertidumbre a través de esta teoría da lugar a una nueva forma de pensar que reúne el rigor del razonamiento y la imaginación inherente a la borrosidad.

Dentro de los conjuntos booleanos se asigna un valor 0 o 1 para indicar la pertenencia a dicho conjunto, sin embargo, Liern (2014) indica que los valores que pueden adoptarse deben ser dentro de un rango de $[0,1]$, lo que permitirá apreciar el grado de pertinencia de los elementos.

Gil Lafuente (1997) señala que un conjunto referencias es aquel que se presenta por la letra mayúscula *E*, el cual reúne todos los elementos que constituirán la base o referencia de las operaciones posteriores, de tal modo que el conjunto es toda colección de objetos bien definidos y distintos entre sí, representándose de la siguiente forma:

$$E = \{a, b, c, d, e, f\}$$

En donde los elementos a,b,c,d,e,f forman parte del conjunto referencial *E*.

En los conjuntos clásicos es frecuente que el conjunto referencial se presente de la siguiente forma:

$$E = \frac{a\ b\ c\ d\ e\ f}{1\ 1\ 1\ 1\ 1\ 1}$$

Donde los elementos que poseen la propiedad se les asigna un valor de 1 (por lo que pertenecen al conjunto referencial) y cuando no la poseen se asigna un 0. Partiendo del conjunto referencial y de una determinada propiedad, se puede generar un subconjunto de referencia representado de la siguiente manera:

$$E = \{a, b, e\}$$

Y por lo tanto

$$E = \frac{a\ b\ c\ d\ e\ f}{1\ 1\ 0\ 0\ 1\ 0}$$

Indicando con 1 si cumple o posee la característica y 0 en caso contrario. Por lo que el subconjunto *A* de *E* se representa como conjunto de pares ordenados, tantos como los elementos de *E*, donde el primer componente es un elemento en *E* y el segundo es un valor de la función característica. Representándose:

$$A \subset E \ \longrightarrow A = \{x; \mu_A(x)\} \forall x \in A$$

$$\mu_A(x) \rightarrow \{\begin{matrix} 1\ si\ x \in A \\ 0\ si\ x \notin A \end{matrix}$$

De tal modo que dentro de los subconjuntos ordinarios la pertenencia de un elemento al subconjunto es de estar o no estar, sin embargo, dentro de los subconjuntos borrosos se plantea que la función característica de pertenencia $\mu_A(x)$ pueda tomar valores dentro del segmento [*0,1*].

$$\mu_A : E \rightarrow [0,1]$$

$$x \rightarrow \mu_A(x)$$

Asignándose un valor cercano a la unidad cuando el elemento se considere que posee la característica, por lo que subconjunto borroso se representa de la siguiente forma:

$$A = \frac{a \quad b \quad c \quad d \quad e}{0.2 \;\; 0.5 \;\; 0.3 \;\; 0.6 \;\; 0.1}$$

$$A = [(x; \mu_A(x))] \quad \forall\, x \in E$$

En donde $\mu_A(x)$ es el grado de pertenencia de *x* a *A* y [*0,1*] es el espacio de pertenencia.

## 3.2 Operadores de agregación de información

### *3.2.1 El operador Ordered Weighted Average (OWA)*

El operador OWA fue introducido por Yager (1988). Proporciona una familia parametrizada de operadores de agregación que se han utilizada en muchas aplicaciones (Emrouznejad and Marra, 2014; Yager & Kacprzyk, 1997; Yager *et al*., 2011). A continuación, se da la definición del operador OWA introducida por Yager (1988).

Un operador OWA de dimensión *n* es una aplicación $F: R^n \rightarrow R$ con un vector de asociación de peso $w = [w_1, w_2, \dots, w_n]^T$así que $w_j \in [0,1]$, $1 \leq i \leq n$ y

$$\sum_{i=1}^{n} w_{i=} w_1 + w_2 + \cdots + w_n = 1, \tag{20}$$

donde

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j\, b_j, \tag{21}$$

siendo $b_j$ el *j*th es el elemento más grande de la colección $a_1, a_2, \dots, a_n$ y $a_1, a_2, \dots, a_n$ son los argumentos que integran la formulación.

Obsérvese que se distingue la diferencia entre el operador OWA descendiente (DOWA) y el operador ascendiente OWA (AOWA). Los pesos de estos operadores está relacionado por $w_j = w^*_{n-j+1}$, donde $w_j$ es el *j*th de peso del operador DOWA y $w^*_{n-j+1}$ el *j*th peso del operador AOWA.

Las decisiones dentro de los operadores OWA se pueden dar bajo cuatro criterios

1. Criterio optimista. Se basa en el supuesto de que se presentara el estado más favorable, de tal modo que se debe seleccionar el resultado más favorable de cada alternativa y de los resultados obtenidos seleccionar el más favorable de todos. De tal modo que este criterio se basa en una máxima que se formula

$$Decisión = Max\{E_i\} = Max[Max\{Max\{a_j\}\}]$$

2. Criterio pesimista o de Wald. Sostiene que el decisor debe seleccionar la alternativa que proporcione mayor nivel de seguridad, de tal modo que nuestra decisión debe ser el resultado más favorable de entre los más desfavorables para cada alternativa. A este método se le conoce como max min y su fórmula es

$$Decisión = Max\{E_i\} = Max[Min\{a_j\}]$$

3. Criterio de Hurwics. Consiste en ponderar con un coeficiente optimista y otro pesimista al mejor y peor caso respectivamente, posteriormente se suman los valores y se escoge aquella alternativa que propone un mayor resultado. La fórmula de este criterio es

$$Decisión = Max\{E_j\} = Max[\alpha Max\{a_j\} + (1-\alpha)Min\{a_j\}]$$

Donde $\alpha + (1-\alpha) = 1$.

4. Criterio de Laplace. Se basa en el principio de razón insuficiente, de tal modo que se asocia un mismo grado de probabilidad a los distintos escenarios, siempre que no se presenten indicios de lo contrario. La fórmula es

$$Decisión = Max\{E_j\} = Max[(^1/_n)\sum_{j=1}^{n} a_j]$$

*3.2.2 El operador Heavy OWA (HOWA)*

El operador Heavy OWA (HOWA) introducido por Yager (2002) es una extensión del operador OWA. La motivación para el uso de este operador es porque hay situaciones en las que la información disponible no puede ser agregada mediante el uso de las medias habituales delimitadas por el máximo y el mínimo. En este caso, la diferencia con el operador OWA es que la suma de los pesos se le permite estar entre 1 y n, en lugar de limitarse a 1. Con esto, obtenemos una amplia clase de operadores de agregación que incluyen los operadores medios y operador total. A continuación, se proporciona una definición del operador HOWA sugerido por Yager (2002).

Un operador de agregación heavy, es una extensión al operador OWA que permite al vector de peso ir hasta *n.* Por lo que un operador HOWA es una aplicación $R^n \rightarrow R$ que están asociados a un vector de peso *w* que $w_j \in [0,1]$ y $1 \leq \sum_{j=1}^{n} w_j \leq n$, por lo que:

$$HOWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j\, b_j, \qquad (22)$$

donde $b_j$ el *j*th elemento más grande de la colección $a_1 a_2, \dots, a_n$.

El operador HOWA es monótono y conmutativo. Obsérvese que el operador HOWA no está limitada por el mínimo y el máximo. En este caso, está limitada por el mínimo

y el operador total, que representa la suma de todos los argumentos. Podemos distinguir entre el operador descendente HOWA (DHOWA) y el operador ascendente HOWA (AHOWA).

Obsérvese que es posible expandir el peso utilizando un vector cuyo rango vaya de $-\infty$ a $\infty$. Por lo que el vector *w* se convierte en $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$. Permitiendo al vector de peso ir de $-\infty$ a $\infty$, podemos drásticamente sub o sobre estimar el resultado de la media móvil de acuerdo con el vector de ponderación utilizado.

Cuando se analiza la magnitud del vector $|W|$ para el operador HOWA, Yager (2002) denota que el peso no depende del proceso de reorden de los argumentos e introduce un parámetro característico llamada el valor beta de vector *W.* Este se define como $\beta(W) = (|W| - 1)/(n - 1)$. Ya que $|W| \in [1, n]$, entonces $\beta \in [0,1]$. Por lo que si $\beta = 1$, se obtiene el operador total y si $\beta = 0$, se obtiene el operador OWA.

También se nota que si $w_j = 1/n$ para todas las i, se obtiene el operador heavy weighted average (HWA), el cual se define a continuación.

Un operador HWA es una aplicación $R^n \rightarrow R$ que tiene asociado un peso *H*, por lo que:

$$HWA\ (a_1, a_2, \dots, a_n) = \left(\sum_{j=1}^{n} a_j\right) H \tag{23}$$

Obsérvese que es posible expandir el peso *H* de $-\infty$ a $\infty$.

*3.2.3 Operadores de agregación móviles*

Una media móvil (MA por sus siglas en ingle) es un promedio usual que se mueve sobre una parte de un total. Generalmente, la media móvil se puede ver como un

operador de agregación móvil. Este método es conocido por resolver problemas de series de tiempo y ha sido aplicado extensivamente en economía y estadística (Evans, 2002). La principal ventaja en este contexto es la posibilidad de pronosticar resultados futuros basados en datos históricos. Las medias móviles, de acuerdo a Kenney y Keeping (1962), se definen como.

Una media móvil se definen como una secuencia $\{a_i\}_{i=1}^{N}$, donde la media móvil $n$ es una nueva secuencia $\{s_i\}_{i=1}^{N-n+1}$ definida desde $a_i$ tomando el promedio aritmético de una secuencia de $n$ términos, por lo que.

$$s_i = \frac{1}{n} \sum_{j=i}^{i+n-1} a_j, \tag{24}$$

La media movil usual puede extenderse utilizando medias ponderadas, obteniéndose las *weighted moving average* (WMA). Estas se definen a continuación.

Un operador *weighted moving average* (WMA) de dimensión *m,* de acuerdo a Merigó & Yager (2013) es una aplicación $WMA: R^m \rightarrow R$ que tiene un vector de peso asociado *W* de dimensión *m* con $W = \sum_{i=1+t}^{m+t} w_i = 1$ y $w_i \in [0,1]$, por lo que:

$$WMA\,(a_{1+t}, a_{2+t}, \dots, a_{m+t}) = \sum_{i=1+t}^{m+t} w_i a_i, \tag{25}$$

donde $a_i$ es el argumento *i*th, *m* es el número de argumentos considerado de la muestra total y t indica los movimientos hechos en promedio desde el análisis inicial. Nótese que si $w_i = 1/m$ para todas las i, el WMA se convierte en MA.

Otra extensión de las medias móviles se puede obtener si se combina con el operador OWA, generándose el *ordered weighted moving average* (OWMA), Merigó & Yager (2013) la definen como.

Un operador ordered weighted moving average (OWMA) de dimensión *m* es una aplicación $OWMA: R^m \rightarrow R$ que tiene asociado un vector de peso *W* de dimensión *m* con $W = \sum_{j=1+t}^{m+t} w_j = 1$ y $w_j \in [0,1]$, por lo que:

$$OWMA\,(a_{1+t}, a_{2+t}, \dots, a_{m+t}) = \sum_{j=1+t}^{m+t} w_j b_j, \tag{26}$$

donde $b_j$ es el *j*th argumento más grande de $a_i$, *m* es el total de argumentos considerados de la muestra total y *t* indica el número de movimientos hechos en el promedio desde el análisis inicial.

# CAPÍTULO IV. Metodología

*4.1 Esquema metodológico*

Con la finalidad de presentar de forma simplificada lo que se realizara en la presente investigación, la cual tiene como objetivo identificar la efectividad de los modelos econométricos basados en fundamentales económicos para pronosticar la evolución futura del tipo de cambio, a través del desempeño esperado de las variables exógenas, apoyado de operadores de agregación heavy y la metodología de los Efectos Olvidados, se presenta el cuadro esquemático en la Figura 4.

**Figura 4. Esquema metodológico**
Elaboración propia

El proceso en el que se trabajara el esquema anterior es el siguiente:

1. Para la determinación de las variables que afectan al tipo de cambio, se utilizan aquellas que se encuentran dentro de los modelos tradicionales basados en fundamentales económicos, los cuales son: a) Paridad de Poder y Compra (PPP), b) Paridad en Tasas de Interés (PTI) y c) Balanza de Pago (BoP).

Adicionalmente se entrevistaron a 5 expertos para determinar mediante el uso de expertones (Kaufmann, 1988) y la metodología de los efectos olvidados (Kaufmann y Gil-Aluja, 1988) las variables ocultas que impactan el tipo de cambio y que no se encuentran consideradas en los modelos fundamentales. Ambas técnicas se definen en el punto 4.2 y 4.3 del presente apartado respectivamente.

2. Se elaboró una base de datos históricas de 1994 al 2015, de las variables de los fundamentales económicos y de los efectos olvidados encontrados.

3. Mediante el uso del software STATA se formulan 9 modelos econométricos diferentes, los cuales son:
a) Modelo PPP para los años 2015, 2016 y 2017[5].
b) Modelo PTI para los años 2015, 2016 y 2017.
c) Modelo BoP para los años 2015, 2016 y 2017.

4. Cada uno de los modelos econométricos se analiza con la finalidad de identificar su pertinencia estadística, dentro de las pruebas que se realizan se encuentran: nivel de significancia, autocorrelación y heteroscedasticidad.

[5] Los modelos consideran la siguiente información: a) 2015: 1994-2014, b) 2016: 1994-2015 y c) 2017: 1994-2016.

5. Se determinan los valores futuros de las variables exógenas que componen cada uno de los modelos econométricos con la finalidad de pronosticar el tipo de cambio futuro. Estos cálculos se realizan utilizando dos técnicas: series de tiempo y el operador de agregación de información *Heavy Ordered Weighted Moving Average* (HOWMA). Estas dos últimas técnicas se encuentran descrita en el apartado 4.3.

6. Se analizan los errores de pronostico mediante un comparativo del error promedio y el uso de las técnicas: *mean absolute percentage error* (MAPE), *mean absolute deviation* (MAD) y *mean squared deviation* (MSD) para los resultados obtenidos por la técnica de series de tiempo y los operadores de agregación *heavy*.

### *4.2 Expertón*

El expertón es una herramienta que utiliza la teórica de conjuntos borrosos y el análisis de posibilidad para unificar la información de diferentes expertos, mediante el uso de una función de distribución acumulativa, obtenida mediante expresiones lingüísticas de un grupo de expertos en una escala endecadaria [0,1] (Kaufmann, 1988).

De tal forma, Kaufmann (1988) define al expertón como una extensión del conjunto probabilístico, en donde la probabilidad de cada $\alpha$-*corte* es reemplazada por un intervalo de probabilidades de acuerdo con los expertos. La formulación es la siguiente.

$$\forall a \in E: [a^{j*}(a)], [a_{j*}(a)] \subset [0,1] \tag{27}$$

donde $\subset$ significa el conjunto de inclusión y *j* el experto.

Derivado de que el expertón es una extensión de los conjuntos probabilísticos, este cuenta con las mismas características, estas son:

1) Propiedad de monotonía creciente horizontal no estricta[6].
2) Propiedad de monotonía creciente vertical no estricta, salvo en el nivel 0 que siempre toma el valor 1.

De tal manera que

1) $\forall \alpha \in [0, 1]: \alpha_1(a) \leq \alpha_2(a) en \ [\alpha_1(a), \alpha_2(a)]$ (28)

2) $\forall \alpha \alpha' \in [0, 1]: (a' > a) \Rightarrow (a_1(a)) \leq \alpha_1(a'), a_2(a) < a_2(a')$ (29)

3) $(a = 0) \Rightarrow (a_1(a) = 1, \ a_2(a) = 1)$ (30)

Si se considera que la valuación de cada experto expresa el nivel de verdad en una cantidad de *$\alpha$-cortes* comprendida entre 0 y 1.

De acuerdo con Kaufmann (1988) y Álvarez & León (2016) para la construcción de un expertón se deben seguir los siguientes tres pasos:

1) Se deben evaluar las variables, esto en base a un número $\alpha \in [0, 1]$ o bien a través de intervalos de confianza.
2) Las opiniones individuales deben convertirse en una opinión global del grupo de expertos
3) Se genera la estadística y se aplica la ley de acumulación complementaria

La importancia del expertón radica en que se puede conocer la distribución y la tendencia de las opiniones subjetivas y al final agregarlas para generar una opinión general del grupo de expertos.

[6] Esto significa que la función característica de pertenencia de la función de pendiente positiva es menor o igual a la función característica de la función de pendiente negativa (Álvarez & León, 2016)

*4.3 Metodología de los efectos olvidados*

La metodología de los Efectos Olvidados, desarrollada por Kaufmann & Gil Aluja (1988), se soporta en el supuesto de la existencia de dos conjuntos:

$$A = \left\{\frac{a_i}{i} = 1,2, \dots, n\right\}$$

$$B = \left\{\frac{b_j}{j} = 1,2, \dots, m\right\}$$

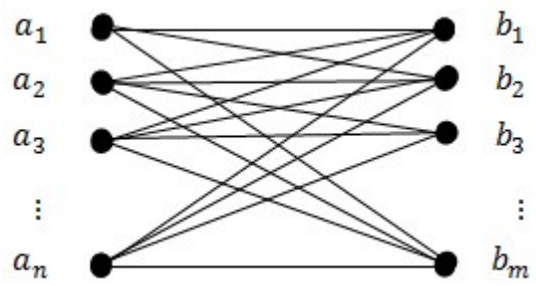
Se conjetura que prevalece una incidencia de $a_i$ sobre $b_j$ si el valor de la función característica de pertenencia del par $(a_i, b_j)$ está valuado dentro del intervalo $[0,1]$, es decir:

$$\forall(a_i, b_j) \Longrightarrow \mu(a_i, b_j) \in [0,1]$$

El conjunto de pares de elementos valuados se conoce como matriz de incidencia directa, la cual muestra la relación causa-efecto en diferentes grados, provocadas por los correspondientes del conjunto *A* (causas) y el conjunto *B* (efectos), expresándose de forma matricial siguiente:

| | | $b_1$ | $b_2$ | $b_3$ | … | $b_m$ |
|---|---|---|---|---|---|---|
| | $a_1$ | $\mu_{a_1b_1}$ | $\mu_{a_1b_2}$ | $\mu_{a_1b_3}$ | … | $\mu_{a_1b_m}$ |
| | $a_2$ | $\mu_{a_2b_1}$ | $\mu_{a_2b_2}$ | $\mu_{a_2b_3}$ | … | $\mu_{a_2b_m}$ |
| M= | $a_3$ | $\mu_{a_3b_1}$ | $\mu_{a_3b_2}$ | $\mu_{a_3b_3}$ | … | $\mu_{a_3b_m}$ |
| | ⋮ | … | … | … | … | … |
| | $a_n$ | $\mu_{a_nb_1}$ | $\mu_{a_nb_2}$ | $\mu_{a_nb_3}$ | … | $\mu_{a_nb_m}$ |

La matriz anterior puede ser representada por un grafo de incidencia asociado, donde en caso de que un elemento del conjunto *A* tenga una pertenencia nula, no exista unión con el conjunto *B*.




Determinadas las incidencias directas entre los conjuntos *A* y *B*, se procede a la detección de los efectos olvidados. Posteriormente, se presume que hay un tercer conjunto de elementos, denominado *C*, expresado cómo sigue:

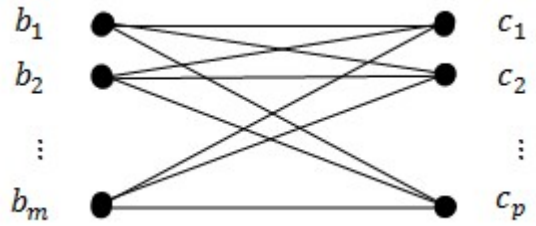
$$C = \left\{ \frac{C_k}{k} = 1,2, \dots, k \right\}$$

Este conjunto está constituido por elementos que son efectos del conjunto *B*, teniendo una matriz de incidencia entre ellos expresada de la forma siguiente:

N=

| | $c_1$ | $c_2$ | $c_3$ | … | $c_m$ |
|---|---|---|---|---|---|
| $b_1$ | $\mu_{b_1c_1}$ | $\mu_{b_1c_2}$ | $\mu_{b_1c_3}$ | … | $\mu_{b_1c_m}$ |
| $b_2$ | $\mu_{b_2c_1}$ | $\mu_{b_2c_2}$ | $\mu_{b_2c_3}$ | … | $\mu_{b_2c_m}$ |
| $b_3$ | $\mu_{b_3c_1}$ | $\mu_{b_3c_2}$ | $\mu_{b_3c_3}$ | … | $\mu_{b_3c_m}$ |
| ⋮ | … | … | … | … | … |
| $b_n$ | $\mu_{b_nc_1}$ | $\mu_{b_nc_2}$ | $\mu_{b_nc_3}$ | … | $\mu_{b_nc_m}$ |

El grafo de incidencia asociado de esta matriz toma la forma:




Se tienes dos matrices de incidencia, las cuales engloban, como elementos comunes, el conjunto *B*, esta relación se expresa como sigue:

$$M \subset A\text{x}B \text{ y } N \subset B\text{x}C$$

Para detectar las relaciones entre los conjuntos *A* y *C*, utilizando el del *B*, se utiliza el operador maxmin (expresado por el símbolo∘), generando una nueva matriz de incidencia.

$$M \circ N = P$$

$$P \subset A\text{x}C$$

Esta nueva relación se formula así:

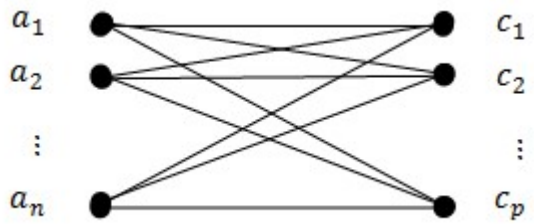
$$\forall(a_i, c_z \in A\text{x}C)$$

$$\mu(a_i, c_z)M \circ N = \forall_{bj}(\mu M(a_i, b_j) \wedge \mu N(b_j, c_z)$$

La matriz de incidencia resultante de realizar la operación maxmin a las matrices anteriores es:

$$P = \begin{array}{c|ccccc} & c_1 & c_2 & c_3 & \dots & c_m \\ \hline a_1 & \mu_{a_1c_1} & \mu_{a_1c_2} & \mu_{a_1c_3} & \dots & \mu_{a_1c_m} \\ a_2 & \mu_{a_2c_1} & \mu_{a_2c_2} & \mu_{a_2c_3} & \dots & \mu_{a_2c_m} \\ a_3 & \mu_{a_3c_1} & \mu_{a_3c_2} & \mu_{a_3c_3} & \dots & \mu_{a_3c_m} \\ \vdots & \dots & \dots & \dots & \dots & \dots \\ a_n & \mu_{a_nc_1} & \mu_{a_nc_2} & \mu_{a_nc_3} & \dots & \mu_{a_nc_m} \end{array}$$

El grafo de incidencia asociado, se expresa de la siguiente forma:




La matriz *P* define las relaciones de causalidad entre los elementos de los conjuntos *A* y *C*, en la intensidad o grado que lleva al considerar los pertenecientes al *B*.

Para asignar los valores ubicados en el intervalo $[0,1]$, Kaufmann & Gil Aluja (1988) proponen una escala semántica endecadaria constituida por 11 valores $[0,0.1,0.2,0.3,\dots,1]$. Ello, apuntan Gil-Lafuente & Barcellos de Paula (2010), facilita su adaptación y tratamiento, ya que, las personas están acostumbradas a pensar y trabajar de forma decimal.

### *4.3 Operador Heavy Ordered Weighted Moving Average (HOWMA)*[7]

El operador HOWMA es una extensión de los operadores OWA y consiste en combinar en un solo operador las características del operador Heavy OWA (HOWA) con la de las medias móviles. En general, este operador usa las medias móviles con un vector asociado de peso cuyo rango es $-\infty$ a $\infty$. Esto es útil cuando conocemos que el resultado de la media móvil será mayor o menor basado en las expectativas del futuro (León *et al.,* 2016).

Un operador HOWMA es definido como una secuencia $\{a_i\}_{i=1}^{N}$, en donde se obtiene una nueva secuencia $\{s_i\}_{i=1}^{N-n+1}$ que es multiplicada por un vector de peso heavy, de tal forma

$$HOWMA(s_i) = \sum_{j=1+t}^{m+t} w_j b_j, \tag{31}$$

donde $b_j$ es el *j*th elemento más grande de la colección $\mathrm{a}_1, \mathrm{a}_2, \dots, \mathrm{a}_\mathrm{n}$, y *W* es un vector de peso asociado de dimensión *m* con $W: 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ y $w_i \in [0,1]$. Obsérvese que el vector de peso puede expandirse de $-\infty$ a $\infty$. Por lo que el vector de peso *w* se convierte en $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

[7] La explicación detallada del operador y sus características se publicaron en el artículo *Induced Heavy Moving Average* (Ver apéndice 2).

Se identifica que permitiendo que el vector de peso sea mayor a uno, se acepta la posibilidad de que alguna parte de la información es independiente. Desde el paso de reorden, se distingue entre el operador HOWMA descendiente (DHOWMA) y el operador HOWMA ascendiente (AHOWMA) utilizando $w_j = w^*_{n-j+1}$ donde $w_j$ es el *j*th peso de DHOWMA y $w^*_{n-j+1}$ el *j*th peso de AHOWMA.

