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TESIS:
PRONÓSTICO DE VOLATILIDAD MEDIANTE MODELOS GARCH Y OPERADORES OWA: EL CASO DEL TIPO DE CAMBIO PESO MEXICANO-DÓLAR ESTADOUNIDENSE.

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INTRODUCCIÓN

El mercado de divisas es el mercado financiero más grande y liquido del mundo. La plataforma GAIN capital (2020) reporta movimientos promedios diarios en alrededor de 6.6 billones de dólares diarios. El tipo de cambio es de interés no solo de inversores, sino también de empresas y responsables políticos, quienes toman decisiones basadas en su valor (León-Catro et al., 2016; Auer y Schoenle, 2016; Avdjiev et al., 2019). Adicionalmente, este afecta en el valor relativo de los bienes nacionales y extranjeros, lo cual, impacta en variables macroeconómicas como balanza comercial, afectando nuevamente al tipo de cambio de forma cíclica (Jeelani et al., 2019).

Por un lado, la ruptura del acuerdo de Bretton-Woods en 1971 inició la adopción de regímenes flotantes en el mundo provocando variaciones continuas del precio nominal de las monedas (Asteriou et al., 2016; Ouyang et al., 2016). Por otro lado, las crisis financieras globales como la de 2008, han generado una significante inestabilidad en el valor de los bienes financieros (Stiglitz, 2010; Pinho et al., 2016; Kholer, 2019).

La variabilidad cambiaria se ha vuelto un movimiento normal en los mercados financieros, los cuales se acentúan en épocas de incertidumbre económica. La tabla 1 muestra la volatilidad de China, Japón, México, Brasil, Canadá y la Unión Europea con respecto al dólar estadounidense en los últimos años. Países como Japón y México tienen las monedas con más alta volatilidad, presentando valores de hasta 170% y 36% respectivamente. Mientras que la Unión Europea y Canadá experimentan la más baja volatilidad de alrededor del 1%.

Tabla 1: Volatilidades de tipo de cambio con respecto al dólar estadounidense

	2014-1	2014-2	2015-1	2015-2	2016-1	2016-2	2017-1	2017-2	2018-1	2018-2	2019-1	2019-2
China	1.90%	1.23%	1.17%	2.83%	2.28%	2.87%	1.67%	3.26%	4.18%	3.07%	3.23%	3.05%
UE	0.68%	0.93%	1.59%	1.21%	1.10%	1.02%	1.02%	0.95%	1.02%	0.67%	0.50%	0.57%
Japón	56.01%	116.08%	92.23%	101.22%	156.30%	170.60%	117.67%	92.01%	90.84%	70.18%	53.04%	52.25%
México	8.07%	11.36%	15.01%	20.40%	29.60%	36.11%	26.69%	24.73%	26.72%	28.70%	17.19%	18.70%
Brasil	2.43%	3.78%	6.54%	7.92%	6.58%	4.99%	3.54%	3.66%	4.81%	7.84%	4.11%	6.37%
Canadá	0.77%	0.69%	1.61%	1.19%	1.62%	1.04%	1.02%	1.15%	1.05%	0.92%	0.69%	0.61%

Elaboración propia a partir de datos de la Federal Reserve.

La volatilidad de tipo de cambio es experimentada por cada país en diferentes niveles acorde a las características político-económicas de cada uno. Los movimientos de los precios en activos financieros se originan en un enfoque basado en eventos, una noticia o anuncio impactan en su valor (Andersen et al., 2007; Ayadi et al., 2019). Según Mishkin (2008) los principales factores que influyen en la variación del tipo de cambio a largo plazo son: 1) los niveles relativos de precios, 2) los aranceles y las cuotas 3) las preferencias por bienes nacionales frente a bienes extranjeros y 4) la productividad. Mientras que a corto plazo se consideran las decisiones de mantener activos nacionales o extranjeros por parte de los agentes económicos en los mercados. Por su parte, Della Corte et al. (2016) consideran que las variaciones en el tipo de cambio son un reflejo de una serie de decisiones de los agentes financieros representados por especuladores e inversionistas.

El diferencial de precios entre una economía y el exterior es una causa de la variabilidad cambiaria. En este sentido, factores como los diferenciales de inflación y de tasa de interés entre países genera volatilidad en el tipo de cambio (Clavelina, 2018; Jeelani et al., 2019). La política monetaria de otros países se convierte en un factor que impacta en el precio de las monedas, esto genera una transferencia de volatilidad (spillovers) entre tasas de interés y tipos de cambio (Fisher y Chairman, 2015; Hunf, 2019). Adicionalmente, la transferencia de volatilidad se puede presentar con la variación de otras monedas (Wen y Wang, 2020) o la inestabilidad en otros mercados financieros (Jebran y Lqbal, 2016).

En este sentido, la fluctuación en otros precios y variables macroeconómicas especificas también pueden afectar el tipo de cambio. Lagunés y Pérez (2016) encuentran existencia de volatilidad cuando se presentan cambios en el precio internacional del petróleo, producto interno bruto (PIB) y las reservas internacionales. Por su parte Clavelina (2018) demuestra la influencia de las exportaciones e importaciones, la productividad, las perspectivas de crecimiento y la posible revisión a la baja de la nota crediticia del país.

En el caso de México, luego de la caída de su régimen de tipo de cambio ajustable a fines de 1994, se ha mantenido un tipo de flotación bastante libre. Sin embargo, en diferentes etapas ha surgido la necesidad de intervención por parte de las autoridades en el mercado de divisas. La volatilidad del tipo de cambio en escenarios con intervención gubernamental tiende a ser

menor que en regímenes completamente flexibles, sin embargo, esta suele marcarse en épocas de inestabilidad económica-financiera (Ouyang et al., 2016).