El operador HOWMA es monótona y conmutativo para el operador DHOWMA y el AHOWMA. Es monótona porque si $a_i \geq d_i$, para todas *i*, entonces $HOWMA(a_1, \dots, a_n) \geq HOWMA(d_1, \dots, d_n)$. Es conmutativa porque cualquier permutación de los argumentos tiene la misma evaluación. Nótese
que el operador HOWMA está limitada por el mínimo y el operador total el cual varía entre 1 y *n*. Si el vector de peso tiene como rango $-\infty$ and $\infty$, el operador HOWMA no tiene límites

Cuando se analiza la magnitud del vector de peso $|W|$ para el operador HOWMA, se utiliza la metodología usada por Yager (2002) para el operador HOWA. Como el futuro no depende del reorden de los argumentos, la formulación es la misma que la del operador HOWA. Por lo que es definida como $\beta(W) = (|W| - 1)/(n - 1)$. Desde $|W| \in [1, n]$, entonces $\beta \in [0,1]$. Por lo que si $\beta = 1$, se obtiene el operador total y si $\beta = 0$ se obtiene la media móvil.

# CAPÍTULO V. Resultados

En este capítulo se presentan los resultados de utilizar operadores de agregación heavy para la determinación del valor de las variables exógenas que integran los modelos econométricos y la metodología de los efectos olvidados para la detección e inclusión de las variables ocultas en los modelos tradicionales con la finalidad de pronosticar el tipo de cambio USD/MXN basados en los fundamentales económicos para los años 2015-2017.

## 5.1 Modelos econométricos para el pronóstico de tipo de cambio USD/MXN

Los modelos econométricos basados en las tres teorías fundamentales son: Paridad de Poder y Compra (PPP), Paridad de Tasas de Interés (IRP por sus siglas en inglés) y Balanza de Pagos (BoP). Dichos modelos fueron generados con la siguiente información histórica: a) 2015: Enero 1994 - Diciembre 2014, b) 2016: Enero 1994 – Diciembre 2015 y c) 2017: Enero 1994 – Diciembre 2016.

Cada modelo utiliza las variables definidas en la sección 2.2 del presente documento. Adicionalmente se incluyen dos variables: 1) $e_{t-1}$ que se refiere a la variable dependiente con un desfase[8] y 2) $v_t$ representando la volatilidad del periodo. (Ver Tabla 2-4). Los modelos son los siguientes (Ver Tabla 5-7).

Tabla 2. Variables del modelo PPP

| Variable | Nombre | Tipo de dato[9] |
|---|---|---|
| $\boldsymbol{e_t}$ | Tipo de cambio | Logaritmo del valor nominal |
| $\boldsymbol{v_t}$ | Volatilidad | Logaritmo del valor nominal |
| $\boldsymbol{e_{t-1}}$ | Tipo de cambio con un desfase | Logaritmo del valor nominal |
| $\boldsymbol{pci_t^{MEX}}$ | Índice del precio al consumidor en México | Logaritmo del valor nominal |
| $\boldsymbol{pci_t^{USA}}$ | Índice del precio al consumidor en Estados Unidos | Logaritmo del valor nominal |

Fuente. Elaboración propia

[8] La variable dependiente con un desfase se utiliza porque el valor del tipo de cambio se encuentra significativamente influenciado por el valor al final del periodo anterior.
[9] El logaritmo del valor nominal se utiliza en los modelos PPP e IRP con la finalidad de linealizar la relación.

Tabla 3. Variables en el modelo IRP

| Variable | Nombre | Tipo de dato |
|---|---|---|
| $\boldsymbol{e_t}$ | Tipo de cambio | Logaritmo del valor nominal |
| $\boldsymbol{v_t}$ | Volatilidad | Logaritmo del valor nominal |
| $\boldsymbol{e_{t-1}}$ | Tipo de cambio con desfase | Logaritmo del valor nominal |
| $\boldsymbol{r_t^{MEX}}$ | Tasas de interés en México | Logaritmo del valor nominal |
| $\boldsymbol{r_t^{USA}}$ | Tasas de interés en Estados unidos | Logaritmo del valor nominal |

Fuente. Elaboración propia.

Tabla 4. Variables para el modelo BoP

| Variable | Nombre | Tipo de dato[10] |
|---|---|---|
| $\boldsymbol{E_t}$ | Tipo de cambio | Valor nominal |
| $\boldsymbol{V_t}$ | Volatilidad | Valor nominal |
| $\boldsymbol{E_{t-1}}$ | Tipo de cambio con desfase | Valor nominal |
| $\boldsymbol{FPI_t^{MEX}}$ | Inversión extranjera en cartera en México | Valor nominal |
| $\boldsymbol{FDI_t^{MEX}}$ | Inversión extranjera directa en México | Valor nominal |
| $\boldsymbol{R_t^{MEX}}$ | Reservas internacionales en México | Valor nominal |
| $\boldsymbol{BT_t^{MEX}}$ | Balanza comercial en México | Valor nominal |

Fuente. Elaboración propia.

Tabla 5. Modelo PPP para los años 2015 - 2017[11]

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| $\boldsymbol{v_t}$ | $\beta_1 = 0.0234641$ | $\beta_1 = 0.0230913$ | $\beta_1 = 0.0237885$ |
| $\boldsymbol{e_{t-1}}$ | $\beta_2 = 0.9141334$ | $\beta_2 = 0.9302370$ | $\beta_2 = 0.9428252$ |
| $\boldsymbol{pci_t^{MEX}}$ | $\beta_3 = -0.0194534$ | $\beta_3 = 0.0101966$ | $\beta_3 = 0.0260894$ |
| $\boldsymbol{pci_t^{USA}}$ | $\beta_4 = 0.0407992$ | $\beta_4 = 0.0242960$ | $\beta_4 = 0.0127441$ |
| *Constante* | $\beta_0 = 0.1066104$ | $\beta_0 = 0.0527191$ | $\beta_0 = 0.0264180$ |
| $\boldsymbol{r^2}$ | 0.9899 | 0.9906 | 0.9915 |
| $\boldsymbol{r^2\ ajustada}$ | 0.9897 | 0.0991 | 0.9914 |

Fuente. Elaboración propia.

[10] La transformación logarítmica no se realiza dentro del modelo BoP debido a que tiene datos negativos en la serie histórica de las variables que lo integran.

[11] La fórmula para el modelo PPP es: $e_t = \beta_0 + \beta_1 v_t + \beta_2 e_{t-1} + \beta_3 pci_t^{MEX} - \beta_4 pci_t^{USA}$

Analizando el nivel de significancia individual[12] para cada año, para el caso de la constante, $pci_t^{MEX}$ y $pci_t^{USA}$ la hipótesis nula es aceptada, siendo rechazada para $e_{t-1}$ y $v_t$ (Ver Anexo 1). Por otra parte, para el modelo completo la hipótesis nula es rechazada (Ver Anexo 4).[13]

En términos de multicolinealidad[14] tiene niveles altos para $e_{t-1}$, $pci_t^{MEX}$ y $pci_t^{USA}$ y baja para $v_t$[15] (Ver anexo 5). Los valores del estadístico Durbin-Watson (D-W) indican que no existe autocorrelación[16] a niveles de significancia 0.05 y 0.01 (Ver Anexo 8)

Tabla 6. Modelo IRP para los años 2015 - 2017[17]

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| $\boldsymbol{v_t}$ | $\alpha_1 =$ 0.2254560 | $\alpha_1 =$ 0.2243830 | $\alpha_1 =$ 0.0237093 |
| $\boldsymbol{e_{t-1}}$ | $\alpha_1 =$ 0.9604932 | $\alpha_1 =$ 0.9618081 | $\alpha_1 =$ 0.9644360 |
| $\boldsymbol{r_t^{MEX}}$ | $\alpha_1 =$ -0.0022782 | $\alpha_1 =$ -0.0022228 | $\alpha_1 =$ -0.0031719 |
| $\boldsymbol{r_t^{USA}}$ | $\alpha_1 =$ -0.0001871 | $\alpha_1 =$ -0.0007320 | $\alpha_1 =$ -0.0001735 |
| *Constante* | $\alpha_1 =$ 0.0983360 | $\alpha_1 =$ 0.8955890 | $\alpha_1 =$ 0.9064130 |
| $\boldsymbol{r^2}$ | 0.9896 | 0.9905 | 0.9914 |
| $\boldsymbol{r^2}$ ***ajustada*** | 0.9895 | 0.9903 | 0.9913 |

Fuente. Elaboración propia.

[12] El test de significancia se realiza mediante el análisis del estadístico P (Ver Anexo 1-3) en donde la hipótesis nula es rechazada: $P < 0.05$ y aceptada cuando $P > 0.05$.

[13] Un aspecto importante que considerar en el estudio de las variables que no tienen un nivel no significativo es que estas son estudiadas con la finalidad de mantener la formulación teórica original para cada modelo, en este sentido, aun cuando la variable no contribuye relevantemente a la ecuación es tomada en cuenta.

[14] La multicolinealidad en un modelo econométrico ocurre cuando los predictores del modelo están correlacionados con otros predictores. El análisis de los Factores de Inflación de la Varianza (VIF por sus siglas en inglés) se utiliza para determinar la presencia del mismo.

[15] Es importante notar que la presencia de multicolinealidad no es relevante para el presente estudio, debido a que el objetivo de los modelos es el pronóstico y no la simplicidad. De tal modo que se utiliza la formulación teórica completa, ya que no se pretende examinar su significado individual ni eliminar algunas de ellas que disminuyan el valor explicativo del modelo.

[16] La no presencia autocorrelación nos indica que los errores del modelo son aleatorios, es decir, no existe una tendencia dentro de los mismos.

[17] La fórmula para el modelo IRP es: $e_t = \alpha_0 + \alpha_1 v_t + \alpha_2\, tc_{lag} + \alpha_3\, r_t^{MEX} - \alpha_4 r_t^{USA}$

Examinando los niveles de significancia, se detecta que, para todos los años, en el caso de $r_t^{MEX}$y $r_t^{USA}$ la hipótesis nula es aceptada, siendo rechazada para la constante, $e_{t-1}$ y $v_t$ (Ver Anexo 2). Por otro lado, para el modelo completo (Ver Anexo 4), la hipótesis nula es rechazada.

La presencia de multicolinealidad es baja para $r_t^{MEX}$, $r_t^{USA}$, $e_{t-1}$ y $v_t$ (Ver Anexo 6). Finalmente, los valores D-W indican que no existe autocorrelación para los niveles de significancia 0.05 y 0.0.1 (Ver Anexo 8).

Tabla 7. Modelo BoP para los años 2015 - 2017[18]

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| $\boldsymbol{V_t}$ | $\theta_1 = 4.5873750$ | $\theta_1 = 4.6158630$ | $\theta_1 = 4.9582940$ |
| $\boldsymbol{E_{t-1}}$ | $\theta_2 = 0.9780775$ | $\theta_2 = 0.9778967$ | $\theta_2 = 0.9791574$ |
| $\boldsymbol{FPI_t^{MEX}}$ | $\theta_3 =$ - 0.0000227 | $\theta_3 =$ - 0.0000228 | $\theta_3 =$ - 0.0000170 |
| $\boldsymbol{FDI_t^{MEX}}$ | $\theta_4 =$ - 0.0000162 | $\theta_4 =$ - 0.0000157 | $\theta_4 =$ - 0.0000132 |
| $\boldsymbol{R_t^{MEX}}$ | $\theta_5 = 0.0000023$ | $\theta_5 = 0.0000024$ | $\theta_5 = 0.0000021$ |
| $\boldsymbol{BT_t^{MEX}}$ | $\theta_6 =$ - 0.0000411 | $\theta_6 =$ - 0.0000434 | $\theta_6 =$ - 0.0000540 |
| *Constante* | $\theta_0 = 0.1738603$ | $\theta_0 = 0.1629673$ | $\theta_0 = 0.1406526$ |
| $\boldsymbol{r^2}$ | 0.9876 | 0.9896 | 0.9902 |
| $\boldsymbol{r^2\ ajustada}$ | 0.9873 | 0.9894 | 0.9900 |

Fuente. Elaboración propia.

Analizando el nivel de significancia para el año 2015 la hipótesis nula es aceptada para la constante y $BT_t^{MEX}$, en contraste, es rechazada para las variables: $V_t$, $E_{t-1}$, $FPI_t^{MEX}$ $FDI_t^{MEX}$ y $R_t^{MEX}$. En 2016, es aceptada para la constante y $FPI_t^{MEX}$, rechazándose para $V_t$, $E_{t-1}$, $FDI_t^{MEX}$ $R_t^{MEX}$ y $BT_t^{MEX}$. Finalmente, para 2017 es aceptada para la constante y, rechazada para $V_t$, $E_{t-1}$, $FPI_t^{MEX}$ $R_t^{MEX}$, $FDI_t^{MEX}$ y $BT_t^{MEX}$ (Ver Anexo 3). Por otro lado, se identifica que para el modelo completo la hipótesis nula es rechazada (Ver Anexo 4).

[18] The formula for the BoP model is: $e_t = \theta_0 + \theta_1 V_t + \theta_2\, tc_{lag} - \theta_3 FPI_t^{MEX} - \theta_4 FDI_t^{MEX} - \theta_5\, R_t^{MEX} - \theta_6\, BT_t^{MEX}$

En términos de multicolinealidad, es baja para la $V_t$, $E_{t-1}$, $FPI_t^{MEX}$ $R_t^{MEX}$, $FDI_t^{MEX}$ y $BT_t^{MEX}$ (Ver anexo 7). De acuerdo con los valores D-W no existe autocorrelación para los niveles de significancia 0.05 y 0.01 (Ver Anexo 8).

### 5.2 Pronostico del tipo de cambio USD/MXN 2015-2017 con modelos tradicionales

Para pronosticar, primeramente, se utiliza las series de tiempo de descomposición multiplicativa, en la cual se determina la tendencia, temporalidad, ciclo e irregularidad (Ver Anexo 9). Adicionalmente, el operador HOWMA se utiliza para pronosticar el futuro de las variables con una secuencia $n = 6$.

De acuerdo con el criterio de la política económica general, publicada por el Banco de México (Banxico) y la Secretaria de Hacienda y Crédito Público (SHCP), las siguientes expectativas se tienen para los fundamentales económicos.

a) Incremento para $pci_{USA}$, $pci_{MEX}$, $r_{USA}$, $r_{MEX}$, *FPI* y *FDI;*

b) Disminución de *R* y *BT.*

c) Depreciación de $e_t$ e incremento de *V*.

Con esta información, el vector de peso $w = [0.05,0.05,0.15,0.15,0.4]$ es considerado para las variables con expectativa de incremento y $w = [0.05,0.05,0.15,0.15,0.3]$ para aquellas con expectativa de disminución. Adicionalmente, un criterio optimista se consideró en la etapa de ordenación del operador.

Con los cálculos realizados, se presenta el pronóstico para los años 2015[19]. y 2016 (Ver Tabla 8-11). Adicionalmente, para el año 2017 se realiza un comparativo entre el pronóstico del tipo de cambio publicado por la Bolsa Mexicana de Derivados (MEXDER) y el obtenido por los modelos mediante series de tiempo y operadores HOWMA (Ver Tabla 12-13).

Tabla 8. Pronostico del tipo de cambio USD/MXN 2015 con series de tiempo

| Tiempo | Tipo de cambio spot $e_t$ | Modelo PPP $\widehat{e_t}$ | Error | Modelo IRP $\widehat{e_t}$ | Error | Modelo BoP $\widehat{e_t}$ | Error |
|---|---|---|---|---|---|---|---|
| 01-15 | 14.6808 | 15.6435 | 0.9627 | 15.6757 | 0.9949 | 15.6344 | 0.9536 |
| 02-15 | 14.9230 | 15.5716 | 0.6486 | 15.6272 | 0.7042 | 15.8959 | 0.9729 |
| 03-15 | 15.2136 | 15.1264 | -0.0872 | 15.2111 | -0.0025 | 15.4197 | 0.2061 |
| 04-15 | 15.2208 | 15.1832 | -0.0376 | 15.2666 | 0.0458 | 15.4226 | 0.2018 |
| 05-15 | 15.2475 | 15.3492 | 0.1017 | 15.4326 | 0.1851 | 15.6026 | 0.3551 |
| 06-15 | 15.4692 | 15.6489 | 0.1797 | 15.7162 | 0.2470 | 15.8848 | 0.4156 |
| 07-15 | 15.9225 | 15.8342 | -0.0883 | 15.8889 | -0.0336 | 15.9721 | 0.0496 |
| 08-15 | 16.5032 | 15.6062 | -0.8970 | 15.6731 | -0.8301 | 15.6735 | -0.8297 |
| 09-15 | 16.8519 | 15.6222 | -1.2297 | 15.6922 | -1.1597 | 15.8393 | -1.0126 |
| 10-15 | 16.5813 | 15.7903 | -0.7910 | 15.8567 | -0.7246 | 15.7810 | -0.8003 |
| 11-15 | 16.6325 | 15.6985 | -0.9340 | 15.7631 | -0.8694 | 15.7541 | -0.8784 |
| 12-15 | 17.0365 | 16.0393 | -0.9972 | 16.0779 | -0.9586 | 16.2798 | -0.7567 |
| **Promedio** | **15.8569** | **15.5928** | **-0.2641** | **15.6568** | **-0.2001** | **15.7633** | **-0.0936** |

Fuente. Elaboración propia.

[19] Los resultados para el año 2015 se publicaron en el artículo *Exchange rate USD/MXN forecast through econometric models, time series and HOWMA operators* (Ver Apéndice 3).

Tabla 9. Pronostico del tipo de cambio USD/MXN 2015 con operadores HOWMA

| Fecha | Tipo de cambio spot $e_t$ | Modelo PPP $\widehat{e_t}$ | Error | Modelo IRP $\widehat{e_t}$ | Error | Modelo BoP $\widehat{e_t}$ | Error |
|---|---|---|---|---|---|---|---|
| 01-15 | 14.6808 | 14.4197 | -0.2611 | 14.4255 | -0.2553 | 14.5561 | -0.1360 |
| 02-15 | 14.9230 | 14.4971 | -0.4259 | 14.4952 | -0.4278 | 14.5631 | -0.3629 |
| 03-15 | 15.2136 | 14.8056 | -0.4080 | 14.8092 | -0.4044 | 14.8342 | -0.3734 |
| 04-15 | 15.2208 | 15.1007 | -0.1201 | 15.1098 | -0.1110 | 15.1095 | -0.0940 |
| 05-15 | 15.2475 | 15.4455 | 0.1980 | 15.4622 | 0.2147 | 15.4608 | 0.2407 |
| 06-15 | 15.4692 | 15.7565 | 0.2873 | 15.7792 | 0.3100 | 15.7469 | 0.3221 |
| 07-15 | 15.9225 | 16.0647 | 0.1422 | 16.0965 | 0.1740 | 16.0656 | 0.2047 |
| 08-15 | 16.5032 | 16.3817 | -0.1215 | 16.4208 | -0.0824 | 16.3778 | -0.0523 |
| 09-15 | 16.8519 | 16.7113 | -0.1406 | 16.7585 | -0.0934 | 16.7060 | -0.0603 |
| 10-15 | 16.5813 | 17.0465 | 0.4652 | 17.1021 | 0.5208 | 17.0394 | 0.5572 |
| 11-15 | 16.6325 | 17.3888 | 0.7563 | 17.4533 | 0.8208 | 17.3816 | 0.8627 |
| 12-15 | 17.0365 | 17.7368 | 0.7003 | 17.8103 | 0.7738 | 17.7273 | 0.8198 |
| **Promedio** | **15.8569** | **15.9462** | **0.0893** | **15.9769** | **0.1200** | **15.9640** | **0.1071** |

Fuente. Elaboración propia

Tabla 10. Pronostico del tipo de cambio USD/MXN 2016 con series de tiempo.

| Tiempo | Tipo de cambio spot $e_t$ | Modelo PPP $\widehat{e_t}$ | Error | Modelo IRP $\widehat{e_t}$ | Error | Modelo BoP $\widehat{e_t}$ | Error |
|---|---|---|---|---|---|---|---|
| 01-16 | 18.0255 | 17.0488 | -0.9767 | 17.1546 | -0.8709 | 17.1498 | -0.8758 |
| 02-16 | 18.4777 | 16.8809 | -1.5968 | 16.9827 | -1.4949 | 17.0689 | -1.4088 |
| 03-16 | 17.6923 | 17.0252 | -0.6671 | 17.1248 | -0.5675 | 17.1437 | -0.5486 |
| 04-16 | 17.4905 | 17.1666 | -0.3239 | 17.2738 | -0.2167 | 17.3333 | -0.1572 |
| 05-16 | 18.0980 | 17.4955 | -0.6025 | 17.6169 | -0.4811 | 17.6715 | -0.4266 |
| 06-16 | 18.6506 | 17.6676 | -0.9830 | 17.7936 | -0.8570 | 17.9268 | -0.7237 |
| 07-16 | 18.5862 | 17.3921 | -1.1941 | 17.5030 | -1.0832 | 17.7274 | -0.8588 |
| 08-16 | 18.4715 | 17.5371 | -0.9344 | 17.6466 | -0.8249 | 17.8680 | -0.6035 |
| 09-16 | 19.1955 | 17.8172 | -1.3783 | 17.9344 | -1.2611 | 18.0945 | -1.1010 |
| 10-16 | 18.9157 | 17.8131 | -1.1026 | 17.9220 | -0.9937 | 17.9934 | -0.9223 |
| 11-16 | 20.0371 | 18.0194 | -2.0177 | 18.1422 | -1.8949 | 18.2600 | -1.7771 |
| 12-16 | 20.5156 | 17.9193 | -2.5963 | 18.0382 | -2.4774 | 18.1306 | -2.3850 |
| **Promedio** | **18.6797** | **17.4819** | **-1.1978** | **17.5944** | **-1.0852** | **17.6973** | **-0.9824** |

Fuente. Elaboración propia

Tabla 11. Pronostico del tipo de cambio USD/MXN 2016 con operadores HOWMA

| Tiempo | Tipo de cambio spot $e_t$ | Modelo PPP $\widehat{e_t}$ | Error | Modelo IRP $\widehat{e_t}$ | Error | Modelo BoP $\widehat{e_t}$ | Error |
|---|---|---|---|---|---|---|---|
| **01-16** | 18.0255 | 16.9908 | -1.0348 | 17.1103 | -0.9152 | 17.0096 | -1.0160 |
| **02-16** | 18.4777 | 17.6152 | -0.8625 | 17.7435 | -0.7342 | 17.6267 | -0.8510 |
| **03-16** | 17.6923 | 17.9259 | 0.2336 | 18.0531 | 0.3608 | 17.9444 | 0.2521 |
| **04-16** | 17.4905 | 18.2969 | 0.8063 | 18.4220 | 0.9314 | 18.2786 | 0.7880 |
| **05-16** | 18.0980 | 18.6428 | 0.5447 | 18.7637 | 0.6657 | 18.5983 | 0.5003 |
| **06-16** | 18.6506 | 19.0387 | 0.3881 | 19.1588 | 0.5082 | 18.9734 | 0.3229 |
| **07-16** | 18.5862 | 19.4318 | 0.8456 | 19.5539 | 0.9677 | 19.3622 | 0.7759 |
| **08-16** | 18.4715 | 19.8395 | 1.3680 | 19.9613 | 1.4898 | 19.7537 | 1.2822 |
| **09-16** | 19.1955 | 20.2452 | 1.0497 | 20.3665 | 1.1710 | 20.1448 | 0.9493 |
| **10-16** | 18.9157 | 20.6634 | 1.7477 | 20.7843 | 1.8686 | 20.5480 | 1.6323 |
| **11-16** | 20.0371 | 21.0890 | 1.0519 | 21.2094 | 1.1723 | 20.9599 | 0.9228 |
| **12-16** | 20.5156 | 21.5250 | 1.0094 | 21.6451 | 1.1294 | 21.3819 | 0.8662 |
| **Promedio** | 18.6797 | 19.2753 | 0.5956 | 19.3977 | 0.7180 | 19.2151 | 0.5354 |

Fuente. Elaboración propia.

Tabla 12. Pronostico del tipo de cambio USD/MXN 2017 con series de tiempo

| Tiempo | Tipo de cambio MEXDER | Modelo PPP $\widehat{e_t}$ | Error | Modelo IRP $\widehat{e_t}$ | Error | Modelo BoP $\widehat{e_t}$ | Error |
|---|---|---|---|---|---|---|---|
| **01-17** | 20.7818 | 18.4543 | -2.3275 | 18.6534 | -2.1284 | 17.1498 | -3.6320 |
| **02-17** | 20.8672 | 17.9187 | -2.9485 | 18.0962 | -2.7710 | 17.0689 | -3.7983 |
| **03-17** | 20.9365 | 17.9072 | -3.0293 | 18.0782 | -2.8583 | 17.1437 | -3.7928 |
| **04-17** | 21.0369 | 18.0168 | -3.0201 | 18.1912 | -2.8457 | 17.3333 | -3.7036 |
| **05-17** | 21.1246 | 18.1620 | -2.9626 | 18.3392 | -2.7854 | 17.6715 | -3.4531 |
| **06-17** | 21.2382 | 18.5056 | -2.7326 | 18.6927 | -2.5455 | 17.9268 | -3.3114 |
| **07-17** | 21.3293 | 18.7231 | -2.6062 | 18.9137 | -2.4156 | 17.7274 | -3.6019 |
| **08-17** | 21.4207 | 18.4370 | -2.9837 | 18.6133 | -2.8074 | 17.8680 | -3.5527 |
| **09-17** | 21.5369 | 18.4719 | -3.0650 | 18.6435 | -2.8934 | 18.0945 | -3.4424 |
| **10-17** | 21.6337 | 18.8572 | -2.7765 | 19.0374 | -2.5963 | 17.9934 | -3.6403 |
| **11-17** | 21.7345 | 18.7124 | -3.0221 | 18.8909 | -2.8436 | 18.2600 | -3.4745 |
| **12-17** | 21.8622 | 19.0072 | -2.8550 | 19.1883 | -2.6739 | 18.1306 | -3.7316 |
| **Promedio** | 21.2919 | 18.4311 | -2.8608 | 18.6115 | -2.6804 | 17.6973 | -3.5945 |

Fuente. Elaboración propia.

Tabla 13. Pronostico del tipo de cambio USD/MXN 2017 con operadores HOWMA

| Tiempo | Tipo de cambio MEXDER | Modelo PPP $\widehat{e_t}$ | Error | Modelo IRP $\widehat{e_t}$ | Error | Modelo BoP $\widehat{e_t}$ | Error |
|---|---|---|---|---|---|---|---|
| **01-17** | 20.7818 | 20.0285 | -0.7533 | 20.1849 | -0.5969 | 19.8278 | -0.9540 |
| **02-17** | 20.8672 | 20.2859 | -0.5813 | 20.4473 | -0.4199 | 20.0535 | -0.8137 |
| **03-17** | 20.9365 | 20.6899 | -0.2466 | 20.8290 | -0.1075 | 20.4116 | -0.5249 |
| **04-17** | 21.0369 | 20.9185 | -0.1184 | 21.0437 | 0.0068 | 20.6017 | -0.4352 |
| **05-17** | 21.1246 | 21.4049 | 0.2803 | 21.5240 | 0.3994 | 21.0536 | -0.0710 |
| **06-17** | 21.2382 | 21.4909 | 0.2527 | 21.5936 | 0.3554 | 21.1299 | -0.1083 |
| **07-17** | 21.3293 | 21.5661 | 0.2368 | 21.6522 | 0.3229 | 21.1584 | -0.1709 |
| **08-17** | 21.4207 | 21.8677 | 0.4470 | 21.9423 | 0.5216 | 21.4325 | 0.0118 |
| **09-17** | 21.5369 | 22.6596 | 1.1227 | 22.7345 | 1.1976 | 22.2041 | 0.6672 |
| **10-17** | 21.6337 | 22.4716 | 0.8379 | 22.5211 | 0.8874 | 21.9762 | 0.3425 |
| **11-17** | 21.7345 | 22.7753 | 1.0408 | 22.8120 | 1.0775 | 22.2494 | 0.5149 |
| **12-17** | 21.8622 | 23.0084 | 1.1462 | 23.0300 | 1.1678 | 22.4500 | 0.5878 |
| **Promedio** | 21.2919 | 21.5973 | 0.3054 | 21.6929 | 0.4010 | 21.2124 | -0.0795 |

Fuente. Elaboración propia.

Para evaluar los errores obtenidos por los pronósticos para cada uno de los modelos y técnicas, los resultados se analizan mediante el error de porcentaje absoluto medio (MAPE por sus siglas en inglés), desviación absoluta media (MAD por sus siglas en inglés) y la desviación media cuadrada (MSD por sus siglas en inglés), cuyas formulas son las siguientes.

$$MAPE = \frac{\sum(|y_t - \hat{y}_t|/y_t}{n} \text{x}100,$$

$$MAD = \frac{\sum|y_t - \hat{y}_t|}{n},$$

$$MSD = \frac{\sum|y_t - \hat{y}_t|^2}{n},$$

donde $y_t$ es el valor actual, $\hat{y}_t$ es el valor pronosticado y *n* es el número de observaciones. Con estas fórmulas los siguientes resultados son obtenidos (Ver Tabla 14-16).

Table 14. Evaluación de los errores para el año 2015

| | Series de tiempo | | | HOWMA | | |
|---|---|---|---|---|---|---|
| | **PPP** | **IRP** | **BoP** | **PPP** | **IRP** | **BoP** |
| **MAD** | 0.5796 | 0.5630 | 0.6194 | 0.3355 | 0.3490 | 0.3405 |
| **MSD** | 0.5180 | 0.4848 | 0.4949 | 0.1571 | 0.1796 | 0.1863 |
| **MAPE** | 3.9477 | 3.8347 | 4.2189 | 2.0988 | 2.3775 | 2.3194 |

Fuente. Elaboración propia.

Table 15. Evaluación de los errores para el año 2016

| | Series de tiempo | | | HOWMA | | |
|---|---|---|---|---|---|---|
| | **PPP** | **IRP** | **BoP** | **PPP** | **IRP** | **BoP** |
| **MAD** | 1.1978 | 1.0853 | 0.9824 | 0.9118 | 0.9929 | 0.8466 |
| **MSD** | 1.8008 | 1.5407 | 1.3162 | 0.9862 | 1.1454 | 0.8508 |
| **MAPE** | 6.6449 | 6.0208 | 5.4499 | 5.0586 | 5.5081 | 4.6966 |

Fuente. Elaboración propia.

Table 16. Evaluación de los errores para el año 2017

| | Series de tiempo | | | HOWMA | | |
|---|---|---|---|---|---|---|
| | **PPP** | **IRP** | **BoP** | **PPP** | **IRP** | **BoP** |
| **MAD** | 2.8608 | 2.6804 | 3.5945 | 0.5887 | 0.9929 | 0.8466 |
| **MSD** | 8.2279 | 7.2317 | 12.9420 | 0.4782 | 1.1454 | 0.8508 |
| **MAPE** | 13.7657 | 12.8977 | 17.2966 | 2.8326 | 5.5081 | 4.6966 |

Fuente. Elaboración propia.

Con la información obtenida por los tres métodos de evaluación de errores, se observa que los resultados derivados del uso de operadores HOWMA son mejores para los tres modelos y tres años de pronóstico. Esto demuestra la eficiencia del operador. Adicionalmente, como se observa en el proceso de cálculo, es posible generar diferentes escenarios modificando los valores del vector de peso. Esto no es posible con las series de tiempo, ya que el valor de las variables exógenas en

este último peso depende solamente de la información histórica, por lo que permite un resultado único y no pueden generarse diversos escenarios.

Es importante notar que para el 2016, el error es mayor que para el año 2015 cuando el escenario económico era más estable. Finalmente, para el año 2017, el MEXDER predice una paridad cambiaria de 21.8622, en contraste con el uso de operadores HOWMA que estima que oscile entre 22.45 y 23.00.

### 5.3 Pronostico del tipo de cambio USD/MXN 2015 con modelos incorporando efectos olvidados.

Para generar los modelos con efectos olvidados se utilizó información obtenida de cinco expertos del mercado financiero, los cuales dieron su opinión individual y esta fue unificada mediante el uso de expertos, posteriormente con las matrices globales se procedió a utilizar la metodología de los efectos olvidados para determinar la incidencia de segundo grado y se llega a la conclusión que el precio del petróleo es una elemento oculto con un impacto relevante en las variables que pronostican el tipo de cambio[20].

Posteriormente, se generan nuevamente los modelos de pronóstico para el año 2015 de los tres modelos tradicionales (PPP, IRP y BoP) incluyendo las dos variables adicionales mencionadas en el punto 5.2 (Tipo de cambio con desfase y volatilidad) y la variable olvidada (Precio del petróleo). Se presentan los resultados utilizando series de tiempo y operadores HOWMA (Ver tabla 17-18)[21].

[20] El detalle del proceso de obtención de los efectos olvidados se encuentra explicado en el artículo *Forgotten Effects in Exchange Rate Forecasting Models* (Ver Apéndice 4)
[21] La información específica de los modelos y resultados están publicados en el artículo *Forgotten effects and heavy moving averages in exchange rate forecasting* (Ver Apéndice 5)

Tabla 17. Pronostico del tipo de cambio USD/MXN 2015 con series de tiempo[22]

| Tiempo | Tipo de cambio spot $e_t$ | Modelo PPP VO $\widehat{e_t}$ | Error | Modelo IRP VO $\widehat{e_t}$ | Error | Modelo BoP VO $\widehat{e_t}$ | Error |
|---|---|---|---|---|---|---|---|
| **01-17** | 14.6808 | 15.8839 | 1.2031 | 15.6071 | 0.9263 | 15.8697 | 1.1889 |
| **02-17** | 14.923 | 15.842 | 0.919 | 15.5329 | 0.6099 | 15.952 | 1.029 |
| **03-17** | 15.2136 | 15.4178 | 0.2042 | 15.0923 | -0.1213 | 15.5446 | 0.331 |
| **04-17** | 15.2208 | 15.4674 | 0.2466 | 15.1519 | -0.0689 | 15.6086 | 0.3878 |
| **05-17** | 15.2475 | 15.6321 | 0.3846 | 15.318 | 0.0705 | 15.7326 | 0.4851 |
| **06-17** | 15.4692 | 15.9168 | 0.4476 | 15.6176 | 0.1484 | 15.9896 | 0.5204 |
| **07-17** | 15.9225 | 16.0907 | 0.1682 | 15.8022 | -0.1203 | 16.1437 | 0.2212 |
| **08-17** | 16.5032 | 15.8782 | -0.625 | 15.5731 | -0.9301 | 15.9175 | -0.5857 |
| **09-17** | 16.8519 | 15.891 | -0.9609 | 15.5931 | -1.2588 | 16.0088 | -0.8431 |
| **10-17** | 16.5813 | 16.0594 | -0.5219 | 15.7599 | -0.8214 | 16.001 | -0.5803 |
| **11-17** | 16.6325 | 15.9705 | -0.662 | 15.6662 | -0.9663 | 15.972 | -0.6605 |
| **12-17** | 17.0365 | 16.3045 | -0.732 | 15.9985 | -1.038 | 16.4927 | -0.5437 |
| **Promedio** | 15.8569 | 15.8629 | 0.006 | 15.5594 | -0.2975 | 15.9361 | 0.0792 |

Fuente. Elaboración propia.

[22] La abreviación VO se refiere a que el modelo incluye la Variable Oculta.

Tabla 18. Pronostico del tipo de cambio USD/MXN 2015 con operadores HOWMA

| Tiempo | Tipo de cambio spot $e_t$ | Modelo PPP VO $\widehat{e_t}$ | Error | Modelo IRP VO $\widehat{e_t}$ | Error | Modelo BoP VO $\widehat{e_t}$ | Error |
|---|---|---|---|---|---|---|---|
| **01-17** | 14.6808 | 14.5507 | -0.1301 | 14.4156 | -0.2652 | 14.82 | 0.1392 |
| **02-17** | 14.923 | 14.6505 | -0.2725 | 14.4829 | -0.4401 | 14.8369 | -0.0861 |
| **03-17** | 15.2136 | 14.9846 | -0.229 | 14.7938 | -0.4198 | 15.1216 | -0.092 |
| **04-17** | 15.2208 | 15.305 | 0.0842 | 15.092 | -0.1288 | 15.4122 | 0.1914 |
| **05-17** | 15.2475 | 15.6786 | 0.4311 | 15.4418 | 0.1943 | 15.7696 | 0.5221 |
| **06-17** | 15.4692 | 16.0171 | 0.5479 | 15.7569 | 0.2877 | 16.0768 | 0.6076 |
| **07-17** | 15.9225 | 16.3512 | 0.4287 | 16.0739 | 0.1514 | 16.4181 | 0.4956 |
| **08-17** | 16.5032 | 16.6976 | 0.1944 | 16.3962 | -0.107 | 16.7447 | 0.2415 |
| **09-17** | 16.8519 | 17.0577 | 0.2058 | 16.7321 | -0.1198 | 17.0888 | 0.2369 |
| **10-17** | 16.5813 | 17.4243 | 0.8429 | 17.0738 | 0.4925 | 17.4392 | 0.8579 |
| **11-17** | 16.6325 | 17.799 | 1.1666 | 17.4232 | 0.7907 | 17.7992 | 1.1667 |
| **12-17** | 17.0365 | 18.1806 | 1.1442 | 17.7783 | 0.7418 | 18.164 | 1.1275 |
| **Promedio** | 15.8569 | 16.2248 | 0.3679 | 15.9551 | 0.0982 | 16.3076 | 0.4507 |

Fuente. Elaboración propia.

Los errores se evalúan mediante las técnicas MAPE, MSD y MAD detalladas en el punto 5.2, los resultados se presentan contrastando series de tiempo y operadores HOWMA mediante modelos tradicionales y modelos con efecto olvidado (Ver Tabla 19-20).

Table 19. Evaluación de los errores para el año 2015 con series de tiempo

| | **Tradicional** | | | **Con efecto olvidado** | | |
|---|---|---|---|---|---|---|
| | **PPP** | **IRP** | **BoP** | **PPP** | **IRP** | **BoP** |
| **MAD** | 0.5796 | 0.5630 | 0.6194 | 0.4227 | 0.5900 | 0.6147 |
| **MSD** | 0.5180 | 0.4848 | 0.4949 | 0.25914 | 0.5355 | 0.4508 |
| **MAPE** | 3.9477 | 3.8347 | 4.2189 | 2.6844 | 3.6488 | 3.9055 |

Fuente. Elaboración propia.

Table 20. Evaluación de los errores para el año 2015 con operadores HOWMA

| | Tradicional | | | Con efecto olvidado | | |
|---|---|---|---|---|---|---|
| | **PPP** | **IRP** | **BoP** | **PPP** | **IRP** | **BoP** |
| **MAD** | 0.3355 | 0.3490 | 0.3405 | 0.4731 | 0.3449 | 0.4803 |
| **MSD** | 0.1571 | 0.1796 | 0.1863 | 0.3567 | 0.1703 | 0.3701 |
| **MAPE** | 2.0988 | 2.3775 | 2.3194 | 2.9203 | 2.1569 | 2.9606 |

Fuente. Elaboración propia.

La relevancia del uso de variables olvidadas en los modelos es que permiten la incorporación de nuevos elementos a los modelos que en las formulaciones clásicas no son considerados. Con los resultados obtenidos se observa que para los modelos con series de tiempo el error disminuyo para los tres modelos, mientras que, los resultados con operadores HOWMA solamente mejoraron para el modelo IRP.

Se puede afirmar que con la inclusión de operadores de agregación heavy y la metodología de los efectos olvidados primeramente nos permitimos generar nuevos escenarios que considerando solamente series de tiempo y modelos tradicionales no pudieran ser observados, con esto se puede dar una gama más amplia de posibilidades y visualizar el futuro mediante el conocimiento y expectativa del tomador de decisiones y/o expertos del mercado financiero.

# CAPÍTULO VI. Conclusiones

## 6.1 Conclusiones generales

La adopción del régimen de libre flotación, combinado con diversos factores internos y externos generan cambios importantes en la paridad cambiaria, siendo la más importante para el caso específico de México el tipo de cambio USD/MXN. Por lo que generar estructuras metodológicas que permitan disminuir el riesgo mediante un pronóstico certero se vuelve de suma relevancia.

La literatura en cuestión del pronóstico del tipo de cambio es basta, existiendo autores que afirman que esta se asemeja a un camino aleatorio, mientras otros consideran que los fundamentales económicos sirven como base para proyecciones a largo plazo. En este sentido, no existe una metodología clara y única para la paridad cambiaria, dificultando la posibilidad de adoptar algún criterio especifico por parte de las organizaciones.

Ante esto el objetivo de la presente investigación cobra relevancia, de tal forma que basados en la literatura y los resultados obtenidos se considera un acierto haber incluido operadores de agregación heavy y la metodología de los efectos olvidados en los modelos econométricos basados en fundamentales económicos, ya que permitieron disminuir el error de pronóstico para el caso del tipo de cambio USD/MXN para los años 2015, 2016 y 2017.

Uno de los principales hallazgos fue el desarrollo un nuevo operador de agregación que permitiera incluir la información histórica a través de una media móvil y adicionarla con las expectativas, conocimiento y experiencia del tomador de decisiones en el pronóstico de las variables exógenas.

El uso del operador HOWMA ayudó a identificar nuevos escenarios que las series de tiempo tradicionales no permitían al solamente incluir información histórica, esto permite identificar nuevas estrategias para mitigar el riesgo en base a los pronósticos obtenidos con los modelos econométricos basados en los fundamentales económicos, así lo demuestran los resultados. Por lo que resulta ser una herramienta que permite al tomador de decisiones contar con la flexibilidad de un vector de peso ante las expectativas individuales.

La aplicación de la metodología de los efectos olvidados y el uso de expertones fue útil para identificar que otras variables eran relevantes adicionales a los fundamentales económicos y pudieran adicionarse a los modelos econométricos, siendo la variable con mayor olvido el precio del petróleo.

Analizando los resultados obtenidos se puede afirmar la hipótesis de la presente investigación y dar respuesta a la pregunta central y específicas, ya que se identifica que el uso de operadores de agregación heavy y la metodología de los efectos olvidados permitieron mejorar el error de pronóstico en los modelos econométricos basados en fundamentales económicos, lo cual permitirá al tomador de decisión diseñar estrategias con una mayor certidumbre.

*Recomendaciones*

Una de las principales recomendaciones de la presente investigación es la realización del cálculo sistemático del pronóstico del tipo de cambio. Lo anterior, debido a que las proyecciones presentadas se realizaron solamente considerando el escenario de enero (inicio de año), por lo que realizar el proceso mensual, semanal o diario permitirá agregar información actualizada al modelo y generar nuevos escenarios basados en las realidades más recientes.

*Limitaciones*

La presente investigación tiene como limitación que los resultados que se obtuvieron son a través de la información de expertos y/o tomadores de decisión específicos, de tal forma que las matrices causa-efecto y los vectores de peso pueden tener variaciones derivadas de un cambio en los agentes que proporcionaron los datos.

Otra limitante es que en la presente investigación se formulan modelos econométricos solamente considerando las teorías clásicas basadas en fundamentales económicos, de tal forma que una inclusión de diferentes variables exógenas o el uso de otro tipo de modelos generaría resultados diferentes.

Las afirmaciones plasmadas en el documento son aplicables solamente para el tipo de cambio USD/MXN y para los años 2015, 2016 y 2017, por lo que no pudieran ser generalizadas para diferentes años o diversos tipos de cambio, de tal forma que los resultados son aplicables al caso concreto estudiado.

Para finalizar, es importante señalar que con los trabajos realizados se ha permitido ampliar el conocimiento y con ello impulsar la realización de nuevas investigaciones, las cuales permitan generar nuevas aplicaciones y operadores que permitan reducir la incertidumbre a la que se enfrentan los decisores en el mundo actual.

## 5.2 Futuras líneas de investigación

Durante el desarrollo de la presente tesis doctoral se han podido percibir diferentes futuras líneas de investigación, dentro de las cuales se encuentran:

a) El uso de operadores de agregación de información, específicamente el operador OWA, para el cálculo de la volatilidad en el tipo de cambio, fenómeno que ha sido relevante para los modelos de pronóstico y que tiene un impacto significativo para las proyecciones.

b) Dentro de la tesis se ha logrado recopilar información obtenida por diversos expertos durante los periodos de estudio, por lo que si se continúa alimentado dicha base de datos existe la posibilidad de realizar una red neuronal y así generar nuevos cálculos y modelos.

c) Una de técnicas utilizadas para el pronóstico se encuentran los modelos GARCH, los cuales pudieran expandir su formulación con la inclusión de operadores OWA o alguna de las extensiones existentes del mismo.

d) Existen fenómenos no considerados por los modelos econométricos que suelen causar movimientos importantes en la paridad cambiaria, de tal forma, que la adición de una variable dummy ante la aparición de dichas situaciones pudiera mejorar el pronóstico y dar una idea del impacto de dicho evento.

e) En el caso específico de Sinaloa, la agricultura juega un rol importante dentro de las actividades económicas estatales, de tal manera, que el uso de modelos de pronóstico para el tipo de cambio y el valor futuro de la tonelada del maíz permitiría generar una estrategia de disminución de riesgo que permitiera aumentar la rentabilidad del sector.

f) Finalmente, la aplicación de operadores OWA y generación de nuevas extensiones en diferentes áreas administrativas y financieras, que permitan incluir dentro de los procesos el conocimiento, expectativa y experiencia de los tomadores de decisión a las formulaciones actuales.

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# Anexos

Anexo 1.
Test de significancia para el modelo PPP

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| **Constante** | 0.131 | 0.421 | 0.668 |
| $e_{t-1}$ | 0.000 | 0.000 | 0.000 |
| $v$ | 0.000 | 0.000 | 0.000 |
| $pci_{EUA}$ | 0.625 | 0.784 | 0.458 |
| $pci_{MEX}$ | 0.066 | 0.238 | 0.491 |

Fuente. Elaboración propia.

Anexo 2.
Test de significancia para el modelo IRP

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| **Constante** | 0.000 | 0.000 | 0.000 |
| $e_{t-1}$ | 0.000 | 0.000 | 0.000 |
| $v$ | 0.000 | 0.000 | 0.000 |
| $r_{EUA}$ | 0.917 | 0.484 | 0.628 |
| $r_{MEX}$ | 0.627 | 0.917 | 0.669 |

Fuente. Elaboración propia.

Anexo 3.
Test de significancia para el modelo BoP

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| **Constante** | 0.066 | 0.119 | 0.074 |
| $E_{t-1}$ | 0.000 | 0.000 | 0.000 |
| $V$ | 0.000 | 0.000 | 0.000 |
| $BT$ | 0.052 | 0.009 | 0.031 |
| $FDI$ | 0.000 | 0.003 | 0.000 |
| $FPI$ | 0.020 | 0.058 | 0.021 |
| $R$ | 0.001 | 0.001 | 0.000 |

Fuente. Elaboración propia.

Anexo 4.
Test de significancia para los modelos completos

| Modelo | 2015 | 2016 | 2017 |
|---|---|---|---|
| **PPP** | 0.000 | 0.000 | 0.000 |
| **IRP** | 0.000 | 0.000 | 0.000 |
| **BoP** | 0.000 | 0.000 | 0.000 |

Fuente. Elaboración propia.

Anexo 5.
VIF para el modelo PPP

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| $e_{t-1}$ | 12.42 | 12.07 | 12.42 |
| $v$ | 1.07 | 1.08 | 11.45 |
| $pci_{EUA}$ | 8.97 | 8.82 | 5.49 |
| $pci_{MEX}$ | 28.28 | 26.59 | 5.45 |

Fuente. Elaboración propia.

Annex 6.
VIF para el modelo IRP

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| $e_{t-1}$ | 1.85 | 2.01 | 1.96 |
| $v$ | 1.15 | 1.14 | 2.70 |
| $r_{EUA}$ | 3.22 | 3.26 | 4.78 |
| $r_{MEX}$ | 3.01 | 3.26 | 3.29 |

Fuente. Elaboración propia.

Anexo 7.
VIF para el modelo BoP

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| $e_{t-1}$ | 2.68 | 3.12 | 3.28 |
| $V$ | 1.03 | 1.03 | 1.03 |
| $BT$ | 1.02 | 1.05 | 1.04 |
| $FDI$ | 2.17 | 2.10 | 1.93 |
| $FPI$ | 1.39 | 1.42 | 1.36 |
| $R$ | 3.64 | 4.14 | 4.38 |

Fuente. Elaboración propia.

Anexo 8.
Test Durbin-Watson para todos los modelos

| Modelo | 2015 | 2016 | 2017 |
|---|---|---|---|
| **PPP** | 1.9071 | 1.8910 | 1.9062 |
| **IRP** | 1.9356 | 1.9159 | 1.9288 |
| **BoP** | 1.9137 | 1.9214 | 1.8895 |

Fuente. Elaboración propia.

Anexo 9.
Tipo de tendencia y formula para las variables[23]

| Variable | 2015 | 2016 | 2017 |
|---|---|---|---|
| $e_{t-1}$ | $6.3461 * 1.00352^t$ | $6.3450 * 1.00352^t$ | $6.3138 * 1.00358^t$ |
| $V$ | $0.007 * 1.00125^t$ | $0.0074 * 1.00110^t$ | $0.0073 * 1.00135^t$ |
| $pci_{EUA}$ | $147.085 * 1.00201^t$ | $147.937 * 1.00197^t$ | $148.453 * 1.00193^t$ |
| $pci_{MEX}$ | $31.0370 * 1.00601^t$ | $32.0835 * 1.00574^t$ | $32.6300 * 1.00555^t$ |
| $r_{EUA}$ | $16.359 * 0.978471^t$ | $17.216 * 0.977784^t$ | $14.810 * 0.979421^t$ |
| $r_{MEX}$ | $29.3549 * 0.990829^t$ | $29.3015 * 0.990834^t$ | $28.2408 * 0.991239^t$ |
| $BT$ | $-17 - 10.13t + 0.0402t^2$ | $-126 - 6.22t + 0.0204t^2$ | $-192 - 4.26t + 0.01122t^2$ |
| $FPI$ | $-157 - 17.7t + 0.2128t^2$ | $-854 + 0.8t + 0.1305t^2$ | $-1159 + 9.8t + 0.0883t^2$ |
| $FDI$ | $2840.4 * 1.00396^t$ | $2841.02 * 1.00399^t$ | $2881.36 * 1.00383^t$ |
| $R$ | $13.6034 * 1.01074^t$ | $13781.2 * 1.01066^t$ | $13974.2 * 1.01050^t$ |

Fuente. Elaboración propia.

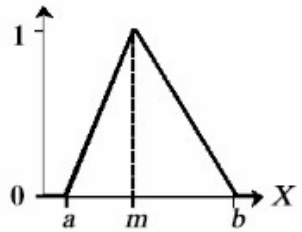
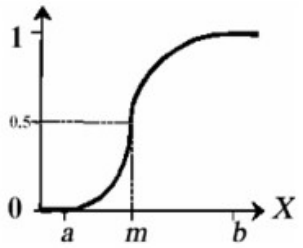
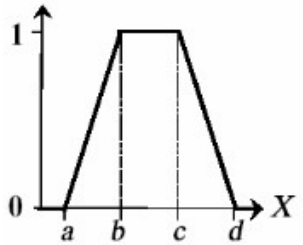
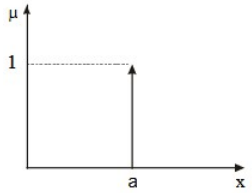
[23] En este anexo, la formula de tendencia para cada variable es diferente para cada año. Se identifica que la mayoría de las fórmulas de tendencia son exponenciales y algunas otras cuadráticas. Lo anterior porque cuando las variables tienen datos positivos y negativos la formula exponencial no puede realizarse.

# Apéndices

Apéndice 1. Tipos y operaciones básicos de los conjuntos borrosos

De acuerdo con Kilir & Yuan (1995) y Zimmermann (2011), los tipos básicos de conjuntos borrosos y las características de su función de pertenencia son.

Tabla A1. Tipos básicos de conjuntos borrosos

| **Tipo de conjunto borroso y características de la función de pertenencia** | **Representación grafica** |
|---|---|
| Forma triangular $$A(x)=\begin{cases} 0 & si \quad x \le a \\ (x-a)/(m-a) & si \quad x \in (a,m] \\ (b-x)/(b-m) & si \quad x \in (m,b) \\ 1 & si \quad x \ge b \end{cases}$$ |   |
| Forma S $$A(x)=\begin{cases} 0 & si \quad x \le a \\ 2\{(x-a)/(b-a)\}^2 & si \quad x \in (a,m] \\ 1-2\{(x-a)/(b-a)\}^2 & si \quad x \in (m,b) \\ 1 & si \quad x \ge b \end{cases}$$ |   |
| Forma trapezoidal $$A(x)=\begin{cases} 0 & si \quad (x \le a) o (x \ge d) \\ (x-a)/(b-a) & si \quad x \in (a,b] \\ 1 & si \quad x \in (b,c) \\ (d-x)/(d-c) & si \quad x \in (b,d) \end{cases}$$ |   |
| Forma singleton $$A(x)=\begin{cases} 1 & x = a \\ 0 & x \ne a \end{cases}$$ |   |

Fuente. Elaboración propia con información de Kiliar & Yuan (1995).

1.2 Operaciones básicas

Supóngase dos conjuntos borrosos *A y B* en un universo *U*, siendo $\{u\}$ un elemento genérico de *U* con una función de pertenencia $\mu_A$ y $\mu_B$ respectivamente, las operaciones básicas son (Lee, 1990; Dick *et al.,* 2016).

a) Unión. La función de pertenencia $\mu_A \cup \mu_B$ de la unión $A \cup B$ es definido por todos los puntos $u \in U$ por

$$\mu_{A\cup B}(u) = max\{\mu_A(u), \mu_B(u)\}.$$

b) Intersección. La función de pertenencia $\mu_A \cap \mu_B$ de la intersección $A \cap B$ es definida para todos los puntos en $u \in U$ por

$$\mu_{A\cap B}(u) = min\{\mu_A(u), \mu_B(u)\}.$$

c) Complemento. La función de pertenencia $\mu_A$ como complemento de un conjunto borroso *A* es definido para todos los puntos en $u \in U$ por

$$\mu_{\bar{A}}(u) = 1 - \mu_A(u).$$

d) Producto cartesiano. Si $A_1, \dots, A_n$ son conjuntos borrosos de $U_1, \dots, U_n$, respectivamente, el producto cartesiano de $A_1, \dots, A_n$ es un conjunto borroso en el producto del espacio $U_1 \times \dots \times U_n$ con una función de pertenencia

$$\mu_{A_1 \mathrm{X} \dots \mathrm{X} A_n}(u_1, \dots, u_n) = min\{\mu_{A_1}(u_1), \dots, \mu_{A_n}(u_n)\}$$

o

$$\mu_{A_1 \times \dots \times A_n}(u_1, \dots, u_n) = \mu_{A_1}(u_1) \cdot \mu_{A_2}(u_2), \dots, \mu_{A_n}(u_n)$$

Apéndice 2.

# Induced Heavy Moving Averages

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This paper presents the induced heavy ordered weighted moving average (IHOWMA) operator. It is an aggregation operator that uses the main characteristics of three well-known techniques: the moving average, induced operator, and heavy aggregation operator. This operator provides a parameterized family of aggregation operators that include the minimum, the maximum, and total operator as special cases. It can be used in a selection process, considering that not all decision makers have the same knowledge and expectations of the future. The main properties of this operator are studied including a wide range of families of IHOWMA operators, such as the heavy ordered weighted moving average operator and uncertain induced heavy ordered weighted moving average operator. The IHOWMA operator is also extended using generalized and quasi-arithmetic means. An example in an investment selection process is also presented. © 2017 Wiley Periodicals, Inc.

## 1. INTRODUCTION

In problems related to the economy, which has high uncertainty and volatility, inclusion of knowledge and expectations of the decision maker about future scenarios is an important way to reduce risk related to the decision. A common aggregation method is the ordered weighted averaging (OWA) operator introduced by Yager.[1] The OWA operator has been used in many applications[2–4] and extended under a wide range of frameworks.[5,6]

This paper focuses on the induced OWA (IOWA), heavy OWA (HOWA), and moving average (MA). The IOWA operator was introduced by Yager and Filev.[7] In this operator, the reordering step is not developed with values of the arguments. Instead it is induced by another mechanism such that the ordered position of arguments depends upon values of their associated order-inducing variables.[22] This operator was extended by using fuzzy numbers,[8] intuitionistic fuzzy information,[9] and generalized and quasi-arithmetic means.[10]

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The HOWA operator[11] is an operator that allows the weighting vector to range from 1 to $\infty$ or even from $-\infty$ to $\infty$. With this characteristic, it is possible to add expectations and knowledge of the decision maker to the aggregation information process. This operator has been studied using fuzzy measures[12] and fuzzy numbers.[13] Extensions have been developed by Merigó et al.[14]

Moving average is an average of some part of the whole sample. With this technique, it is possible to consider historical data to forecast a future value. Making comparisons when modifying the sample is also possible. This method is usually used to solve time series smoothing problems and is very common in economics and statistics.[15]

This paper analyzes the use of moving averages with induced heavy aggregation operators in a selection process. The main advantage of this operator is that it unifies historical data with expectations of the future and knowledge of the decision maker, leading to improvement of the decision process. This operator is called induced heavy ordered weighted moving average (IHOWMA).

The main concepts of this new extension have been developed, along with a range of particular cases including the heavy ordered weighted moving average (HOWMA) operator and the uncertain induced heavy ordered weighted moving average (UIHOWMA) operator, which is used in problems with interval numbers.

A generalization of the IHOWMA operator is presented using generalized and quasi arithmetic means.[10,16,17] The main advantage of this operator is that the same formulation includes arithmetic, geometric, or quadratic aggregations. Several extensions using different types of interval and fuzzy numbers were suggested by Merigó and Casanovas.[16,17]

An application of the new approach in an investment selection process is developed. We use percent of profit information from three types of investment from February 2016 to April 2016. In the problem, we identify interval numbers, which is why the UIHOWMA operator is used to select the most profitable option considering characteristics of the decision maker.

The remainder of the paper is organized as follows. In Section 2, we review moving averages and some aggregation operators. Section 3 introduces the IHOWMA operator, and Section 4 develops the generalized IHOWMA operator. Section 5 explains the steps to use induced heavy aggregation operators in a selection process, and Section 6 presents the use of the IHOWMA operator in an investment selection process. Section 7 summarizes the main conclusions of the paper.

## 2. PRELIMINARIES

In this section, we briefly review some basic concepts that will be used throughout the paper. We analyze the interval numbers, OWA operator, UOWA operator, heavy aggregation operators, induced aggregation operators, moving averages, and generalized aggregation operators.

### 2.1. Interval Numbers

Interval numbers, introduced by Moore,[18] is a technique that represents uncertainty in a simple way. They can be expressed in different forms. For example, assume a triplet $(a_1, a_2, a_3)$, where $a_1$ and $a_3$ are considered the minimum and the maximum of the interval and $a_2$ is the value with the highest probability or possibility. In the following sections, we define uncertain aggregation operators as those operators that use interval numbers in the analysis.

To rank alternatives using interval numbers, we establish criteria as follows:[17,19]

(a) For doublet, calculate the arithmetic mean of the interval: $(c_1 + c_2)/2$.
(b) For triples and more, calculate a weighted average that gives more importance to central values, that is, $(c_1 + 2c_2 + c_3)/4$.
(c) For 4-tuples we could calculate: $(c_1 + 2c_2 + 2c_3 + c_4)/6$.
(d) And so on.

### 2.2. OWA Operator

The OWA operator was introduced by Yager,[1] and it is an aggregation operator that has many applications.[2,16,19] It is defined as follows:

DEFINITION 1. *An OWA operator of dimension n is a mapping* $OWA : R^n \to R$ *with an associated weight vector W of dimension n such that* $\sum_{j=1}^{n} w_j = 1$ *and* $w_j \in [0, 1]$, *according to the following formula:*

$$OWA\ (a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

*where* $b_j$ *is the jth largest element of the collection* $a_i$.

It is possible to distinguish descending OWA (DOWA) and ascending (AOWA) operators. The difference lies in weights that are related by $w_j = w^*_{n-j+1}$, where $w_j$ is the $j$th weight of the DOWA operator and $w^*_{n-j+1}$ the $j$th weight of the AOWA operator.

### 2.3. Uncertain OWA Operator

The uncertain OWA (UOWA) operator was introduced by Xu and Da.[20] The main characteristic of this operator is uncertainty; this can be included in the operator by using interval numbers. It can be defined as follows:

DEFINITION 2. *Let* $\Omega$ *be the set of interval numbers. An UOWA operator of dimension n is a mapping* $UOWA : \Omega^n \to \Omega$ *that has an associated weighting vector W of dimension n such that* $w_j \in [0, 1]$ *and* $\sum_{j=1}^{n} w_j = 1$, *according to the following*

*formula:*

$$UOWA\ (\tilde{a}_1, \tilde{a}_2, .., \tilde{a}_3) = \sum_{j=1}^{n} w_j \tilde{b}_j, \tag{2}$$

*where $\tilde{b}_j$ is the jth largest of the $\tilde{a}_i$ and the $\tilde{a}_i$ are interval numbers.*

As in usual interval numbers, we can distinguish the uncertain maximum, minimum, and average. Additionally, as in the case of the OWA operator, we have descending UOWA and ascending UOWA operators.

### 2.4. Heavy Aggregation Operators (HOWA, UHOWA, and HWA)

The HOWA operator[11] is an extension of the traditional OWA, where the main characteristic of this new operator is in the weight vector, which is not bounded by the sum of 1, but instead bounded according to available information and expectations and knowledge of the decision maker. It can vary from 1 to $n$ or even from $-\infty$ to $\infty$. This operator can be defined as follows:

DEFINITION 3. *A HOWA operator is a mapping HOWA : $R^n \rightarrow R$ which is associated with a weight vector w, where $w_j \in [0, 1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, such that*

$$HOWA\ (a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j\ , \tag{3}$$

*where $b_j$ is the jth largest element of the collection $a_i$.*

The characteristics of the HOWA operator are: monotonic and commutative, but it is not bounded by the minimum and the maximum operators. Additionally, similar to OWA, we can distinguish between descending HOWA and ascending HOWA operators.

With the possibility to expand the weighting vector from $-\infty$ to $\infty$, we can drastically under- or overestimate the results of the HOWA operator, considering new scenarios according to information of the decision maker and some expectations of the future of the case in study.

Yager[11] introduced a characteristic for the HOWA operator, that is related to the weighting vector and noted that this vector does not depend on reordering of the argument and introduced the beta value of the vector $W$. This beta value can be defined as $\beta\ (W) = (|W| - 1)/(n - 1)$. Note that if $\beta = 1$ we obtain the total operator and if $\beta = 0$ we obtain the usual OWA operator.

It is possible to use interval numbers in arguments of the HOWA operator, resulting in the uncertain heavy OWA (UHOWA) operator. Merigó and Casanovas[21] define it as follows:

DEFINITION 4. *Let $\Omega$ be the set of interval numbers. An UHOWA operator of dimension n is a mapping UHOWA : $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W*

*of dimension n such that* $w_j \in [0, 1]$ *and* $1 \leq \sum_{j=1}^{n} w_j \leq n$, *then*

$$UHOWA\ (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j \tilde{b}_j\ . \tag{4}$$

Note that the HOWA operator can convert to a heavy moving average (HWA) if $w_j = 1/n$. Merigó and Casanovas[17] define it as follows:

DEFINITION 5. *A HWA operator of dimension n is a mapping* $HWA : R^n \to R$ *that has an associated weighting vector W of dimension n with* $w_j \in [0, 1]$ *and* $1 \leq \sum_{i=1}^{n} w_i \leq n$, *such that*

$$HWA\ (a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i a_i\ , \tag{5}$$

*where* $a_i$ *is the ith argument of the aggregation. Also note that it is possible to expand the weighting vector from* $-\infty$ *to* $\infty$.

### 2.5. Induced Aggregation Operators

The IOWA operator was introduced by Yager and Filev[7] as an extension of the OWA operator. Its main difference is that the reordering step is not developed with values of the arguments $a_i$. In this case, the reordering step is developed with order inducing variables. The IOWA operator can be defined as follows.

DEFINITION 6. *An IOWA operator of dimension n is an application* $IOWA : R^n \times R^n \to R$ *that has a weighting vector associated, W of dimension n where the sum of the weights is 1 and* $w_j \in [0, 1]$, *where an induced set of ordering variables is included* $(u_i)$ *such that the formula is*

$$IOWA\ (\langle u_1, a_1\rangle, \langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j, \tag{6}$$

*where* $b_j$ *is the* $a_i$ *value of the OWA pair* $\langle u_i, a_i\rangle$ *having the jth largest* $u_i$. $u_i$ *is the order inducing variable, and* $a_i$ *is the argument variable.*

From a generalized perspective of the reordering step, we can distinguish between the descending IOWA (DIOWA) operator and the ascending IOWA (AIOWA) operator. The weight of these operators is related to $w_j = w^*_{n-j+1}$, where $w_j$ is the $j$th weight of the DIOWA and $w^*_{n-j+1}$ the $j$th weight of the AIOWA operator.

The IOWA operator is an averaging operator. This is reflected by the fact that the operator is monotonic, commutative, bounded, and idempotent, both for the DIOWA and the AIOWA operator. Note that the OWA operator is obtained when $u_i = a_i$, for all $i$.[7,22]

### 2.6. Moving Averages

A moving average is a usual average that moves toward some part of the sample. This method has been used in economics and statistics to solve time series smoothing problems.[15] The moving average, according to Kenney and Keeping,[23] can be defined as follows:

DEFINITION 7. *Moving averages are defined as a sequence given* $\{a_i\}_{i=1}^{N}$, *where a moving average n is a new sequence* $\{s_i\}_{i=1}^{N-n+1}$ *defined from* $a_i$ *taking the arithmetic mean of the sequence of n terms, such that*

$$s_i = \frac{1}{n} \sum_{j=i}^{i+n-1} a_j, \tag{7}$$

The usual moving average can be extended by using weighted averages, obtaining the weighted moving average (WMA). Merigó and Yager[24] defined it as follows:

DEFINITION 8. *A WMA of dimension m is a mapping* $WMA : R^m \to R$ *that has an associated weighting vector W of dimension m with* $W = \sum_{i=1+t}^{m+t} w_i = 1$ *and* $w_i \in [0, 1]$, *such that*

$$WMA\ (a_{1+t}, a_{2+t}, \ldots, a_{m+t}) = \sum_{i=1+t}^{m+t} w_i a_i\ , \tag{8}$$

*where* $a_i$ *is the ith argument, m is the total number of arguments considered from the whole sample, and t indicates the movement performed in the average from the initial analysis. Note that if* $w_i = 1/m$ *for all i, the WMA becomes the MA aggregation.*

The moving average can also be combined with the OWA operator generating the ordered weighted moving average (OWMA). Merigó and Yager[24] defined it as follows:

DEFINITION 9. *An OWMA of dimension m is a mapping* $OWMA : R^m \to R$ *that has an associated weighting vector W of dimension m with* $W = \sum_{j=1+t}^{m+t} w_j = 1$ *and* $w_j \in [0, 1]$, *such that*

$$OWMA\ (a_{1+t}, a_{2+t}, \ldots, a_{m+t}) = \sum_{j=1+t}^{m+t} w_j b_j\ , \tag{9}$$

*where* $b_j$ *is the jth largest argument of the* $a_i$, *m is the total number of arguments considered from the whole sample, and t indicates the movement in the average from the initial analysis.*

The OWMA can also be extended using the IOWA operator and its characteristic of an induced weighted vector resulting in the IOWMA operator. Merigó and Yager[24] defined it as follows:

DEFINITION 10. *An IOWMA of dimension m is a mapping IOWMA* : $R^M \times R^M \to R$ *that has an associated weighting vector W of dimension m with* $W = \sum_{j=1+t}^{m+t} w_j = 1$ *and* $w_j \in [0, 1]$, *such that*

$$IOWMA\ (\langle u_{1+t}, a_{1+t}\rangle, \langle u_{2+t}, a_{2+t}\rangle, \ldots, \langle u_{m+t}, a_{m+t}\rangle) = \sum_{j=1+t}^{m+t} w_j b_j, \qquad (10)$$

*where* $b_j$ *is the* $a_i$ *value of the IOWMA pair* $u_i, a_i$ *having the jth largest* $u_i$, $u_i$ *is the order inducing variable,* $a_i$ *is the argument variable, m is the total number of arguments considered from the whole sample, and t indicates movement in the average from the initial analysis.*

### 2.7. Generalized Aggregation Operators

Generalized aggregation operators are those that provide a general formulation, including a wide range of particular cases, such as the generalized and quasi-arithmetic mean. Note that we focus on the quasi-arithmetic mean because it includes the generalized as a particular case.[10,25,26] The weighted quasi-arithmetic mean (Quasi-WA) is defined by Merigó and Yager[24] as follows.

DEFINITION 11. *A Quasi-WA operator of dimension n is a mapping* $Quasi - WA$ : $R^n \to R$ *that has an associated weighting vector W of dimension n with* $\sum_{i=1}^{n} w_i = 1$ *and* $w_i \in [0, 1]$, *such that*

$$Quasi - WA\ (a_1, a_2, \ldots, a_n) = g^{-1}\left(\sum_{i=1}^{n} w_i g(a_i)\right), \qquad (11)$$

*where g(b) is a strictly continuous monotone function. Note that if* $w_i = 1/n$ *for all i, the QWA becomes the simple quasi-arithmetic mean. Moreover, if* $g(a) = a^{\lambda}$, *the QWA becomes the weighted generalized mean.*

The QWA operator can be combined with the OWA operator to obtain the ordered weighted quasi-arithmetic mean (Quasi-OWA). This operator is defined by Fodor et al.[27] as follows:.

DEFINITION 12. *A Quasi-OWA operator of dimension n is a mapping* $Quasi - OWA$ : $R^n \to R$ *that has an associated weighting vector W of dimension n with* $\sum_{j=1}^{n} g(b_j) = 1$ *and* $w_j \in [0, 1]$, *and such that*

$$Quasi - OWA\ (a_1, a_2, \ldots, a_3) = g^{-1}\left(\sum_{j=1}^{n} w_j g(b_j)\right), \qquad (12)$$

*where $b_j$ is the jth largest of the $a_i$ and $g(b)$ is a strictly continuous monotone function.*

Note that if we induce the reordering step of the weighting vector we obtain the Quasi-IOWA operator. Merigó and Gil-Lafuente[10] defined it as follows:

DEFINITION 13. *A Quasi-IOWA operator of dimension n is a mapping $Quasi - IOWA : R^n x\ R^n \rightarrow R$ that has an associated weighting vector W of dimension n with $w_j \in [0, 1]$ and $W = \sum_{j=1}^{n} w_j = 1$, such that*

$$Quasi - IOWA\left(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle\right) = g^{-1}\left(\sum_{j=1}^{n} w_j g\left(b_j\right)\right), \tag{13}$$

*where $b_j$ is the $a_i$ value of the Quasi-IOWA pair $u_i, a_i$ having the jth largest $u_i$, $u_i$ is the order inducing variable, $a_i$ is the argument, and $g(b)$ is a strictly continuous monotonic function.*

Also the IOWMA operator can be generalized, and similar to the other operators the Quasi-IOWMA is presented because it includes the generalized IOWMA as a particular case.

DEFINITION 14. *An induced order weighted quasi-arithmetic moving average (Quasi-IOWMA) of dimension m is a mapping $Quasi - IOWMA : R^n \times\ R^n \rightarrow R$ that has an associated weighting vector W of dimension m with $W = \sum_{j=1+t}^{m+t} w_j = 1$ and $w_j \in [0, 1]$, such that*

$$Quasi - IOWMA\left(\langle u_{1+t}, a_{1+t}\rangle, \ldots, \langle u_{n+t}, a_{n+t}\rangle\right) = g^{-1}\left(\sum_{j=1+t}^{m+t} w_j g\left(b_j\right)\right), \tag{14}$$

*where $b_j$ is the $a_i$ value of the IOWMA pair $u_i, a_i$ having the jth largest $u_i$, $u_i$ is the order inducing variable, $a_i$ is the argument variable, m is the total number of arguments considered from the whole sample, t indicates the movement in the average from the initial analysis, and $g(b)$ is a strictly continuous monotone function.*

It is important to note that if $w_i = 1/m$ for all $i$, the Quasi-IOWMA operator becomes the Quasi-MA operator. Furthermore, if $g\ (a) = a^{\lambda}$, the Quasi-IOWMA becomes the generalized weighted moving average.

## 3. INDUCED HEAVY MOVING AVERAGES

### 3.1. Theoretical Foundations

The IHOWMA operator is an extension of the OWA operator and combines characteristics of the IOWA operator and the HOWA operator with the moving

average. This operator uses a moving average with an associated weighting vector that ranges from $-\infty$ to $\infty$; it also uses order-inducing variables in the reordering of information. It can be defined as follows.

DEFINITION 15. *A IHOWMA operator is defined as a given sequence* $\{a_i\}_{i=1}^{N}$, *where a new sequence* $\{s_i\}_{i=1}^{N-m+1}$ *is multiplied by a heavy weighting vector, such that*

$$IHOWMA\ (\langle u_{1+t}, a_{1+t}\rangle, \langle u_{2+t}, a_{2+t}\rangle, \ldots, \langle u_{n+t}, a_{m+t}\rangle) = \sum_{j=1+t}^{m+t} w_j b_j\ , \quad (15)$$

*where* $b_j$ *is jth element that has the largest value of* $u_i$, $u_i$ *is the order inducing variable, and W is an associated weighting vector of dimension m with* $W : 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ *and* $w_i \in [0, 1]$. *Observe that we can also expand the weighting vector from* $-\infty$ *to* $\infty$. *Thus, the weighting vector w becomes* $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

Note that by allowing the weighting vector to range from $-\infty$ to $\infty$ it is possible to under- or overestimate the result based on the information available and knowledge of the decision maker. Because of the reordering step, similar to the OWA operator, it is possible to distinguish between descending and ascending IHOWMA operators.

The characteristics of the IHOWMA operator are as follows:

(a) It is monotonic because if $a_i \geq d_i$, for all $i$, then $IHOWMA\ (a_1, \ldots, a_n) \geq IHOWMA(d_1, \ldots, d_n)$.
(b) It is commutative because any permutation of the argument has the same evaluation.
(c) It can be bounded if the weight vector ranges from 1 to $\infty$, and it can consider that if the weight vector ranges from $-\infty$ to $\infty$, the IHOWMA operator is not bounded.
(d) The characteristic of the beta value explained in Definition 3 (HOWA operator) also applies to the IHOWMA operator.

When a problem is based on interval numbers, the uncertain IHOWMA (UI-HOWMA) operator is obtained. This operator can be defined as follows:

DEFINITION 16. *Let* $\Omega$ *be the set of interval numbers. An UIHOWMA operator of dimension n is a mapping* $UIHOWMA : \Omega^n \times \Omega^n \to \Omega$ *that acts on the sequence* $\{s_i\}_{i=1}^{N-n+1}$, *which is multiplied by a heavy weighting vector, according to*

$$UIHOWA\ (\langle\langle u_{1+t}, \tilde{a}_{1+t}\rangle, \langle u_{2+t}, \tilde{a}_{2+t}\rangle, \ldots, \langle u_{n+t}, \tilde{a}_{m+t}\rangle) = \sum_{j=1+t}^{m+t} w_j \tilde{b}_j\ , \quad (16)$$

*where* $\tilde{b}_j$ *is the jth element that has the largest value* $u_i$, $u_i$ *is the order inducing variable,* $\tilde{a}_i$ *are interval numbers, and W is an associated weighting vector of dimension m with* $W : 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ *and* $w_i \in [0, 1]$.

Note that if the weighting vector is not induced, then the IHOWMA operator becomes the HOWMA operator, such that

DEFINITION 17. *A HOWMA operator is defined as a given sequence* $\{a_i\}_{i=1}^{N}$, *where a new sequence* $\{s_i\}_{i=1}^{N-n+1}$ *is multiplied by a heavy weighting vector, such that*

$$HOWMA\ (s_i) = \sum_{j=1+t}^{m+t} w_j b_j\ , \tag{17}$$

*where* $b_j$ *is the jth largest element of the collection* $a_1, a_2, \ldots, a_n$ *and W is an associated weighting vector of dimension m with* $W : 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ *and* $w_i \in [0, 1]$. *Observe that we can also expand the weighting vector from* $-\infty$ *to* $\infty$. *Thus, the weighting vector w becomes* $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

### 3.2. Families of the IHOWMA Operator

In this section, different families of the IHOWMA operator are presented. We demonstrate a wide range of particular cases that can be used in the IHOWMA operator, leading to different results. These particular cases are found by analyzing the coefficient $\beta$, so a wide range of particular cases are presented by giving different values and interpretations of the $\beta$ value. By choosing different values for the weight vector, we obtain different types of aggregation operators. Some of the families of the IHOWMA operator are as follows:

(a) If $\beta = 0$ the IHOWMA operator becomes the IOWMA operator.
(b) When $w_n = 1$, $w_j = 0$, for all $j \neq n$ and $\beta = 0$, we obtain the minimum IOWMA. If $w_1 = 1$, $w_j = 0$, for all $j \neq n$ and $\beta = 0$, we obtain the maximum IOWMA. Finally, the total operator is obtained when $\beta = 1$.
(c) The IOWMA operator becomes the MA aggregation when $w_i = 1/m$ for all $i$.
(d) The arithmetic mean IHOWMA: If $w_j = 1/n$ for all $j$ and $p_i = 1/n$ for all $i$.
(e) The median IHOWMA: if $n$ is odd we assign $w_{(n+1)/2} = 1$ and $w_{j*} = 0$ for all others. If $n$ is even we assign, $w_{n/2} = w_{(n/2)+1} = 0.5$ and $w_{j*} = 0$ for all others.
(f) *The weighted median IHOWMA*: Select the argument $b_k$ that has the $k$th largest argument such that the sum of the weights from 1 to $k$ is equal to or higher than 0.5 and the sum of the weights from 1 to $k-1$ is less than 0.5.
(g) *The general olympic-IHOWMA operator*: if $w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $w_{j*} = 1/(n-2k)$, where $k < n/2$.
(h) *The centered-IHOWMA*: if it is symmetric, it strongly decays from the center to the maximum and the minimum, and it is inclusive.

To understand this approach, we present a simple numerical example of the UIHOWMA operator.

*Example* 1. Assume the following income for three different companies shown in Table I in an aggregation process:

Assume an ordered inducing variable $U = (C_1 = (10, 15, 5)\ ,\ C_2 = (5, 10, 15),\ C_3 = (15, 10, 5))$, and the following weighting vector $W = (0.3, 0.3, 0.4)$ . If we want to forecast the income for April based on the information below using the IHOWMA operator, the results will be

$$C_1 = (0.3 * 70{,}000) + (0.3 * 80{,}000) + (0.4 * 83{,}000) = 78{,}200$$

**Table I.** Available data

| | January | February | March |
|---|---|---|---|
| $C_1$ | 80,000 | 90,000 | 87,000 |
| $C_2$ | 70,000 | 85,000 | 91,000 |
| $C_2$ | 83,000 | 92,000 | 86,000 |

$$C_2 = (0.3 * 92{,}000) + (0.3 * 85{,}000) + (0.4 * 90{,}000) = 89{,}100$$

$$C_3 = (0.3 * 87{,}000) + (0.3 * 91{,}000) + (0.4 * 86{,}000) = 87{,}800$$

We see from this information that the company with better forecast income will be $C_2 > C_3 > C_1$.

## 4. GENERALIZED IHOWA OPERATOR

The IHOWMA operator provides a wide range of cases using generalized and quasi-arithmetic means.[24,28,29] In this section, we present the Quasi-HOWMA operator and the Quasi-IHOWMA operator because they include the generalized mean as a particular case.

DEFINITION 18. *A Quasi-HOWMA operator of dimension n is a mapping HOWMA : $R^n \to R$ that has an associated weighting vector W of dimension n that is defined by a given sequence $\{a_i\}_{i=1}^{N}$, where you obtain a new sequence $\{s_i\}_{i=1}^{N-n+1}$, which is multiplied by a heavy weighting vector, such that*

$$Quasi - HOWMA\ (s_i) = g^{-1} \sum_{j=1+t}^{m+t} w_j g\left(b_j\right), \tag{18}$$

*where $g(a_i)$ is a strictly continuous monotonic function and $b_j$ is the jth largest element of the collection $a_1, a_2, \ldots, a_n$, W is an associated weighting vector of dimension m with $W : 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ and $w_i \in [0, 1]$, and $g(b_j)$ is a strictly continuous monotone function.*

DEFINITION 19. *A Quasi-IHOWMA operator is defined as a given sequence $\{a_i\}_{i=1}^{N}$, where you obtain a new sequence $\{s_i\}_{i=1}^{N-m+1}$ that is multiplied by a heavy weighting vector, such that*

$$\begin{aligned} &Quasi - IHOWMA\left(\langle u_1, a_{1+t}\rangle, \langle u_2, a_{2+t}\rangle, \ldots, \langle u_n, a_{m+t}\rangle\right) \\ &= g^{-1} \sum_{j=1+t}^{m+t} w_j g\left(b_j\right), \end{aligned} \tag{19}$$

*where $g(b_j)$ is a continuous monotonic function and $b_j$ is jth element that has the largest value of $u_i$, $u_i$ is the order inducing variable, and W is an associated*

**Table II.** Families of generalized IHOWMA operators

| Particular case | Quasi-IHOWMA | Quasi-HOWMA |
|---|---|---|
| $u_i = \frac{1}{n}$, for all $i$ | Quasi-arithmetic heavy moving average (Quasi-HOWMA) | Quasi-arithmetic heavy moving average (Quasi-HMA) |
| $g(b) = b^{\lambda}$ | Generalized IHOWMA | Generalized HOWMA |
| $g(b) = b$ | IHOWMA | HOWMA |
| $g(b) = b^2$ | Heavy ordered weighted moving quadratic average (IHOWMQA) | Heavy ordered weighted moving quadratic average (HOWMQA) |
| $g(b) \to b^{\lambda}$, $for\ \lambda \to 0$ | Heavy ordered weighted moving geometric average (IHOWMGA) | Heavy ordered weighted moving geometric average (HOWMGA) |
| $g(b) = b^{-1}$ | Heavy ordered weighted moving harmonic average (IHOWMHA) | Heavy ordered weighted moving harmonic average (HOWMHA) |
| $g(b) = b^3$ | Heavy ordered weighted moving cubic average (IHOWMCA) | Heavy ordered weighted moving cubic average (HOWMCA) |
| $g(b) \to b^{\lambda}$, for $\lambda \to \infty$ | Heavy maximum | Heavy maximum |
| $g(b) \to b^{\lambda}$, for $\lambda \to \infty$ | Heavy minimum | Heavy minimum |

*weighting vector of dimension m with* $W : 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ *and* $w_i \in [0, 1]$. *Observe that we can also expand the weighting vector from* $-\infty$ *to* $\infty$. *Thus, the weighting vector w becomes* $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

Note that if interval numbers are used, the Quasi-UIHOWMA operator is obtained. It can be defined as follows:

DEFINITION 20. *Let* $\Omega$ *be the set of interval numbers. A Quasi-UIHOWMA operator of dimension n is a mapping* $Quasi - UIHOWMA : \Omega^n \times \Omega^n \to \Omega$ *that acts on the sequence* $\{s_i\}_{i=1}^{N-n+1}$, *which is multiplied by a heavy weighting vector, according to*

$$Quasi - IHOWA(\langle(u_{1+t}, \tilde{a}_{1+t}\rangle, \langle u_{2+t}, \tilde{a}_{2+t}\rangle, \ldots, \langle u_{n+t}, \tilde{a}_{m+t}\rangle) = \sum_{j=1+t}^{m+t} w_j g\left(\tilde{b}_j\right), \tag{20}$$

*where* $\tilde{b}_j$ *is the jth element that has the largest value* $u_i$, $u_i$ *is the order inducing variable,* $\tilde{a}_i$ *are interval numbers, W is an associated weighting vector of dimension m with* $W : 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ *and* $w_i \in [0, 1]$, *and* $g(\tilde{b}_j)$ *is a strict, continuous monotonic function.*

In Table II, we briefly present some of the main particular cases of the Quasi-IHOWMA operator and Quasi-HOWMA operator.

The same families of the generalized IHOWMA operator apply to the UI-HOWMA operator. The difference is that now these aggregation operators address interval numbers.

## 5. INVESTMENT SELECTION WITH INDUCED HEAVY MOVING AVERAGES

In an investment selection process, aversion to loss and the risk of each choice play an important role for the investor, so including these characteristic in the selection process will generate the best investment according to their needs, skills, and knowledge of the financial market.[30–32]

The use of information aggregation operators can integrate all of these characteristics in the selection process, such that use of UIHOWMA operators helps decision makers to select an appropriate choice taking in account personal characteristics and expectations for the future of the financial market.

To use UIHOWMA operators in the selection process, there are a number of steps to obtain the best results. These steps are as follows:

Step 1. Identify alternatives available, in this case, different types of investment $(A_1, A_2, A_3)$.

Step 2. Choose the number of historical months to consider, for this case three months.

Step 3. Analyze the data using the opinion of several experts and form collective results that summarize information given by the experts.

Step 4. Calculate the weighting vector $W = (w_{1+t}, .., w_{m+t})$

Step 5. Calculate the order-inducing variable $U = (u_{1+t}, \ldots, u_{m+t})$

Step 6. Use the IHOWMA (or UIHOWMA) operator for each of the criteria $(c_1, c_2, c_3)$ for alternatives identified in Step 1.

Step 7. Adopt a decision based on results found using different aggregation operators. In the case of uncertain aggregation operators, use the ranking methods presented in Section 2.1.

## 6. SELECTION PROCESS FOR THREE INVESTMENT OPTIONS

In this section, we investigate a simple numerical example to understand the approach suggested in this article. The example shows a multiperson decision-making problem in the selection of investments.

Step 1. We consider the following alternatives for investment

$$a_1 \ Tbills_{MEX}$$

$$a_2 \ Sight\ Savings$$

$$a_3 \ Investment\ fund$$

Step 2. The historical profit information for each alternative is presented as a 3-tuplet because each investment option has a minimum, median, and maximum percent of profit according to the quantity of the investment.

Step 3. Since the historical data are imprecise, three experts evaluate the data and provide their own opinion (see Tables III–V). Assume that opinions of the three experts are equally important.

**Table III.** Historical profit of the investments: Expert 1

| Month | $Tbills_{MEX}$ | *Sight saving* | *Investment fund* |
|---|---|---|---|
| April 2016 | (0.39,0.41,0.46) | (0.41,0.43,0.48) | (0.30,0.47,0.64) |
| March 2016 | (0.34,0.38,0.42) | (0.24,0.26,0.28) | (0.21,0.34,0.50) |
| February 2016 | (0.30,0.34,0.38) | (0.15,0.19,0.24) | (0.13,0.33,0.42) |

**Table IV.** Historical profit of the investments: Expert 2

| Month | $Tbills_{MEX}$ | *Sight saving* | *Investment fund* |
|---|---|---|---|
| April 2016 | (0.37,0.42,0.45) | (0.42,0.47,0.49) | (0.26,0.51,0.59) |
| March 2016 | (0.35,0.36,0.40) | (0.23,0.24,0.29) | (0.20,0.36,0.51) |
| February 2016 | (0.29,0.32,0.37) | (0.16,0.20,0.25) | (0.14,0.30,0.40) |

**Table V.** Historical profit of the investments: Expert 3

| Month | $Tbills_{MEX}$ | *Sight saving* | *Investment fund* |
|---|---|---|---|
| April 2016 | (0.35,0.40,0.44) | (0.40,0.45,0.46) | (0.25,0.52,0.60) |
| March 2016 | (0.36,0.37,0.41) | (0.22,0.25,0.27) | (0.19,0.35,0.52) |
| February 2016 | (0.31,0.33,0.36) | (0.17,0.21,0.26) | (0.18,0.30,0.38) |

**Table VI.** Historical profit of the investments: Collective results

| Month | $Tbills_{MEX}$ | *Sight saving* | *Investment fund* |
|---|---|---|---|
| April 2016 | (0.37,0.41,0.45) | (0.42,0.45,0.48) | (0.27,0.50,0.61) |
| March 2016 | (0.35,0.37,0.41) | (0.23,0.25,0.28) | (0.20,0.35,0.51) |
| February 2016 | (0.30,0.33,0.37) | (0.16,0.20,0.25) | (0.15,0.31,0.40) |

Information from the experts is aggregated using the arithmetic mean ($W = 1/3, 1/3, 1/3$) forming the collective results shown in Table VI.

Steps 4 and 5. Note that $n = 3$ because the decision maker believes that these are the months that still hold important information, the ordered inducing variables $U = (A_1 = (5, 10, 20), A_2 = (10, 5, 20), A_3 = (5, 20, 10))$ and the weighted vector is $W = (0.30, 0.35, 0.40) = 1.05$ because the future scenario of the investment is optimistic.

Step 6. Using the above information, the uncertain moving average (UMA), uncertain heavy moving average (UHMA) operator, uncertain heavy ordered weighted moving average (UHOWMA) operator, and UIHOWMA operator are applied to generate various selection scenarios (see Table VII).

Step 7. Unify the results of interval numbers according to the procedure explained in Section 2.1. (see Table VIII). Rank alternatives according to the results (see Table IX).

Note that by using the UIHOWMA operator, the order of the alternatives changes because of inclusion of the induced weighted vector instead of the regular one. In this sense, the ranking of alternatives includes more information about the

**Table VII.** Results using aggregation operators

| Operator | $Tbills_{MEX}$ | *Sight saving* | *Investment fund* |
|---|---|---|---|
| UMA | (0.34,0.37,0.41) | (0.27,0.30,0.34) | (0.21,0.39,0.51) |
| UHMA | (0.36,0.39,0.43) | (0.28,0.32,0.35) | (0.22,0.41,0.53) |
| UHOWMA | (0.35,0.38,0.43) | (0.27,0.30,0.34) | (0.21,0.40,0.52) |
| UIHOWMA | (0.36,0.39,0.43) | (0.29,0.32,0.36) | (0.22,0.41,0.54) |

Abbreviations: UMA, the uncertain moving average operator; UHMA, uncertain heavy moving average operator; UHOWMA, uncertain heavy ordered weighted moving average operator; UIHOWMA, uncertain induced heavy ordered weighted moving average operator.

**Table VIII.** Unification of the interval numbers

| Operator | $Tbills_{MEX}$ | *Sight saving* | *Investment fund* |
|---|---|---|---|
| UMA | 0.3725 | 0.3016 | 0.3716 |
| UHMA | 0.3911 | 0.3167 | 0.39025 |
| UHOWMA | 0.3872 | 0.3043 | 0.3813 |
| UIHOWMA | 0.3950 | 0.3192 | 0.3961 |

Abbreviations: UMA, the uncertain moving average operator; UHMA, uncertain heavy moving average operator; UHOWMA, uncertain heavy ordered weighted moving average operator; UIHOWMA, uncertain induced heavy ordered weighted moving average operator.

**Table IX.** Ranking of the alternatives

| Operator | Ranking |
|---|---|
| UMA | $Tbills_{MEX}$ > *Investment fund* > *Sight saving* |
| UHMA | $Tbills_{MEX}$ > *Investment fund* > *Sight saving* |
| UHOWMA | $Tbills_{MEX}$ > *Investment fund* > *Sight saving* |
| UIHOWMA | *Investment fund* > $Tbills_{MEX}$ > *Sight saving* |

Abbreviations: UMA, the uncertain moving average operator; UHMA, uncertain heavy moving average operator; UHOWMA, uncertain heavy ordered weighted moving average operator; UIHOWMA, uncertain induced heavy ordered weighted moving average operator.

decision maker instead of only historical data. With the use of the UIHOWMA operator, knowledge, expectations, and characteristics of the decision maker can be added making results more complex and specialized.

## 7. CONCLUSIONS

This paper has introduced a new extension of the OWA operator called the IHOWMA operator. This operator uses the main characteristics of three techniques: the moving average, IOWA operator, and heavy OWA (HOWMA) operator. This operator uses historical information and combines it with a weighted vector, reflecting the expectations of the decision maker. Note that the reordering step in the process is induced.

We analyze this new operator by providing its definition and studying some of its main properties. We developed a wide range of families of the IHOWMA operator, such as the arithmetic mean, median, weighted median, olympic mean, and generalized IHOWMA. Other interesting cases were described including the HOWMA and UIHOWMA, where interval numbers are used.

An application of the new approach to the selection process has also been developed. We observe that using the UIHOWMA operator instead of other operators leads to different results about the expectation of the future of alternatives. Using the UIHOWMA operator, the decision maker will have a more complete vision of situations.

In future research, we expect to develop new extensions of the OWA operator[33] by considering a probabilistic heavy ordered weighted moving average operator including characteristics such as fuzzy sets,[34,35] distance measure,[36–38] or the use of expertons in a group decision-making problem.[39]

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Apéndice 3.

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# EXCHANGE RATE USD/MXN FORECAST THROUGH ECONOMETRIC MODELS, TIME SERIES AND HOWMA OPERATORS

*Abstract. This paper aims to provide models that can predict the exchange rate and generate future scenarios of this variable, this because exchange risk management has become a strategic activity of the corporate governance. Also the study aims to expand the uses of operators like Heavy Ordering Weight Moving Average (HOWMA) in different fields of economy and management.*

*Design/methodology/approach. In this work three fundamental econometric models were used to forecast exchange rate USD/MXN, using 1994 to 2014 data, which are price index, interest rate and balance of payments. Additionally, two variables forecasting techniques were used; these are time series and HOWMA.*

*Findings. Among the results it was found that both methods are effective in middle term forecast, the last one being the one that can introduce uncertainty, expectations of the economy and characteristics of the decider into the models, enabling a range of possible scenarios.*

*Keywords: Exchange rate forecast, Econometric models, Time Series, HOWMA operators.*

**JEL Classification: F31, C51, C22, C53**

## 1. Introduction

Because the high volatility of the exchange rate points Majhi, Panda &Sahoo(2009) it is necessary to generate models that predict the future of it. In this regard Engel, Mark & West (2007) state that the exchange rate is a reflection of basic macroeconomics, as the interest rate, purchasing power parity and trade balance.

The fundamental models to forecast exchange rate is unstable and have not been successful in short term, Cheung*et al.* (2005) found that exchange rate follows a random walk behavior better than models based on macroeconomic fundamentals as the purchasing power parity, parity of interest rate and simple versions of monetary models. For their part Engel*et al.*(2007) found some success in determining the exchange rate in the long term using these variables.

One of the limitations of traditional models to forecast exchange rate, indicate Phillips(2003) and Boyer & Young(2005) is using variables whose behavior is complex, so it is based on simple assumptions of the future behavior of them, so the result is a reflection of the conditions used and exist a possibility of different scenarios.

In this sense it is necessary to use models in which accurate data is not used, because these become ineffective in situations of uncertainty (Gil Aluja, 2004). So considering the opinion of the currency market experts within the models will allow assumes different expectations of the future (Chen, 2011).

In this work, three econometric models based on macroeconomic fundamentals were used, additionally two different techniques for the prediction of each of the variables, time series and operators Heavy Ordering Weight Moving Average (HOWMA), looking to forecast the exchange rate USD/MXN for 2015.

The remainder of the paper is organized as follows. In Section 2, we review some theories of exchange rate determination and techniques used in the paper. In Section 3, we make a brief description of the importance of exchange rate USD/MXN in Mexico. In Section 4, the results of the application of the models is presented. Finally, Section 5 summarized the main conclusions of the paper.

## 2. Methodological approach

The basic concepts that should be known in the current investigation are as follows

**Definition 1**. The theory of Parity Purchasing Power (PPP), indicate Taylor & Taylor(2004) say that exchange rate between two currencies is determined by the change in the price level of the two countries. So that exchange rate adjusts to the inflation differentials. In this way the formula of PPP is:

$$TC_F = \beta_0 + \beta_1 PI_F + \beta_2 PI_D \,, \qquad (1)$$

where $TC_F$ is future exchange rate, $PI_F$ is foreign price index y $PI_D$is domestic price index.

**Definition 2**. The theory of Interest Rate Parity (IRP), said McCallum(1993) that in free money markets, the spread of interest rates should equal the discount or

premium future, so there is parity if the difference between interest rates offsets the forward premium of the stronger currency. So the mathematical relation is:

$$TC_F = \beta_0 + \beta_1 I_F + \beta_2 I_D \quad , \tag{2}$$

where $TC_F$ is future exchange rate, $I_F$ is foreign interest rate and $I_D$ is domestic interest rate.

**Definition 3.** The theory of the Balance of Payments (BoP), raises Dornbusch(1979) that the exchange rate is adjusted to the balance of inflows and outflows from international transactions in goods, services and assets, so that the current account is affected by the exchange rate due to relative Price changes and hence the competitiveness, the capital account on the other hand is affected by expectations of investors and interest rate. The formula that explains this phenomenon is:

$$TC_F = \beta_0 + \beta_1 CAB + \beta_2 FIP + \beta_3 DFI + \beta_4 R \, , \tag{3}$$

where $TC_F$ is future exchange rate, $CAB$ is current account balance, $FIP$ is foreign investment in portfolio, $DFI$ is direct foreign investment y $R$ reserves account.

**Definition 4.** The time series models that express a variable can be decomposed into four elements that are the trend, seasonality, cycle and irregularities, which can be expressed in the following multiplicative form (Fischer & Planas, 2000).

$$Y_t = T_t * S_t * I_t \quad , \tag{4}$$

where $Y_t$ is observed value, $T_t$is trend, $S_t$ is seasonality, $C_t$ is cycle y $I_t$ is irregularity

**Definition 5.** Moving averages, according to Kenney & Keeping(1962), are defined as a given sequence $\{a_i\}_{i=1}^{N}$, where a moving average $n$ is a new sequence $\{s_i\}_{i=1}^{N-n+1}$ defined from de $a_i$ taking the arithmetic mean of the sequence of $n$ terms, such that

$$s_i = \frac{1}{n} \sum_{j=i}^{i+n-1} a_j \, , \tag{5}$$

so a sequence $s_n$ given a moving average $n$ will be

$$S_2 = \frac{1}{2}(a_1 + a_2, a_2 + a_3, \dots, a_{n-1} + a_n),$$

$$S_3 = \frac{1}{3}(a_1 + a_2 + a_3, a_2 + a_3 + a_4, \dots, a_{n-2} + a_{n-1} + a_n) \, ,$$

and so on.

**Definition 6.** The ordered weighted averaging(OWA) operators developed by Yager(1988) allow the decision-maker subject to add information from a data set, in order to obtain a representative of the same value. In this respect an OWA operator of dimension $n$is a mapping$F: R^n \to R$ with an associated weight vector $w = [w_1, w_2, \dots, w_n]^T$ thereby $w_j \in [0,1]$, $1 \leq i \leq n$ and

$$\sum_{i=1}^{n} w_i = w_1 + w_2 + \cdots + w_n = 1\,, \tag{6}$$

where

$$F(a_1, a_2, \dots, a_n) = \sum_{k=1}^{n} w_j\, b_j\,, \tag{7}$$

being $b_j$ the jth largest element of the collection $a_1 a_2, \dots, a_n$.

**Definition 7.** A heavy aggregation operator, points Yager(2002) and Merigo& Casanovas (2011) is an extension to OWA operator that allows the weight vector goes up to $n$. So a HOWA operator is a mapping $R^n \to R$ which are associated to a weight vector $w$ which $w_j \in [0,1]$ y $1 \leq \sum_{j=1}^{n} w_j \leq n$, so that

$$HOWA\,(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j b_j\,, \tag{8}$$

being $b_j$ is the $j$th element largest of the collection $a_1 a_2, \dots, a_n$.

**Definition 8.** A HOWMA is defined as a sequence given $\{a_i\}_{i=1}^{N}$, where you get a new sequence $\{s_i\}_{i=1}^{N-n+1}$ which is multiplied by the heave weight vector, so that

$$HOWMA(s_i) = \sum_{j=1}^{n} w_j b_j\,, \tag{9}$$

being $b_j$ is the $j$th element largest of the collection $s_i$.

### 2.1.1 Considerations

In this research various econometric models were used in order to predict the exchange rate USD/MXN, for this multiple linear regression model were used, where exchange rate was dependent variable and for independent variables were used price index, interest rates and balance of payments (Tsai, 2012; Razmi*et al.*, 2012).

Among the problems encountered in the use of multiple liner regression models for determining a dependent variable is the existence of non-constant variation in the variables. One way to fix it is by transforming the data, which is generating new information using the logarithm of the original variables and generate the model with that data (Bartram & Bodnar, 2007).

Finally, a characteristic of the exchange rate as a variable, is that the changes from month to month are only partial, since the change for next month will be based on the end of the previous month, so that the model should be smoothing using the same variable with a lag in the model (Engel *et al.*, 2007).

## 3. Exchange rate USD/MXN

The result of 1994 crisis in Mexico, expressed Cartens & Werner(2000), forced to leave the default floating regime and adopt a free floating regime against the various world currencies. Those responsible for the fluctuations in the price of currencies, claim Ghosh, Ostry, & Chamon(2015) are the changes in supply and demand of financial markets, influenced by numerous external factors and the regime of free float, which creates a kind of unexpected, obscure and volatile change that threatens the future cash flows of the companies. (Bartram & Bodnar, 2012).

The effect of adopting the system of free floating exchange rate has generated considerable uncertainty in the value of currencies in Mexico, mainly in the exchange rate USD/MXN which from 1994 to 2014 has faced a high variation in price and volatility (See figure 1 and 2).

Currency risk exposure, expressed Chaney(2013) and Chatterjee, Dix-Carneiro, & Vichyanond(2013) is linked to the daily operations of the company, among which include imports, exports, investment and foreign currency loans. In that way foreign exchange risk, note Wu & Chang(2012) leads decision makers to accurately measure the exchange rate, in that way design strategies to reduce it to acceptable levels, that is why determine the future exchange rate becomes necessary.

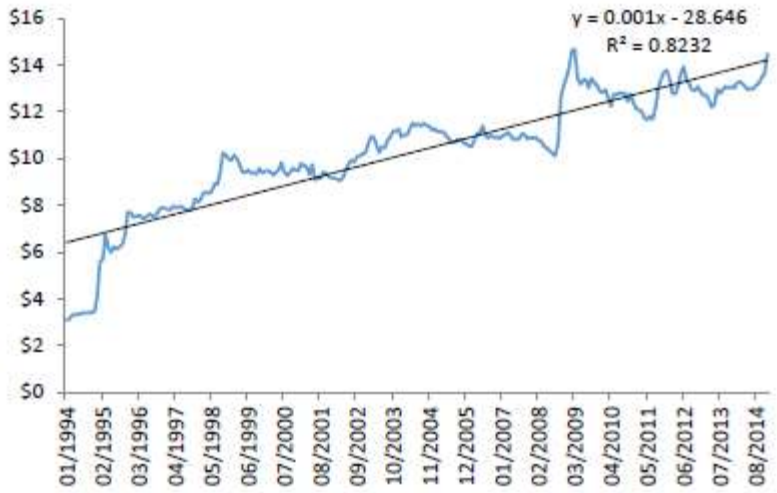


**Figure 1. Spot exchange rate USD/MXN 1994-2014**
Source: Own elaboration with data from Banxico (2015)

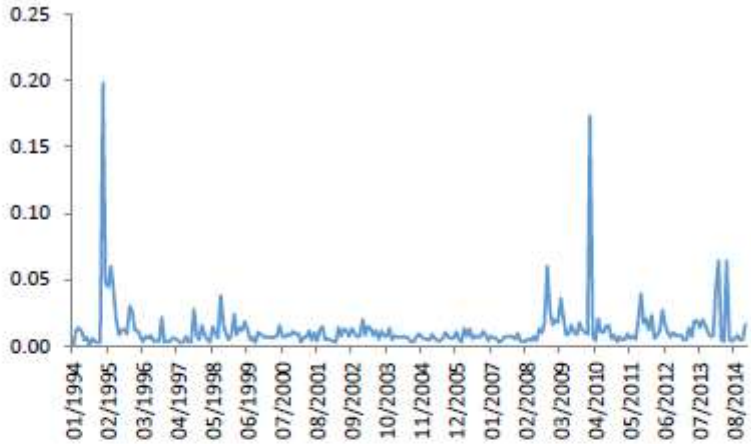


**Figure 2. Exchange rate volatility USD/MXN 1994-2014**
Source: Own elaboration with data from Banxico (2015)

## 4. Application of the model

*Preliminary considerations*

For the PPP and IRP model a data transformation was used, changing the original values with the logarithm of the same. In the BoP model such process was not performed because the current account balance and foreign investment has negative data so it is not possible to use logarithm. Within each model two additional variables were added, the first one is the exchange rate with a lag, this in order to smooth the model, and the second is volatility, in order to identify the effect this has within the forecast rate.

*Fundamental econometric models*

The information used to generate each of these models were from 1994 to 2014 data for each variable that integrated the model, obtaining the following models

PPP model

$$tc_F = 0.107 + 0.914tc_{-1} + 0.0235v - 0.00195pi_F + 0.0408pi_D \quad (10)$$

IRP model

$$tc_F = 0.0908 + 0.960tc_{-1} + 0.0225v - 0.000019i_F - 0.00228i_D \quad (11)$$

BoP model

$$TC_F = 0.175 + 0.978TC_{-1} + 4.59V - 0.000041CAB - 0.000023FIP - 0.000016DFI + 0.000002R \quad (12)$$

Where $tc_F$ is future exchange rate, $tc_{-1}$ is spot exchange rate with one lag, $v$ is volatility, $pi_F$ is foreign price index, $pi_D$is domestic price index, $i_F$ is foreign interest rate, $i_D$ is domestic interest rate, where all of the above variables are expressed in logarithm, $TC_F$ is future exchange rate, , $TC_{-1}$ is future exchange rate with one lag, $V$ is volatility, $CAB$ is current account balance, $FIP$is foreign investment in portfolio, $DFI$ is direct foreign investment y Ris reserved account.

*Econometric analysis*

*Significance test individual and joint*

To each of the models the P value is used to determine the significance of each of the variables under the following assumptions

1. $H_0$ is accepted if the P value is higher than 0.05

1. $H_0$ is rejected if the P value is lower than 0.05

Using P value to determine the significance of each of the variables for each model it was found that within the PPP model, exchange rate with a lag and domestic price index are rejected, but the null hypothesis was accepted for constant, volatility and foreign price index (See annex 1)

Within IRP model the null hypothesis is rejected for constant, exchange rate with a lag and volatility, but is accepted for foreign interest rate and domestic interest rate (See annex 2). Finally, for the BoP model the null hypothesis is rejected for exchange rate with a lag, volatility, foreign investment in portfolio and direct foreign investment and is accepted for constant and current account balance (See annex 3).

For the significance test for the entire models, we find that the null hypothesis for each of them is rejected, meaning that all have accepted significance considering all the variables (See annex 4)

*Autocorrelation residues test*

With the analysis of Durbin-Watson to determine if exist autocorrelation within the residues, it is found that PPP, IRP and BoP model have no autocorrelation in their residues. (See annex 5)

*Multicollinearity Test*

For this test variance inflation factors were used. Within the PPP model was found that exchange rate with a lag has low multicollinearity, volatility moderate and foreign price index and domestic price index have high. (See annex 6)

For IRP model by analyzing the variance inflation factors is detected that exchange rate with a lag and volatility have low multicollinearity, instead foreign interest rate and domestic interest rate have high. (See annex 7). Finally, balance of payments has low multicollinearity for all the variables. (See annex 8).

*Forecast of the variables*

For forecast every variable first it was used multiplicative decomposition time series method, in which determine the type of tendency and its equation, further seasonality, cycle and irregularities for each of the months of the year. (See annex 9)

Moreover, HOWMA operators were used to forecast the future of the variables in which a sequence $n = 6$ was used, this because the decision maker believes that this are the months that still hold important information for the forecast. Additionally, consider a $w = 1.05$, this is due the economic scenario for the 2015 was negative, so the exchange rate USD/MXN will depreciate with a valuation for each of the six months in the moving averages as follow $w = (0.05, 0.15, 0.15, 0.25, 0.40)$, also a maximization criterion was used in the result, that is the scenario that leads to greater exchange rate depreciation. All this based on the information provided by the decision maker.

The result of econometric models with time series and operators HOWMA are seen in Table 1 and 2 respectively.

**Table 1. Forecast of exchange rate using econometric models and time series**

| Time | Spot Exchange rate | PPP model | Error | IRP model | Error | BoP model | Error |
|---|---|---|---|---|---|---|---|
| 01-15 | 14.6808 | 15.6757 | 0.9949 | 15.6435 | 0.9627 | 15.5804 | 0.8996 |
| 02-15 | 14.9230 | 15.6272 | 0.7042 | 15.5716 | 0.6486 | 15.8417 | 0.9187 |
| 03-15 | 15.2136 | 15.2111 | -0.0025 | 15.1264 | -0.0872 | 15.3650 | 0.1514 |
| 04-15 | 15.2208 | 15.2666 | 0.0458 | 15.1832 | -0.0376 | 15.3671 | 0.1463 |
| 05-15 | 15.2475 | 15.4326 | 0.1851 | 15.3492 | 0.1017 | 15.5471 | 0.2996 |
| 06-15 | 15.4692 | 15.7162 | 0.2470 | 15.6489 | 0.1797 | 15.8292 | 0.3600 |
| 07-15 | 15.9225 | 15.8889 | -0.0336 | 15.8342 | -0.0883 | 15.9158 | -0.0067 |
| 08-15 | 16.5032 | 15.6731 | -0.8301 | 15.6062 | -0.8970 | 15.6174 | -0.8858 |
| 09-15 | 16.8519 | 15.6922 | -1.1597 | 15.6222 | -1.2297 | 15.7823 | -1.0696 |
| 10-15 | 16.5813 | 15.8258 | -0.7555 | 15.7580 | -0.8233 | 15.6008 | -0.8905 |
| Average Errors | 15.6614 | 15.6009 | 0.0604 | 15.5343 | -0.1271 | 15.6637 | -0.0077 |

**Table 2. Forecast of exchange rate using econometric models and HOWMA**

| Time | Spot Exchange rate | PPP model | Error | IRP model | Error | BoP model | Error |
|---|---|---|---|---|---|---|---|
| 01-15 | 14.6808 | 14.4197 | - 0.2611 | 14.4255 | -0.2553 | 14.5448 | -0.1360 |
| 02-15 | 14.9230 | 14.4971 | -0.4259 | 14.4952 | -0.4278 | 14.5601 | -0.3629 |
| 03-15 | 15.2136 | 14.8056 | -0.4080 | 14.8092 | -0.4044 | 14.8402 | -0.3734 |
| 04-15 | 15.2208 | 15.1007 | -0.1201 | 15.1098 | -0.1110 | 15.1268 | -0.0940 |
| 05-15 | 15.2475 | 15.4455 | 0.1980 | 15.4622 | 0.2147 | 15.4882 | 0.2407 |
| 06-15 | 15.4692 | 15.7565 | 0.2873 | 15.7792 | 0.3100 | 15.7913 | 0.3221 |
| 07-15 | 15.9225 | 16.0647 | 0.1422 | 16.0965 | 0.1740 | 16.1272 | 0.2047 |
| 08-15 | 16.5032 | 16.3817 | -0.1215 | 16.4208 | -0.0824 | 16.4509 | -0.0523 |
| 09-15 | 16.8519 | 16.7113 | -0.1406 | 16.7585 | -0.0934 | 16.7916 | -0.0603 |
| 10-15 | 16.5813 | 17.0465 | 0.4652 | 17.1021 | 0.5208 | 17.1385 | 0.5572 |
| Average Errors | 15.6614 | 15.6229 | -0.0384 | 15.6459 | -0.0155 | 15.6860 | 0.0246 |

## 5. Conclusions

Globalization, open markets and free floating regime of exchange rate has generated great expectations for companies on the future of the various types of changes. In the case of Mexico's economy, the effect of the USD/MXN exchange is considerable, so is necessary to generate scenarios about the future behavior of the same that let make strategies and decisions within this environment of uncertainty.

In this research three traditional econometric models, PPP, IRP and BoP were used to determine the exchange rate USD/MXN, additionally time series and HOWMA were used to detect the future of each one of the variables that compose the models.

The results show that both techniques have problems detecting shot term exchange rate, but analyzing the medium term with the average error can be seen that is very low, so that econometric models using time series and HOWMA are efficient under these scenarios.

Finally, it is noted that forecast with time series generate one result, leaving aside the possibility of make different scenarios according to the characteristics of the decision maker and changes in the economic expectations among months, but with the HOWMA operators we can add this information to the models, allowing them to adapt to these uncertain scenarios. Future research will be made using different aggregation

operators such probabilistic, generalized, induced, among others, as the inclusion of expertons for generating a weight vector more efficiently. (e.g. Blanco-Mesa, Gil-Lafuente, & Merigó(2015)).

## ANNEXES

**Annex 1. Significance test for PPP model**

| Variable | P Value | Accept or Reject $H_0$ |
|---|---|---|
| Constant | 0.131 | Accept |
| $tc_{-1}$ | 0.000 | Reject |
| $v$ | 0.625 | Accept |
| $pi_F$ | 0.066 | Accept |
| $pi_D$ | 0.000 | Reject |

**Annex 2. Significance test for IRP model**

| Variable | P Value | Accept or Reject $H_0$ |
|---|---|---|
| Constant | 0.000 | Reject |
| $tc_{-1}$ | 0.000 | Reject |
| $v$ | 0.000 | Reject |
| $i_F$ | 0.627 | Accept |
| $i_D$ | 0.917 | Accept |

**Annex 3. Significance test for BoP model**

| Variable | P Value | Accept or Reject $H_0$ |
|---|---|---|
| Constant | 0.066 | Accept |

| | | |
|---|---|---|
| $tc_{-1}$ | 0.000 | Reject |
| $v$ | 0.000 | Reject |
| $CAB$ | 0.052 | Accept |
| $FIP$ | 0.000 | Reject |
| DFI | 0.020 | Reject |
| $R$ | 0.001 | Reject |

**Annex 4. Significance test for the entire model**

| Model | P Value | Accept or Reject $H_0$ |
|---|---|---|
| PPP model | 0.000 | Reject |
| IRP model | 0.000 | Reject |
| BoP model | 0.000 | Reject |

**Annex 5. Durbin-Watson test for the residues of the models**

| Model | Durbin Watson | Result with 1% of significance | Result with 5% of significance |
|---|---|---|---|
| PPP model | 1.9071 | Without autocorrelation | Without autocorrelation |
| IRP model | 1.93561 | Without autocorrelation | Without autocorrelation |
| BoP model | 1.91376 | Without autocorrelation | Without autocorrelation |

**Annex 6. Variance inflation factors for PPP model**

| Variable | VIF | Multicollinearity |
|---|---|---|
| $tc_{-1}$ | 1.077 | Low |
| $v$ | 8.970 | Moderate |
| $pi_F$ | 28.287 | High |
| $pi_D$ | 12.424 | High |

**Annex 7. Variance inflation factors for IRP model**

| Variable | VIF | Multicollinearity |
|---|---|---|
| $tc_{-1}$ | 1.853 | Low |
| $v$ | 1.153 | Low |
| $i_F$ | 3.229 | Moderate |
| $i_D$ | 3.018 | Moderate |

**Annex 8. Variance inflation factors for BoP model**

| Variable | VIF | Multicollinearity |
|---|---|---|
| $TC_{-1}$ | 2.684 | Moderate |
| $V$ | 1.034 | Low |
| $CAB$ | 1.027 | Low |
| $FIP$ | 2.172 | Moderate |
| $DFI$ | 1.391 | Low |
| $R$ | 3.644 | Moderate |

**Annex 9. Trend type and formula for variables**

| Variable | Type of trend | Trend formula |
|---|---|---|
| $TC_{-1}$ | Exponential | $6.3461 * 1.00352^t$ |
| $V$ | Exponential | $0.007 * 1.00125^t$ |
| $PI_F$ | Exponential | $147.085 * 1.00201^t$ |
| $PI_D$ | Exponential | $31.0370 * 1.00601^t$ |
| $i_F$ | Exponential | $16.359 * 0.978471^t$ |
| $i_D$ | Exponential | $29.3549 * 0.990829^t$ |
| $CAB$ | Quadratic | $-17 - 10.13t + 0.0402t^2$ |
| $FIP$ | Quadratic | $-157 - 17.7t + 0.2128t^2$ |
| $DFI$ | Exponential | $2840.4 * 1.00396^t$ |
| $R$ | Exponential | $13603.4 * 1.01074^t$ |

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Apéndice 4.

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Warsaw, July 18, 2017

Dr. Ernesto León Castro
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Dear Dr. León Castro,

Herewith I wish to reconfirm that your paper coauthored with Dr. Dr. Ezequiel Avilés Ochoa, Ana María Gil Lafuente and José María Merigo Lindahl:

**Forgotten Effects in Exchange Rate Forecasting Models**
Ezequiel Avilés Ochoa , Ernesto León Castro, Ana María Gil Lafuente, José María Merigo Lindahl

has been accepted, after a thorough peer review process, to the volume "Complex systems: solutions and challenges in economics, management and engineering", edited by: Christian Berger-Vachon, Ana María Gil Lafuente, Janusz Kacprzyk, Yuriy Kondratenko, José M. Merigo Lindahl and Carlo Morabito, prepared by the international scientific community in honor of Professor Jaime Gil Aluja. The volume will be published soon in the series "Studies in Systems, Decision and Control" by Springer: http://www.springer.com/series/13304. This series is indexed by ISI, SCOPUS, DBLP, Ulrichs, MathSciNet, Current Mathematical Publications, Mathematical Reviews, Zentralblatt Math: MetaPress and Springerlink.

On behalf of all the Editors and myself I wish to thanks you for your very interesting contribution.

Best regards

J. Kacprzyk

**Forgotten effects in exchange rate forecasting models.**

Ezequiel Avilés Ochoa [1], Ernesto León Castro[2], Ana María Gil Lafuente[3], José María Merigo Lindahl[4]
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**Abstract.**
The aim of the article is to identify with the methodology of forgotten effects, hidden variables that influence the behavior of the forward exchange rate US dollar / Mexican peso (USD/MXN) to then incorporate them into a structured model from the postulates of the theory of Purchasing Power Parity (PPP) and, thus, reduce forecast error. The research question that causes the problem is: Is it possible to decrease the prediction error of the PPP model, using the methodology of the forgotten effects to detect and include hidden or forgotten variables? Among the results it was found that the inclusion of hidden or forgotten variables in the PPP model, decreases forecast error for exchange rate USD/MXN in 2015.

**Keywords**. Forgotten effects, forecast exchange rate, economy, mexican peso.

## 1. Introduction

Liberalization policies promoted by the Washington Consensus, from the eighties, have stimulated the growing internationalization of economics. Trade and financial flows, and therefore, operations in foreign exchange market have increased significantly. Nowadays, financial environment in which businesses operate, warn Gil-Aluja (1995; 2001), is volatile and uncertain. Exchange rate in Mexico, say Bazdresch & Werner (2002), is characterized by a systematic positive trend, long periods of relative stability with sudden episodes of high volatility.

In scenarios of uncertainty, said Lustig *et al.* (2011), global volatility increases risk in emerging economies. Interdependence of financial markets emphasizes exchange risk exposure in organizations located in developing countries, including Mexico. Thus, oriented to reduce the degree of exposure, corporate governance is an everyday practice (Allayannis *et al.*, 2001; Bartman & Bodnar, 2005).

Exchange rate volatility, warn Jorion & Sweeney (1996) and Majhi *et al.* (2009), requires decision makers to structure models to predict in advance exchange rate, with an acceptable forecast error. Basic models based on variables such price index, interest rates and balance of payments, have not produced satisfactory results, having similar behavior to a random walk (Taylor, 1995; Cheung *et al.*, 2005).

Inefficiency of these models, said Pastor (2000) and Krugman (2000), is intrinsic since they are a simplification of reality. This, even if the phenomenon under study is based on a group of variables theoretically accepted, some are unknown or forgotten, so their impact and influence it is ignored in the model (Gil-Aluja, 1999).

A methodology to reduce forecast error is the forgotten effects, which link cause and effect through incidence matrix to obtain or retrieve hidden or indirect items not directly considered by experts (Beltrán & Gil-Lafuente, 2005; Vizuete *et al.*, 2013). From the above discussion, the main question of this research is clear: Is it possible to reduce forecast error in PPP model, using forgotten effects methodology to detect and include hidden or forgotten variables?

The objective of this article is to identify, using the methodology of forgotten effects, developed by Kaufmann & Gil-Aluja (1996), the hidden variables that influence the behavior of the forward exchange rate

USD/MXN to then incorporate them into a model structured from the postulates of the PPP theory and, thus, reduce forecast error.

In the research expertons are used to recognize the variables that remain hidden or generate indirect impact in determining the exchange rate, which should be included in an econometric model and, consequently, generate greater efficiency in forecasting forward exchange rate.

The remainder of the paper is organized as follows. In Section 2 we review the PPP model and some fuzzy techniques. Section 3 presents the process and the results of using expertons and forgotten effects methodology to identify hidden variables. Section 4 shows the results of the incorporation of the forgotten variables in the PPP model. Section 5 summarizes the main conclusions of the paper.

## 2. Methodological and theoretical approach

In this section, we review some of the basic concepts that will be used in the paper. We analyze the Purchase Power Parity (PPP) model for exchange rate forecasting, the experton technique and the forgotten effects methodology.

2.1 Exchange rate forecasting techniques

Among the theories used to predict movements in exchange rate highlights the Purchasing Power Parity (PPP), which is defined as:

**Definition 1.** PPP model, in its relative version, postulates that variations in exchange rate in each period must be equal to inflation differential; that is, the weaker currency should depreciate (Dornbusch, 1985; Rogoff, 1996; Taylor & Taylor, 2004).

$$\hat{e} = \pi_{MEX} - \pi_{EUA} \tag{1}$$

where $\hat{e}$ is the change in exchange rate and $\pi$ denotes inflation.

An important aspect is that two variables were added in the model: the first one is the exchange rate with a lag and the second is the volatility that has been the behavior of the exchange rate (Bates, 1996; Taylor, 2001). To generate the values of the variables that make up the model, the multiplicative decomposition time series method was used (Fischer & Planas, 2000).. It is defined as follows.

**Definition 2.** To predict future value of the independent variables in forecast model, multiplicative decomposition time series method was used, is formulated as follows:

$$Y_t = T_t * S_t * C_t * I_t \tag{4}$$

where $Y_t$ is observed value, $T_t$ trend, $S_t$ seasonality, $C_t$ cycle and $I_t$ irregularity.

2.2. Fuzzy techniques for the treatment of uncertainty

All events that surround us are part of a system or subsystems, so any activity is subject to cause-effect incidence (Gil-Aluja, 2004). Despite the control systems, there is always the possibility to forget some causal relationships that are not explicit, obvious or visible (Barcellos de Paula & Gil-Lafuente, 2010). In situations of uncertainty and volatility, Kaufmann & Gil-Aluja (1988) indicates that there are variables that are not detectable immediately, because they are hidden variables that are the result of an accumulation of causes. To identify these forgotten elements, the application of the forgotten effects methodology and experton method allows to identify them. These techniques can be defined as follows.

**Definition 3.** The theory of expertons developed by Kaufmann (1988) is a generalized notion of a random fuzzy event when the probability of a random event on each $\alpha - cut$ is replaced by the confidence interval. It is defined as follows

Let $D$ a finite set of certain alternatives. A group of $q$ expert is requested to express their subjective opinion regarding each element from $D$ in the form of a confidence interval

$$\forall d \in D: [\alpha^{j_*}(a)], [\alpha^{j^*}(a)] \subset [0,1]$$

where $\subset$ is a set of inclusion and $j$ the expert. The property is monotonous, not horizontally and vertically downward, with a distribution network configuration $\wedge$ and $\vee$. So, that:

$$\forall \alpha \in [0,1] a_1(\alpha) \; in \; [a_1(\alpha), a_2(\alpha)]$$

$$\forall \alpha, \alpha' \in [0,1]: (\alpha' > \alpha) \Rightarrow \left(a_1(\alpha) \leq a_1(\alpha'), a_2(\alpha) \leq a_2(\alpha')\right)$$

$$(\alpha) = 0 \Rightarrow (a_1(\alpha) = 1, a_2(\alpha) = 1)$$

**Definition 4.** Forgotten effects methodology, developed by Kaufmann & Gil Aluja (1988), is supported on the assumption of the existence of two sets:

$$A = \left\{\frac{a_i}{i} = 1,2,\dots,n\right\}$$

$$B = \left\{\frac{b_j}{j} = 1,2,\dots,m\right\}$$

It is conjectured that prevails incidence of $a_i$ on $b_j$ if the value of the membership function characteristic of $(a_i, b_j)$ is valued in the range [0,1], i.e.:

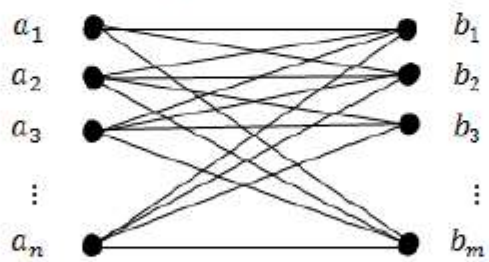
$$\forall(a_i, b_j) \Rightarrow \mu(a_i, b_j) \in [0,1]$$

The set of pair of evaluated elements is known as direct impact matrix, which shows the cause-effect relationship in different degrees, caused by the corresponding assembly A (causes) and the set B (effects)

M=

| | $b_1$ | $b_2$ | $b_3$ | … | $b_m$ |
|---|---|---|---|---|---|
| $a_1$ | $\mu_{a_1b_1}$ | $\mu_{a_1b_2}$ | $\mu_{a_1b_3}$ | … | $\mu_{a_1b_m}$ |
| $a_2$ | $\mu_{a_2b_1}$ | $\mu_{a_2b_2}$ | $\mu_{a_2b_3}$ | … | $\mu_{a_2b_m}$ |
| $a_3$ | $\mu_{a_3b_1}$ | $\mu_{a_3b_2}$ | $\mu_{a_3b_3}$ | … | $\mu_{a_3b_m}$ |
| ⋮ | … | … | … | … | … |
| $a_n$ | $\mu_{a_nb_1}$ | $\mu_{a_nb_2}$ | $\mu_{a_nb_3}$ | … | $\mu_{a_nb_m}$ |

The above matrix can be represented by a graph of incidence associated as follows:




Once detected the direct impact between set A and B, it proceeds to the detection of the forgotten effects. Subsequently, it is assumed that there is a third set of elements, called $C$, expressed as follows:

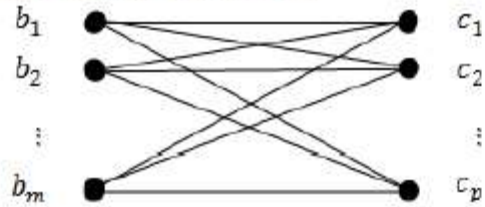
$$C = \left\{\frac{C_k}{k} = 1,2,\dots,k\right\}$$

This set consists of elements that are effects of set $B$, having an incidence matrix expressed as follows:

N=

| | $c_1$ | $c_2$ | $c_3$ | … | $c_m$ |
|---|---|---|---|---|---|
| $b_1$ | $\mu_{b_1c_1}$ | $\mu_{b_1c_2}$ | $\mu_{b_1c_3}$ | … | $\mu_{b_1c_m}$ |
| $b_2$ | $\mu_{b_2c_1}$ | $\mu_{b_2c_2}$ | $\mu_{b_2c_3}$ | … | $\mu_{b_2c_m}$ |
| $b_3$ | $\mu_{b_3c_1}$ | $\mu_{b_3c_2}$ | $\mu_{b_3c_3}$ | … | $\mu_{b_3c_m}$ |
| ⋮ | … | … | … | … | … |
| $b_n$ | $\mu_{b_nc_1}$ | $\mu_{b_nc_2}$ | $\mu_{b_nc_3}$ | … | $\mu_{b_nc_m}$ |

The incidence associated graph of this matrix takes the form:




Now we have two incidence matrixes, which includes, as common element, set $B$, this relationship is expressed as follows:

$$M \subset AxB \text{ and } N \subset BxC$$

To detect relationship between sets $A$ and $C$, using $B$, the max-min operator is used (expressed by symbol $\circ$), generating a new incidence matrix as follows:

$$M \circ N = P$$
$$P \subset AxC$$

This new relationship is formulated

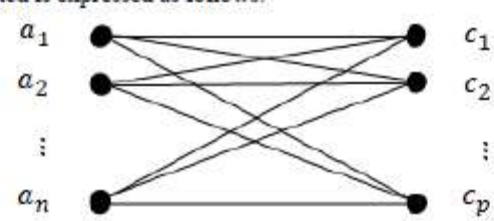
$$\forall (a_i, c_z \in AxC)$$
$$\mu(a_i, c_z) M \circ N = \forall_{bj} (\mu M(a_i, b_j) \wedge \mu N(b_j, c_z)$$

The resulting incidence matrix of performing the operation max-min is

$P=$

| | $c_1$ | $c_2$ | $c_3$ | $\dots$ | $c_m$ |
|---|---|---|---|---|---|
| $a_1$ | $\mu_{a_1c_1}$ | $\mu_{a_1c_2}$ | $\mu_{a_1c_3}$ | $\dots$ | $\mu_{a_1c_m}$ |
| $a_2$ | $\mu_{a_2c_1}$ | $\mu_{a_2c_2}$ | $\mu_{a_2c_3}$ | $\dots$ | $\mu_{a_2c_m}$ |
| $a_3$ | $\mu_{a_3c_1}$ | $\mu_{a_3c_2}$ | $\mu_{a_3c_3}$ | $\dots$ | $\mu_{a_3c_m}$ |
| $\vdots$ | $\dots$ | $\dots$ | $\dots$ | $\dots$ | $\dots$ |
| $a_n$ | $\mu_{a_nc_1}$ | $\mu_{a_nc_2}$ | $\mu_{a_nc_3}$ | $\dots$ | $\mu_{a_nc_m}$ |

The incidence graph associated is expressed as follows:




The $P$ matrix defines the casual relationships between elements of sets $A$ and $C$, in the intensity or degree that leads to consider those belonging to $B$.

To assign values located in the interval [0,1], Kaufmann and Gil-Aluja (1988) propose an endecadarian scale constituted by 11 values [0,0.1,0.2,0.3, ...,1]. This, point Luis & Gil-Lafuente (2011), facilitates their adaptation and treatment since people are used to think and work in decimal form.

**3. Detection of forgotten variables in exchange rate USD/MXN**

In this paper, we consider as variable that explain exchange rate those include in Purchase Power Parity (PPP) model, Interest Rate Parity (IRP) model and Balance of Payment (BoP) model. Also, other macroeconomic variables that have been important in exchange rate such as growth expectations, monetary politics, country risk, global market and oil price are included (Tsai, 1994; Bergholt & Lujala, 2012; Zhang *et al,* 2012; Reboredo, 2012; Blanchard & Adler, 2015; Ghosh *et al.*, 2015).

Based on that information, the variables considered causes are:

a) Growth expectations
b) Monetary politics
c) Country risk
d) Oil price
e) Global markets

Also, the variables considered effects are

a) Foreign inflation
b) Domestic inflation
c) Foreign interest rate
d) Domestic interest rate
e) Trade balance account
f) Foreign domestic investment
g) Foreign portfolio investment
h) International reserves.

The impact of one variable over another is wholly subjective, and therefore hardly measurable and to evaluate this relationship an endecadarian scale was used as follows:

0: no impact
0,1: virtually no effect
0,2: almost no effect
0,3: very low effect
0,4: low effect
0,5: medium effect
0,6: significant effect
0,7: very significant effect
0,8: strong effect
0,9: very strong effect
1: the highest effect

To detect the hidden variables in exchange rate, we integrate the opinions of five experts who have knowledge and information of this topic. In this group three people are financial advisors in forex market and another two are academics that has previous work in this field. With the information provide, the experton technique and the results are presented in table 1-3.

Table 1. Effect-Effect.

| | $PCI_{EUA}$ | $PCI_{MEX}$ | $r_{EUA}$ | $r_{MEX}$ | $BT$ | $FDI$ | $FPI$ | $R$ |
|---|---|---|---|---|---|---|---|---|
| $\pi_{USA}$ | 1 | 0.4 | 0.6 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| $\pi_{MEX}$ | 0.4 | 1 | 0.6 | 0.6 | 0.4 | 0.4 | 0.4 | 0.4 |
| $r_{USA}$ | 0.8 | 0.3 | 1 | 0.6 | 0.2 | 0.4 | 0.5 | 0.3 |
| $r_{MEX}$ | 0.4 | 0.7 | 0.7 | 1 | 0.4 | 0.5 | 0.5 | 0.4 |
| $BT$ | 0.3 | 0.5 | 0.3 | 0.4 | 1 | 0.6 | 0.6 | 0.5 |
| $FDI$ | 0.2 | 0.4 | 0.5 | 0.4 | 0.5 | 1 | 0.6 | 0.5 |
| $FPI$ | 0.3 | 0.3 | 0.4 | 0.5 | 0.5 | 0.6 | 1 | 0.4 |
| $R$ | 0.3 | 0.4 | 0.4 | 0.5 | 0.4 | 0.6 | 0.6 | 1 |

Table 2. Cause-Cause

| | Growth expectations | Monetary politics | Country risk | Oil price | Global markets |
|---|---|---|---|---|---|
| Growth expectations | 1 | 0.8 | 0.5 | 0.7 | 0.7 |
| Monetary politics | 0.8 | 1 | 0.7 | 0.6 | 0.6 |
| Country risk | 0.7 | 0.7 | 1 | 0.5 | 0.7 |
| Oil price | 0.7 | 0.5 | 0.4 | 1 | 0.8 |
| Global markets | 0.5 | 0.4 | 0.3 | 0.7 | 1 |

Table 3. Cause-Effect

| | $PCI_{EUA}$ | $PCI_{MEX}$ | $r_{EUA}$ | $r_{MEX}$ | $BT$ | $FDI$ | $FPI$ | $R$ |
|---|---|---|---|---|---|---|---|---|
| Growth expectations | 0.4 | 0.6 | 0.6 | 0.7 | 0.8 | 0.7 | 0.7 | 0.5 |
| Monetary politics | 0.7 | 0.8 | 0.7 | 0.8 | 0.7 | 0.6 | 0.6 | 0.5 |
| Country risk | 0.3 | 0.6 | 0.5 | 0.8 | 0.5 | 0.7 | 0.7 | 0.5 |
| Oil price | 0.5 | 0.3 | 0.3 | 0.3 | 0.6 | 0.4 | 0.5 | 0.5 |
| Global markets | 0.6 | 0.4 | 0.6 | 0.4 | 0.4 | 0.6 | 0.6 | 0.4 |

where $PCI_{USA}$= price consumer index in USA, $PCI_{MEX}$= price consumer index in México, $r_{USA}$= interest rate in USA, $r_{MEX}$= interest rate in Mexico, $BT$ = Balance of Trade in México, $FDI$ = foreign direct investment in México, $FPI$= foreign portfolio investment in Mexico, $R$= international reserves in Mexico.

Once generated the matrices and unified the information obtained from experts, we proceed to use the forgotten effects methodology. The results are shown in Table 4.

From the information obtained in Table 4, the order of the hidden variables, according to their indirect effects, is as follows: oil price > country risk > global market > growth expectations > monetary risk.

Among the forgotten effects, there are two relationships with 0.4 of indirect effect, these are: a) oil price and interest rate in Mexico, and b) country risk and price consumer index in USA. The former has a second order effect via growth expectations, while the second through monetary policy (see Figure 1-2).

Figure 1. Element interposed between oil prices and interest rate in Mexico.

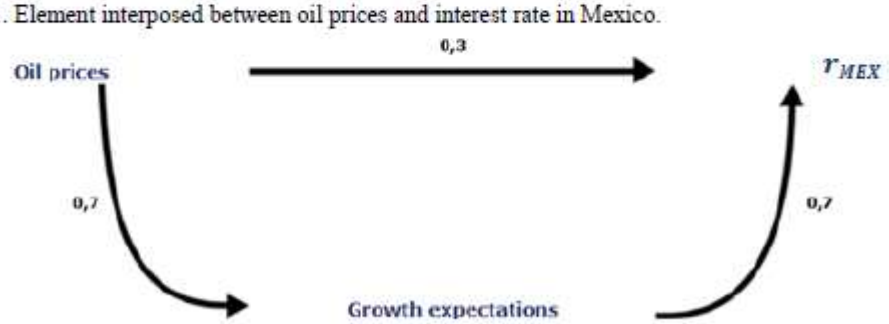


Table 4. Forgotten effects in determining the forward exchange rate

| Cause | Effect | M | $M \circ N$ | M* = $M \circ N \circ B$ | Q = M*-M |
|---|---|---|---|---|---|
| Growth expectations | $PCI_{USA}$ | 0.4 | 0.7 | 0.7 | 0.3 |
| Growth expectations | $PCI_{MEX}$ | 0.6 | 0.8 | 0.8 | 0.2 |
| Growth expectations | $r_{USA}$ | 0.6 | 0.7 | 0.7 | 0.1 |
| Growth expectations | $r_{MEX}$ | 0.7 | 0.8 | 0.8 | 0.1 |
| Growth expectations | $BT$ | 0.8 | 0.8 | 0.8 | 0 |
| Growth expectations | $FDI$ | 0.7 | 0.7 | 0.7 | 0 |
| Growth expectations | $FPI$ | 0.7 | 0.7 | 0.7 | 0 |
| Growth expectations | $R$ | 0.5 | 0.5 | 0.6 | 0.1 |
| Monetary politics | $PCI_{USA}$ | 0.7 | 0.7 | 0.7 | 0 |
| Monetary politics | $PCI_{MEX}$ | 0.8 | 0.8 | 0.8 | 0 |
| Monetary politics | $r_{USA}$ | 0.7 | 0.7 | 0.7 | 0 |
| Monetary politics | $r_{MEX}$ | 0.8 | 0.8 | 0.8 | 0 |
| Monetary politics | $BT$ | 0.7 | 0.8 | 0.8 | 0.1 |
| Monetary politics | $FDI$ | 0.6 | 0.7 | 0.7 | 0.1 |
| Monetary politics | $FPI$ | 0.6 | 0.7 | 0.7 | 0.1 |
| Monetary politics | $R$ | 0.5 | 0.5 | 0.6 | 0.1 |
| Country risk | $PCI_{USA}$ | 0.3 | 0.7 | 0.7 | 0.4 |
| Country risk | $PCI_{MEX}$ | 0.6 | 0.7 | 0.7 | 0.1 |
| Country risk | $r_{USA}$ | 0.5 | 0.7 | 0.7 | 0.2 |
| Country risk | $r_{MEX}$ | 0.8 | 0.8 | 0.8 | 0 |
| Country risk | $BT$ | 0.5 | 0.7 | 0.7 | 0.2 |
| Country risk | $FDI$ | 0.7 | 0.7 | 0.7 | 0 |
| Country risk | $FPI$ | 0.7 | 0.7 | 0.7 | 0 |
| Country risk | $R$ | 0.5 | 0.5 | 0.6 | 0.1 |
| Oil price | $PCI_{USA}$ | 0.5 | 0.6 | 0.6 | 0.1 |
| Oil price | $PCI_{MEX}$ | 0.3 | 0.6 | 0.6 | 0.3 |
| Oil price | $r_{USA}$ | 0.3 | 0.6 | 0.6 | 0.3 |
| Oil price | $r_{MEX}$ | 0.3 | 0.7 | 0.7 | 0.4 |
| Oil price | $BT$ | 0.6 | 0.7 | 0.7 | 0.1 |
| Oil price | $FDI$ | 0.4 | 0.7 | 0.7 | 0.3 |
| Oil price | $FPI$ | 0.5 | 07 | 0.7 | 0.2 |
| Oil price | $R$ | 0.5 | 0.5 | 0.6 | 0.1 |
| Global markets | $PCI_{USA}$ | 0.6 | 0.6 | 0.6 | 0 |
| Global markets | $PCI_{MEX}$ | 0.4 | 0.5 | 0.6 | 0.2 |
| Global markets | $r_{USA}$ | 0.6 | 0.6 | 0.6 | 0 |
| Global markets | $r_{MEX}$ | 0.4 | 0.5 | 0.6 | 0.2 |
| Global markets | $BT$ | 0.4 | 0.6 | 0.6 | 0.2 |
| Global markets | $FDI$ | 0.6 | 0.6 | 0.6 | 0 |
| Global markets | $FPI$ | 0.5 | 0.5 | 0.6 | 0.1 |
| Global markets | $R$ | 0.4 | 0.5 | 0.6 | 0.2 |

Figure 2. Element interposed between country risk and price consumer index in USA.

Note: To develop these relationships, we use FuzzyLog Software. Free available in http://www.fuzzyeconomics.com/jaimegil.html

**4. Incorporation of the forgotten variables in PPP model**

To determine the efficiency of the incorporation of forgotten effects in forecast exchange rate USD/MXN, the results of the PPP model is contrasted against the same model including the forgotten variable oil price. The models are as follow.

PPP model

| Source | SS | df | MS |
|---|---|---|---|
| Model | 4.28244058 | 4 | 1.07061014 |
| Residual | .043882562 | 246 | .000178384 |
| Total | 4.32632314 | 250 | .017305293 |

| | |
|---|---|
| Number of obs | = 251 |
| F( 4, 246) | = 6001.70 |
| Prob > F | = 0.0000 |
| R-squared | = 0.9899 |
| Adj R-squared | = 0.9897 |
| Root MSE | = .01336 |

| tcf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| v | .0234641 | .002765 | 8.49 | 0.000 | .018018 | .0289101 |
| tclag | .9141334 | .0220617 | 41.44 | 0.000 | .8706795 | .9575873 |
| pciUSA | -.0194534 | .0398029 | -0.49 | 0.625 | -.0978513 | .0589445 |
| pciMEX | .0407992 | .0221272 | 1.84 | 0.066 | -.0027837 | .0843821 |
| _cons | .1066104 | .070307 | 1.52 | 0.131 | -.0318702 | .245091 |

PPP model with forgotten effects

| Source | SS | df | MS |
|---|---|---|---|
| Model | 4.28259392 | 5 | .856518785 |
| Residual | .043729218 | 245 | .000178487 |
| Total | 4.32632314 | 250 | .017305293 |

| | |
|---|---|
| Number of obs | = 251 |
| F( 5, 245) | = 4798.78 |
| Prob > F | = 0.0000 |
| R-squared | = 0.9899 |
| Adj R-squared | = 0.9897 |
| Root MSE | = .01336 |

| tcf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| v | .0228157 | .0028529 | 8.00 | 0.000 | .0171964 | .0284349 |
| tclag | .9127896 | .0221156 | 41.27 | 0.000 | .8692286 | .9563505 |
| pciUSA | .0456639 | .0807508 | 0.57 | 0.572 | -.1133904 | .2047182 |
| pciMEX | .0353876 | .0228906 | 1.55 | 0.123 | -.0096999 | .0804751 |
| mme | -.0098227 | .0105975 | -0.93 | 0.355 | -.0306965 | .011051 |
| _cons | -.0161185 | .1499267 | -0.11 | 0.914 | -.3114281 | .2791911 |

where $tc_F$ is forward exchange rate; $tc_{lag}$ is exchange rate with one lag; $v$ is volatility; $pci_{USA}$ is price consumer index in USA; $pci_{MEX}$ is price consumer index in Mexico and $mme$ is mexican crude oil price. All variables are expressed in logarithms.

*Econometric analysis*

P value is used for significance test under the following assumptions:

1. $H_0$ is accepted if the P value is greater than 0.05
2. $H_0$ is rejected if the P value is less than 0.05

For PPP model, $H_0$ is rejected for $tc_{lag}$ y $pci_{MEX}$ and accepted for *constant*, $v$ and $pci_{USA}$ (see annex 1). For PPP model with forgotten effects, $H_0$ is rejected for $tc_{lag}$ and $v$, and accepted for *constant*, $pci_{USA}$, $pci_{MEX}$ and $mme$ (see annex 2). Finally, for the analysis of the complete models, $H_0$ is rejected for all the variables (see annex 3)

To detect autocorrelation, Durbin Watson statistic is used, appearing absence in both models (see annex 4)

Finally, the analysis of Variance Inflation Factors (VIF) is performed to detect multicollinearity. In the PPP model, there is low for $tc_{lag}$, moderate to $v$ and high for $pci_{USA}$ and $pci_{MEX}$ (see annex 5). In the PPP model with forgotten effect is low for $v$ and high for $pci_{USA}$, $pci_{MEX}$, $tc_{lag}$ y $mme$. (see annex 6).

To analyze the results, we use the mean squared deviation (MSD) statistic, that is $MSD = \frac{\Sigma|y_t - \hat{y}_t|^2}{n}$, the results of both models are presented in Table 5.

Table 5. Exchange rate USD/MXN forecasting through econometric models and forgotten effects.

| Time | Spot exchange rate USD/MXN | Time Series | Error | MSD | Time series and FE | Error | MSD |
|---|---|---|---|---|---|---|---|
| 01-15 | 14.6808 | 15.6757 | 0.9949 | 0.9899 | 15.8839 | 1.2031 | 1.4473 |
| 02-15 | 14.9230 | 15.6272 | 0.7042 | 0.4958 | 15.8420 | 0.9190 | 0.8445 |
| 03-15 | 15.2136 | 15.2111 | -0.0025 | 0.0000 | 15.4178 | 0.2042 | 0.0417 |
| 04-15 | 15.2208 | 15.2666 | 0.0458 | 0.0021 | 15.4674 | 0.2466 | 0.0608 |
| 05-15 | 15.2475 | 15.4326 | 0.1851 | 0.0343 | 15.6321 | 0.3846 | 0.1479 |
| 06-15 | 15.4692 | 15.7162 | 0.2470 | 0.0610 | 15.9168 | 0.4476 | 0.2003 |
| 07-15 | 15.9225 | 15.8889 | -0.0336 | 0.0011 | 16.0907 | 0.1682 | 0.0283 |
| 08-15 | 16.5032 | 15.6731 | -0.8301 | 0.6890 | 15.8782 | -0.6250 | 0.3906 |
| 09-15 | 16.8519 | 15.6922 | -1.1597 | 1.3450 | 15.8910 | -0.9609 | 0.9233 |
| 10-15 | 16.5813 | 15.8567 | -0.7246 | 0.5251 | 16.0594 | -0.5219 | 0.2724 |
| 11-15 | 16.6325 | 15.7631 | -0.8694 | 0.7558 | 15.9705 | -0.6620 | 0.4382 |
| 12-15 | 17.0365 | 16.0779 | -0.9586 | 0.9188 | 16.3045 | -0.7320 | 0.5359 |
| Average | 15.8569 | 15.6568 | -0.2001 | 0.4848 | 15.8629 | 0.0060 | 0.4443 |

## 5. Conclusions

The complexity of economic phenomena resulting in the non-inclusion in the forecast models, all statistically significant explanatory variables. To mitigate such failure expert knowledge is used to identify forgotten variables.

Hidden variables that influence the behavior of the forward exchange rate USD/MXN, with noticeable effects are the oil price and country risk. Among the finding it is detected that by including oil price as an additional variable to the PPP model, the forecast error decreases from -0.2001 to 0.0060.

In subsequent research, the aim is to incorporate the forgotten variables in parity of interest rate model and monetary approach to balance of payment model (Dornbusch, 1979; McCallum, 1993; Flood & Rose, 2002). In this way, to analyze the information of experts will be used aggregator operators such as OWA, POWA or IOWA. (Yager & Filev, 1999; Merigó & Gil-Lafuente, 2009; Merigó, 2012).

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**Appendix**

Annex 1. Significance test for PPP model

| Variable | P Value | Accept or reject $H_0$ |
|---|---|---|
| Constant | 0.131 | Accept |
| $tc_{lag}$ | 0.000 | Reject |
| $V$ | 0.625 | Accept |
| $pci_{USA}$ | 0.066 | Accept |
| $pci_{MEX}$ | 0.000 | Reject |

Annex 2. Significance test for PPP model with forgotten effects

| Variable | P Value | Accept or reject $H_0$ |
|---|---|---|
| Constant | 0.914 | Accept |
| $tc_{lag}$ | 0.000 | Reject |
| $V$ | 0.000 | Reject |
| $pci_{EUA}$ | 0.572 | Accept |
| $pci_{MEX}$ | 0.123 | Accept |
| $mme$ | 0.355 | Accept |

Annex 3, Significance test for complete models.

| Model | Valor P | Accept or reject $H_0$ |
|---|---|---|
| PPP model | 0.000 | Reject |
| PPP model with forgotten effects | 0.000 | Reject |

Annex 4. Durbin-Watson test for the residues of the models

| Model | Durbin Watson | Result with 1% of significance | Result with 5% of significance |
|---|---|---|---|
| PPP model | 1.9071 | Without autocorrelation | Without autocorrelation |
| PPP model with forgotten effects | 1.8988 | Without autocorrelation | Without autocorrelation |

Annex 5. Variance inflation factors for PPP model

| Variable | VIF | Multicollinearity |
|---|---|---|
| $tc_{lag}$ | 1.077 | Low |
| $v$ | 8.970 | Moderate |
| $pci_{EUA}$ | 28.287 | High |
| $pci_{MEX}$ | 12.424 | High |

Annex 6. Variance inflation factors for PPP model with forgotten effects

| Variable | VIF | Multicollinearity |
|---|---|---|
| $tc_{lag}$ | 12.478 | High |
| $v$ | 1.146 | Low |
| $pci_{EUA}$ | 36.897 | High |
| $pci_{MEX}$ | 30.255 | High |
| $mme$ | 15.693 | High |

Apéndice 5.

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# Forgotten effects and heavy moving averages in exchange rate forecasting

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*Abstract*— **This paper presents the results of using experton, forgotten effects and heavy moving averages operators in three traditional models based purchasing power parity (PPP) model to forecast exchange rate. Therefore, the use of these methods is to improve the forecast error under scenarios of volatility and uncertainty, such as the financial markets and more precise in exchange rate. The heavy ordered weighted moving average weighted average (HOWMAWA) operator is introduced. This new operator includes the weighted average in the usual heavy ordered weighted moving average (HOWMA) operator, considering a degree of importance for each concept that includes the operator. The use of experton and forgotten effects methodology represents the information of the experts in the field and with that information were obtained hidden variables or second degree relations. The results show that the inclusion of the forgotten effects and heavy moving average operators improve our results and reduce the forecast error.**

***Keywords—Experton, forgotten effects, HOWMAWA operator, econometric forecasting.***

## I. Introduction

One of the most important prices in open economies is the exchange rate, however the growing volatility requires decision makers to structure models to forecast exchange rate with an acceptable forecast error [1,2].

Models based on economic fundamentals such as price index, interest rates and balance of payments, have not produced satisfactory results, having similar behavior to a random walk [3,4] The main problem of these models is that they are a simplification of the reality [5,6]. So, even though there are theoretically accepted variables, there are others whose effect within the phenomenon are forgotten, so they must be identified and then used in the models to forecast.

The forgotten effects methodology introduced by Kaufmann & Gil-Aluja [8]) allows us to identify those relationships of second degree or hidden variables, where the causal relationship is forgotten and with the use of incidence matrix we can get the degree of forgetfulness [9].

In order to make the incidence matrices is necessary to include the knowledge of the experts in the financial markets, to do this we use the experton technique [7]. The experton is a function that used a cumulative distribution and using a decimal scale from 0 to 1 and linguistic expression we can unified the information of a group of experts.

Another technique used in this paper is the heavy ordered weighted moving average (HOWMA) operator that is an extension of the ordered weighted average (OWA) operator introduced by Yager [10]. The main advantage of this operator is that unifies the moving average with a heavy weighted vector [11]. This approach is further extended by unifying the weighted average and the HOWMA operator in the same formulation, considering the degree of importance that each concept has in the aggregation. This new operator is called the heavy ordered weighted moving average weighted average (HOWMAWA) operator. Some of its main properties and particular cases are studied.

An application of these two techniques where used in an econometric forecast problem regarding the exchange rate USD/MXN for 2015. With the use of information from 1994-2014 it was generated the econometric models, then using time series, HOWMA and HOWMAWA operators we forecast the future value of the variables included in the models. After that, we use the forgotten effects methodology to identify the hidden variables. Later they were incorporated in the econometric models, obtaining new results for the future value of the exchange rate.

The remainder of the paper is organized as follows. In Section 2 we review some theories of exchange rate determination and the techniques used in this paper. Section 3 presents the HOWMAWA operator. Section 4 presents the results of the using experton and forgotten effects methodology in exchange rate. Section 5 presents the comparison of using the different techniques in exchange rate USD/MXN 2015 forecast. Finally, Section 6 presents the main conclusions of the paper.

## II. Preliminaries

In this section, we briefly review some basic concepts to be used in the paper. We analyze the theories of exchange rate determination, time series, expertons, forgotten effects methodology and heavy moving average operators.

*2.1 Theories of exchange rate determination and times series*

The theory used to forecast exchange rate are the purchasing power parity (PPP) that can be defined as follows.

**Theory 1**. PPP model postulates that variations in exchange rate in a given period must be equal to inflation differential. It is defined as follows [12]:

$$\hat{e} = \pi_D - \pi_F, \tag{1}$$

where $\hat{e}$ is the change in exchange rate, $\pi_D$ domestic inflation and $\pi_F$ denotes foreign inflation.

In order to predict the future value of the independent variables in the econometric model, multiplicative decomposition time series method was used. It is formulated as follows [13]:

$$Y_t = T_t * S_t * C_t * I_t, \tag{2}$$

where $Y_t$ is observed value, $T_t$ trend, $S_t$ seasonality, $C_t$ cycle and $I_t$ irregularity.

*2.2 Experton*

The theory of expertons [7,8] suggests that in order to obtain realistic data from a phenomenon that is not directly measurable, and aggregated set of valuation given by experts is useful. Kaufmann [7]) indicates that an experton is an extension of probabilistic set, where each $\alpha - cut$, is replaced by a range of probabilities collected by expert opinion [14].

Let assume $E$ is a referential set, finite or not; $r$ a certain number of experts that are asked to give their opinion about each element of E. This is represented as:

$$\forall a \in: \left[a_*^j(x), a_j^*(x)\right] \subset [0,1],$$

where $\subset$ is a set of inclusion and $j$ the $j$th expert.

A static which concerns for each $x \in E$ is the lower and upper bound, for that a cumulative complementary law $F_*(a,x)$ is established for the $a_*^j(x)$ and $F^*(a,x)$ is established for the $a_j^*(x)$. So that:

$$\forall x \in E, \forall x \in [0,1]: \bar{A}(x) = [F_*(a,x), F^*(a,x)],$$

where ~ is the nature of the concept.

The referential set $E$ is the following experton:

$$\forall x \in E, \forall a \in E[0,1]: [F_*(a,x), F^*(a,x)] = 1.$$

The empty experton is then given by:

$$\forall x \in E: [F_*(a,x), F^*(a,x)] = \begin{cases} 1, \alpha = 0 \\ 0, \alpha \neq 0. \end{cases}$$

*2.3 Forgotten effects methodology*

Forgotten effects methodology introduced by Kaufmann & Gil-Aluja [8] is supported on the assumption of the existence of two sets:

$$A = \left\{\frac{a_i}{i} = 1,2,\dots,n\right\},$$

$$B = \left\{\frac{b_j}{j} = 1,2,\dots,m\right\}.$$

It is conjectured that prevails incidence of $a_i$ on $b_j$ if the value of the membership function characteristic of $(a_i, b_j)$ is valued in the range [0,1], i.e.:

$$\forall(a_i, b_j) \Rightarrow \mu(a_i, b_j) \in [0,1].$$

The set of pair of evaluated elements is known as direct impact matrix, which shows the cause-effect relationship in different degrees, caused by the corresponding assembly A (causes) and the set B (effects)

M=

| | $b_1$ | $b_2$ | $b_3$ | … | $b_m$ |
|---|---|---|---|---|---|
| $a_1$ | $\mu_{a_1b_1}$ | $\mu_{a_1b_2}$ | $\mu_{a_1b_3}$ | … | $\mu_{a_1b_m}$ |
| $a_2$ | $\mu_{a_2b_1}$ | $\mu_{a_2b_2}$ | $\mu_{a_2b_3}$ | … | $\mu_{a_2b_m}$ |
| $a_3$ | $\mu_{a_3b_1}$ | $\mu_{a_3b_2}$ | $\mu_{a_3b_3}$ | … | $\mu_{a_3b_m}$ |
| … | … | … | … | … | … |
| $a_n$ | $\mu_{a_nb_1}$ | $\mu_{a_nb_2}$ | $\mu_{a_nb_3}$ | … | $\mu_{a_nb_m}$ |

Now assume that there is a third set of elements, called $C$, expressed as follows:

$$C = \left\{\frac{C_k}{k} = 1,2,\dots,k\right\}.$$

This new set of elements represent effects of the set $B$, having an incidence matrix as follows:

N=

| | $c_1$ | $c_2$ | $c_3$ | … | $c_m$ |
|---|---|---|---|---|---|
| $b_1$ | $\mu_{b_1c_1}$ | $\mu_{b_1c_2}$ | $\mu_{b_1c_3}$ | … | $\mu_{b_1c_m}$ |
| $b_2$ | $\mu_{b_2c_1}$ | $\mu_{b_2c_2}$ | $\mu_{b_2c_3}$ | … | $\mu_{b_2c_m}$ |
| $b_3$ | $\mu_{b_3c_1}$ | $\mu_{b_3c_2}$ | $\mu_{b_3c_3}$ | … | $\mu_{b_3c_m}$ |
| … | … | … | … | … | … |
| $b_n$ | $\mu_{b_nc_1}$ | $\mu_{b_nc_2}$ | $\mu_{b_nc_3}$ | … | $\mu_{b_nc_m}$ |

Note that we have two incidence matrixes, that have as a common element the set $B$. So the relationship of the three sets can be expressed as:

$$M \subset AxB \text{ y } N \subset BxC.$$

With this information we will obtain the forgotten effects in $A$ and $C$, using set $B$ as base. To do this, the max-min operator is used (expressed by symbol $\circ$), generating a new incidence matrix as follows:

$$M \circ N = P,$$

$$P \subset AxC.$$

This new relationship is formulated as:

$$\forall(a_i, c_z \in AxC),$$

$$\mu(a_i, c_z)M \circ N = \forall_{bj}(\mu M(a_i, b_j) \wedge \mu N(b_j, c_z).$$

The resulting incidence matrix of performing the operation max-min is:

$$P = \begin{array}{c|ccccc} & c_1 & c_2 & c_3 & \cdots & c_m \\ \hline a_1 & \mu_{a_1c_1} & \mu_{a_1c_2} & \mu_{a_1c_3} & \cdots & \mu_{a_1c_m} \\ a_2 & \mu_{a_2c_1} & \mu_{a_2c_2} & \mu_{a_2c_3} & \cdots & \mu_{a_2c_m} \\ a_3 & \mu_{a_3c_1} & \mu_{a_3c_2} & \mu_{a_3c_3} & \cdots & \mu_{a_3c_m} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_n & \mu_{a_nc_1} & \mu_{a_nc_2} & \mu_{a_nc_3} & \cdots & \mu_{a_nc_m} \end{array}$$

The $P$ matrix defines the casual relationships between elements of sets $A$ and $C$, in the intensity or degree that leads to consider those belonging to $B$.

In order to assign values to the matrixes, Kaufmann & Gil-Aluja [8] propose a decimal scale constituted by 11 values from 0 to 1, [0,0.1,0.2,0.3, ...,1]. This, point Luis & Gil-Lafuente [15], facilitates their adaptation and treatment since people are used to think and work in decimal form.

*2.4 Heavy moving average operators*

The use of heavy moving averages operators in forecasting can be useful because it combines the historical data through the moving averages and incorporate a weighted vector that goes from $-\infty$ to $\infty$. In this sense we can under or over estimate the results according to the expectations of the experts in the field. In this paper the heavy ordered weighted moving average is used. It can be defined as follows:

**Definition 1.** A HOWMA operator is defined as a given sequence $\{a_i\}_{i=1}^{N}$, where you get a new sequence $\{s_i\}_{i=1}^{N-n+1}$ which is multiplied by a heavy weighting vector, such that:

$$HOWMA(s_i) = \sum_{j=1+t}^{m+t} w_j b_j, \qquad (2)$$

where $b_j$ is the $j$th largest element of the collection $a_1, a_2, \ldots, a_n$, and $W$ is an associated weighting vector of dimension $m$ with $W: 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ and $w_i \in [0,1]$. Observe that here we can also expand the weighting vector from $-\infty$ to $\infty$. Thus, the weighting vector $w$ becomes $-\infty \leq \sum_{i=1+t}^{m+t} w_i \leq \infty$.

If we focus in the reordering step, is possible to distinguish from the descending HOWMA (DHOWMA) operator and the ascending HOWMA (AHOWMA) operator. Also note that if the weighting vector is higher than one, is important to note that the information in some parts is independent.

Among the characteristics of the HOWMA operator we have that it is a monotonic and commutative function because:

a) It is monotonic because if $a_i \geq d_i$, for all $i$, then $HOWMA(a_1, \ldots, a_n) \geq HOWMA(d_1, \ldots, d_n)$.

b) It is commutative because any permutation of the arguments has the same evaluation.

Note that we can identify two types of boundaries in the HOWMA operator depending of the weighting vector. When $W$ ranges from 1 to $\infty$, it is possible to distinguish from the minimum and the total operator but if $W$ ranges from $-\infty$ to $\infty$ the HOWMA operator is not bounded.

The HOWMA operator follows the same parameter of beta value of the weighting vector that uses the HOWA operator [11]. This beta value normalizes the vector $W$ and it is defined as $\beta(W) = (|W| - 1)/(n - 1)$. Since $|W| \in [1, n]$, then $\beta \in [0,1]$. That is why if $\beta = 1$, we get the total operator and if $\beta = 0$, we get the usual moving average.

## III. HOWMAWA OPERATOR

Another extension of the heavy moving averaging operators is presented, combining the HOWMA operator and the weighted averages, this in order to consider in the same formulation the degree of importance that each concept has in the aggregation. This new operator is called the heavy ordered weighted moving average weighted average (HOWMAWA) operator. In order to understand better this operator [16,17] initially the ordered weighted averaging-weighted average (OWAWA) operator [18] and the ordered weighted averaging weighted moving averaging (OWAWMA) operator are presented.

**Definition 2.** An OWAWA operator of dimension $n$ is a mapping $OWAWA: R^n \to R$ that has an associated weighting vector $W$ of dimension $n$ such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$OWAWA(a_1, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j, \qquad (3)$$

where $b_j$ is the $j$th largest of the $a_i$, each argument $a_i$ has an associated weight (WA) $v_i$ with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, $\hat{v}_j \in [0,1]$, $\hat{v}_j = \beta w_j + (1 - \beta) v_j$ with $\beta \in [0,1]$ and $v_j$ is the weight (WA) $v_i$ ordered according to $b_j$, that is, according to the $j$th largest of the $a_i$.

Also, Merigó [18] indicated that if $\beta = 1$, we get the OWA operator and if $\beta = 0$, the WA. The OWAWA operator accomplishes similar properties than the usual aggregation operators. Like in the OWA operator, we can distinguish between the descending and ascending operator.

**Definition 3.** An OWAWMA operator of dimension $m$ is a mapping $OWAWMA: R^m \to R$ that has an associated weighting vector $W$ of dimension $m$ with $W = \sum_{j=1+t}^{m+t} w_j = 1$ and $w_j \in [0,1]$, such that:

$$OWAWMA\,(a_{1+t}, \dots, a_{m+t}) = \sum_{j=1+t}^{m+t} \hat{v}_j\, b_j, \qquad (3)$$

where $b_j$ is the $j$th largest argument of the $a_i$, each argument $a_i$ has an associated weight (WA) $v_i$ with $\sum_{i=1+t}^{m+t} v_i = 1$ and $v_i \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$, $v_j$ is the weight $v_i$ ordered according to $b_j$, that is, according to the $j$th largest of the $a_i$, $m$ is the total number of arguments considered from the whole sample and $t$ indicates the movement done in the average from the initial analysis. With the ideas presented below, it is possible to define the heavy ordered weighted moving average weighted average (HOWMAWA) operator as follows.

**Definition 4.** An HOWMAWA operator of dimension $m$ is a mapping $HOWMAWA: R^m \to R$ that has an associated weighting vector $W$ of dimension $m$ with $W: 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ and $w_j \in [0,1]$, such that:

$$HOWMAWA\,(a_{1+t}, \dots, a_{m+t}) = \sum_{j=1+t}^{m+t} \hat{v}_j\, b_j, \qquad (4)$$

where $b_j$ is the $j$th largest argument of the $a_i$, each argument $a_i$ has an associated weight (WA) $v_i$ with $v_i : 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$ and $v_i \in [0,1]$, $\hat{v}_j = \beta w_j + (1-\beta) v_j$ with $\beta \in [0,1]$, $v_j$ is the weight $v_i$ ordered according to $b_j$, that is, according to the $j$th largest of the $a_i$, $m$ is the total number of arguments considered from the whole sample and $t$ indicates the movement done in the average from the initial analysis. Also note that the characteristics of the HOWMA operator are also applied to the HOWMAWA operator [19], such as:

a) The weighted vector can also expand from from $-\infty$ to $\infty$. Thus, the associated weight (WA) $v_i$ becomes $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

b) It is possible to distinguish between the descending (DHOWMAWA) and ascending (AHOWMAWA) operator

c) This operator is monotonic and commutative. It will be bounded if the weight vector goes from 1 to $\infty$, but is not bounded if the weighting vector goes from $-\infty$ to $\infty$.

d) The beta value of the weighting vector applies the same way it does for the HOWMA operator, so if $\beta = 1$, we get the total operator and if $\beta = 0$, we get the usual moving average.

## IV. Detecting Exchange Rate Forgotten Variables

In order to detect the hidden variables in exchange rate forecast, we integrate the opinions of five people who have knowledge and information of this topic. In this group were three people that are financial advisors in forex market and another two that are academics that has previous work in this field. With the information provide we used the experton technique and the results are presented in Tables 1-3.

Table 1. Effect-Effect.

| | $PCI_{EUA}$ | $PCI_{MEX}$ | $r_{EUA}$ | $r_{MEX}$ | $BT$ | $FDI$ | $FPI$ | $R$ |
|---|---|---|---|---|---|---|---|---|
| $\pi_{EUA}$ | 1 | 0.4 | 0.6 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| $\pi_{MEX}$ | 0.4 | 1 | 0.6 | 0.6 | 0.4 | 0.4 | 0.4 | 0.4 |
| $r_{EUA}$ | 0.8 | 0.3 | 1 | 0.6 | 0.2 | 0.4 | 0.5 | 0.3 |
| $r_{MEX}$ | 0.4 | 0.7 | 0.7 | 1 | 0.4 | 0.5 | 0.5 | 0.4 |
| $BT$ | 0.3 | 0.5 | 0.3 | 0.4 | 1 | 0.6 | 0.6 | 0.5 |
| $FDI$ | 0.2 | 0.4 | 0.5 | 0.4 | 0.5 | 1 | 0.6 | 0.5 |
| $FPI$ | 0.3 | 0.3 | 0.4 | 0.5 | 0.5 | 0.6 | 1 | 0.4 |
| $R$ | 0.3 | 0.4 | 0.4 | 0.5 | 0.4 | 0.6 | 0.6 | 1 |

Table 2. Cause-Cause

| | Growth expectations | Monetary politics | Country risk | Oil price | Global markets |
|---|---|---|---|---|---|
| Growth expectations | 1 | 0.8 | 0.5 | 0.7 | 0.7 |
| Monetary politics | 0.8 | 1 | 0.7 | 0.6 | 0.6 |
| Country risk | 0.7 | 0.7 | 1 | 0.5 | 0.7 |
| Oil price | 0.7 | 0.5 | 0.4 | 1 | 0.8 |
| Global markets | 0.5 | 0.4 | 0.3 | 0.7 | 1 |

Table 3. Cause-Effect

| | $PCI_{EUA}$ | $PCI_{MEX}$ | $r_{EUA}$ | $r_{MEX}$ | $BT$ | $FDI$ | $FPI$ | $R$ |
|---|---|---|---|---|---|---|---|---|
| Growth expectation | 0.4 | 0.6 | 0.6 | 0.7 | 0.8 | 0.7 | 0.7 | 0.5 |
| Monetary politics | 0.7 | 0.8 | 0.7 | 0.8 | 0.7 | 0.6 | 0.6 | 0.5 |
| Country risk | 0.3 | 0.6 | 0.5 | 0.8 | 0.5 | 0.7 | 0.7 | 0.5 |
| Oil price | 0.5 | 0.3 | 0.3 | 0.3 | 0.6 | 0.4 | 0.5 | 0.5 |
| Global markets | 0.6 | 0.4 | 0.6 | 0.4 | 0.4 | 0.6 | 0.6 | 0.4 |

where $PCI_{USA}$= price consumer index in USA, $PCI_{MEX}$= price consumer index in Mexico, $r_{USA}$= interest rate in USA, $r_{MEX}$= interest rate in Mexico, $BT$= balance of trade in Mexico, $FDI$ = foreign direct investment in Mexico, $FPI$= foreign portfolio investment in Mexico, $R$= international reserves in Mexico.

With the information from table 1-3, the forgotten effects methodology developed by Kauffman & Gil-Aluja (1988) was used in order to identify the forgotten variables. The most relevant results are present in Table 7.

Table 7. Forgotten effects in determining the forward exchange rate

| Cause | Effect | M | $M \circ N$ | $M^* = M \circ N \circ B$ | $Q = M^* - M$ |
|---|---|---|---|---|---|
| Growth expectations | $PCI_{USA}$ | 0.4 | 0.7 | 0.7 | 0.3 |
| Country risk | $PCI_{USA}$ | 0.3 | 0.7 | 0.7 | 0.4 |
| Oil price | $PCI_{MEX}$ | 0.3 | 0.6 | 0.6 | 0.3 |
| Oil price | $r_{EUA}$ | 0.3 | 0.6 | 0.6 | 0.3 |
| Oil price | $r_{MEX}$ | 0.3 | 0.7 | 0.7 | 0.4 |
| Oil price | $FDI$ | 0.4 | 0.7 | 0.7 | 0.3 |

Observe that in general the variables with a higher sum of indirect effects are oil price and country risk (1.8 and 1.0 for each one). Analyzing the indirect effects of the variables, note that there are two with values of 0.4, these are: a) oil price in relation with interest rate and country risk with price consumer index in USA. The former has a second order effect via growth expectations, while the second through monetary policy.

Note. To develop these relationships, we use FuzzyLog Software. Free available in http://www.fuzzyeconomics.com/jaimegil.html

## V. Forecast of Exchange Rate USD/MXN for 2015

In order to forecast exchange rate USD/MXN for 2015 we use information from 1994-2014 because was in the first date when Mexico implement floating exchange rate regime. We use the following econometric model:

PPP model

| Source | SS | df | MS |
|---|---|---|---|
| Model | 4.20244058 | 4 | 1.07061014 |
| Residual | .043882562 | 246 | .000178384 |
| Total | 4.32632314 | 250 | .017305293 |

Number of obs = 251
F( 4, 246) = 6001.70
Prob > F = 0.0000
R-squared = 0.9899
Adj R-squared = 0.9897
Root MSE = .01336

| tcf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| v | .0234641 | .002765 | 8.49 | 0.000 | .018018 | .0289101 |
| tclag | .9141334 | .0220617 | 41.44 | 0.000 | .8706795 | .9575873 |
| pciUSA | -.0194534 | .0398029 | -0.49 | 0.625 | -.0978513 | .0589445 |
| pciMEX | .0407992 | .0221272 | 1.84 | 0.066 | -.0027837 | .0843821 |
| _cons | .1066104 | .070307 | 1.52 | 0.131 | -.0318702 | .245091 |

PPP model with forgotten effects

| Source | SS | df | MS |
|---|---|---|---|
| Model | 4.28259392 | 5 | .856518785 |
| Residual | .043729218 | 245 | .000178487 |
| Total | 4.32632314 | 250 | .017305293 |

Number of obs = 251
F( 5, 245) = 4798.78
Prob > F = 0.0000
R-squared = 0.9899
Adj R-squared = 0.9897
Root MSE = .01336

| tcf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| v | .0228157 | .0028529 | 8.00 | 0.000 | .0171964 | .0284349 |
| tclag | .9137896 | .0221156 | 41.27 | 0.000 | .8692288 | .9583505 |
| pciUSA | .0456639 | .0807508 | 0.57 | 0.572 | -.1133904 | .2047182 |
| pciMEX | .0353876 | .0228906 | 1.55 | 0.123 | -.0086999 | .0804751 |
| mme | -.0098227 | .0105975 | -0.93 | 0.355 | -.0306965 | .011051 |
| _cons | -.0163185 | .1499267 | -0.11 | 0.914 | -.3114261 | .2791931 |

where $tc_F$ is forward exchange rate, $tc_{lag}$ is exchange rate with a lag, $v$ is volatility, $pci_{USA}$ is price consumer index in USA, $pci_{MEX}$ is price consumer index in Mexico and $mme$ is Mexican crude oil price. Note that all these variables are expressed in logarithm.

In order to forecast exchange rate based on the econometric equation, we used multiplicative decomposition time series method.

In the case of the HOWMA and HOWMAWA operators, a sequence of $n = 6$, a weighting vector $W = (0.05, 0.15, 0.15, 0.25, 0.40)$ and a weighting vector of the weighted average $V = (0.05, 0.05, 0.10, 0.20, 0.20, 0.40)$ were used based on the information obtained from the experts in the financial market. Also was considered a degree of importance of 70% for the OWA and 30% for the weighted average.

The results of using the different methods can be seen in Tables 4-6. (Note. FE means forgotten effects).

Table 4. USD/MXN forecast using time series

| Time | Spot exchange rate USD/MXN | Time Series | Error | Time series and FE | Error |
|---|---|---|---|---|---|
| 01-15 | 14.6808 | 15.6757 | 0.9949 | 15.8839 | 1.2031 |
| 02-15 | 14.9230 | 15.6272 | 0.7042 | 15.8420 | 0.9190 |
| 03-15 | 15.2136 | 15.2111 | -0.0025 | 15.4178 | 0.2042 |
| 04-15 | 15.2208 | 15.2666 | 0.0458 | 15.4674 | 0.2466 |
| 05-15 | 15.2475 | 15.4326 | 0.1851 | 15.6321 | 0.3846 |
| 06-15 | 15.4692 | 15.7162 | 0.2470 | 15.9168 | 0.4476 |
| 07-15 | 15.9225 | 15.8889 | -0.0336 | 16.0907 | 0.1682 |
| 08-15 | 16.5032 | 15.6731 | -0.8301 | 15.8782 | -0.6250 |
| 09-15 | 16.8519 | 15.6922 | -1.1597 | 15.8910 | -0.9609 |
| 10-15 | 16.5813 | 15.8567 | -0.7246 | 16.0594 | -0.5219 |
| 11-15 | 16.6325 | 15.7631 | -0.8694 | 15.9705 | -0.6620 |
| 12-15 | 17.0365 | 16.0779 | -0.9586 | 16.3045 | -0.7320 |
| Average | 15.8569 | 15.6568 | -0.2001 | 15.8629 | 0.0060 |

Table 5. USD/MXN forecast using HOWMA operator

| Time | Spot exchange rate USD/MXN | HOWMA | Error | HOWMA and FE | Error |
|---|---|---|---|---|---|
| 01-15 | 14.6808 | 14.4197 | -0.2611 | 14.5507 | -0.1301 |
| 02-15 | 14.9230 | 14.4971 | -0.4259 | 14.6505 | -0.2725 |
| 03-15 | 15.2136 | 14.8056 | -0.4080 | 14.9846 | -0.2290 |
| 04-15 | 15.2208 | 15.1007 | -0.1201 | 15.3050 | 0.0842 |
| 05-15 | 15.2475 | 15.4455 | 0.1980 | 15.6786 | 0.4311 |
| 06-15 | 15.4692 | 15.7565 | 0.2873 | 16.0171 | 0.5479 |
| 07-15 | 15.9225 | 16.0647 | 0.1422 | 16.3512 | 0.4287 |
| 08-15 | 16.5032 | 16.3817 | -0.1215 | 16.6976 | 0.1944 |
| 09-15 | 16.8519 | 16.7113 | -0.1406 | 17.0577 | 0.2058 |
| 10-15 | 16.5813 | 17.0465 | 0.4652 | 17.4243 | 0.8429 |
| 11-15 | 16.6325 | 17.3888 | 0.7563 | 17.7990 | 1.1666 |
| 12-15 | 17.0365 | 17.7368 | 0.7003 | 18.1806 | 1.1442 |
| Average | 15.8569 | 15.9462 | 0.0893 | 16.2248 | 0.3679 |

Table 6. USD/MXN forecast using HOWMAWA operator

| Time | Spot exchange rate USD/MXN | HOWMA WA | Error | HOWMA WA and FE | Error |
|---|---|---|---|---|---|
| 01-15 | 14.6808 | 14.2532 | -0.4276 | 14.3696 | -0.3112 |
| 02-15 | 14.9230 | 14.5154 | -0.4076 | 14.6504 | -0.2726 |
| 03-15 | 15.2136 | 14.7428 | -0.4708 | 14.8963 | -0.3173 |
| 04-15 | 15.2208 | 14.9880 | -0.2328 | 15.1604 | -0.0604 |
| 05-15 | 15.2475 | 15.1598 | -0.0877 | 15.3517 | 0.1042 |
| 06-15 | 15.4692 | 15.4292 | -0.0400 | 15.6407 | 0.1715 |
| 07-15 | 15.9225 | 15.6150 | -0.3075 | 15.8422 | -0.0803 |
| 08-15 | 16.5032 | 15.8376 | -0.6656 | 16.0843 | -0.4189 |
| 09-15 | 16.8519 | 16.0610 | -0.7909 | 16.3276 | -0.5243 |
| 10-15 | 16.5813 | 16.5040 | -0.0773 | 16.7944 | 0.2131 |
| 11-15 | 16.6325 | 16.6033 | -0.0291 | 16.9123 | 0.2798 |
| 12-15 | 17.0365 | 16.8365 | -0.2000 | 17.1668 | 0.1303 |
| Average | 15.8569 | 15.5455 | -0.3114 | 15.7664 | -0.0905 |

Note that with the information above we can make decisions based not only in one result, but now we have a range of different scenarios to analyze and improve our vision of the future. It is important to distinguish that with the HOWMA and HOWMAWA operators is easier to generate new scenarios based on changes in the weighting vector and expectations of the future of the decision makers, something that is not possible using time series.

## VI. Conclusions

This paper introduced new hidden variables to the traditional PPP model with the use of two techniques: experton and forgotten effects methodology. Therefore, these techniques use the information of a group of decision makers, that are experts in the field, and with the uses of matrixes we can obtained the second degree effects and with that information the forgotten effects.

Also a new extension of the OWA operator called heavy ordered weighted moving average weighted average (HOWMAWA) operator was introduced. This new operator considers a degree of importance for each concept that is used in the HOWMA operator. We have analyzed this new operator giving its definition, studying its properties and some interesting particular cases have been included.

This new operator and some others have been used in order to forecast exchange rate USD/MXN 2015. As we can note the model improve with the additional hidden variable detected through the forgotten effects methodology and HOWMA and HOWMAWA operators.

In future research, we expect to develop new extension of the OWA operator by considering intuitionistic fuzzy sets [20] or linguistic variables [21,22].

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