La figura 1 muestra el comportamiento de la volatilidad del tipo de cambio peso mexicano-dólar estadounidense en el periodo 1994-2018. Se observa que 1995 fue un año marcado por una alta variabilidad. Lo anterior provocado por la incertidumbre sobre el programa macroeconómico, el problema de liquidez internacional y la salud del sistema financiero (Werner, 1997). Posteriormente se experimentó periodos prolongados de estabilidad, interrumpidos por acontecimientos de importancia internacional como la crisis asiática (1997), la crisis de moratoria de Rusia (1998) y la devaluación del real brasileño (1999).

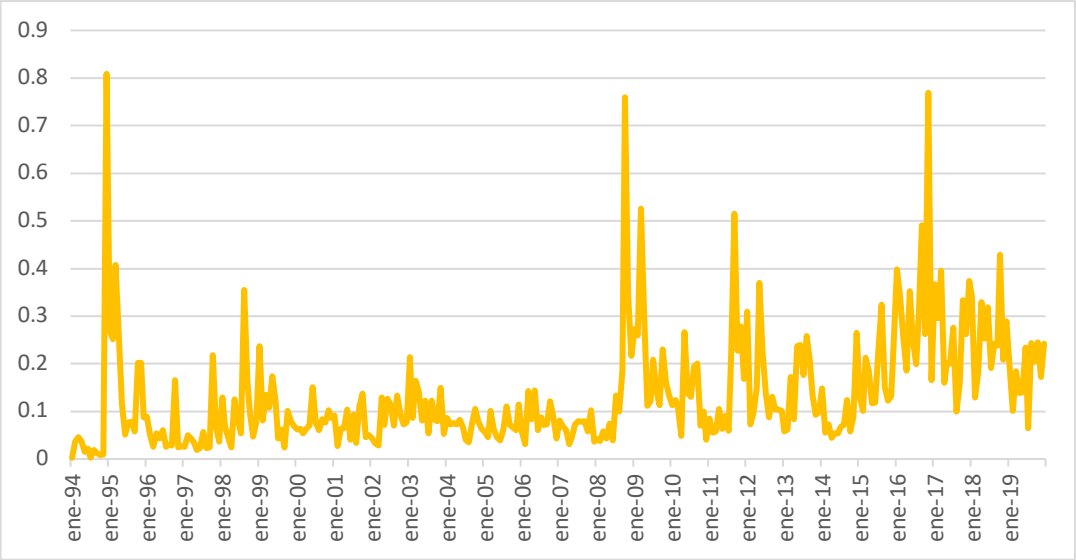

Figura 1. Volatilidad peso mexicano-dólar estadounidense 1994-2019

Fuente: Elaboración propia a partir de datos del tipo de cambio FIX del Banco de México (2020)

Para finales de 2008 y principios de 2009 los mercados financieros internacionales inician un colapso a causa de una serie de problemas en el sector hipotecario de Estados Unidos, provocando así, una crisis global de los mercados financieros. El mercado accionario mundial paso de tener un valor de 51 mil millones de dólares a 22 mil millones, lo cual significó una caída de 52% en 17 meses (Bartram y Bodnar, 2009).

La normalización en la política monetaria estadounidense aunada a la incertidumbre de renegociación del tratado de libre comercio de América del Norte, así como la vulnerabilidad de las finanzas públicas de México y el débil crecimiento económico, han afectado de forma especial al peso en años recientes (IBD, 2016; Clavelina, 2018).

Ante este panorama, un aspecto a considerar entre los economistas, gestores de fondos e inversionistas es conocer por cuanto tiempo los desequilibrios financieros afectan los movimientos en los tipos de cambio (Rossi, 2013; Dash, 2016), tomando en cuenta que la medición del riesgo en el tipo de cambio puede capturarse a través de la variabilidad de sus precios (Markowitz, 1952).

Entre el estudio de la volatilidad, destaca el análisis de modelos Autorregresivos de Heteroscedaticidad Condicional ARCH-GARCH (Bollerslev, 1986; Engle, 1980). Estos modelos reflejan la dependencia no-lineal de la varianza condicional dependiente en el tiempo. Sin embargo, los modelos pueden presentar problemas estructurales en momentos de gran incertidumbre (Hajizadeh et al., 2012; Lahmiri, 2017). Con el objetivo de contrarrestar esto, se han hecho diversas propuestas utilizando metodologías alternas en el modelado y estimación. Procedimientos como redes neuronales (Kristjanpoller y Minutolo, 2015), lógica difusa (D`Urso et al., 2016) y operadores de agregación (Cheng y Chen, 2013) han sido utilizadas.

Utilizar operadores de agregación permite adaptar los modelos y sus datos a los problemas estructurales antes mencionados, pues tienen la característica de agregar información adicional proveniente del exterior. La presente investigación propone utilizar operadores de promediado ponderado ordenado (OWA) (Yager, 1988) en el proceso de estimación de parámetros para un modelo de volatilidad ARCH-GARCH. El operador OWA constituye una familia de operadores de agregación OWA que incluye características como la obtención de mínimos, máximos y medias. Este operador tiene la facilidad de agregar información compleja en situaciones de gran incertidumbre gracias a su característica de reordenamiento y las diversas extensiones que permiten la inserción de información adicional en diferentes formas (Merigó, 2012).

La creación de un método de estimación de parámetros ARCH-GARCH con operadores de agregación OWA permite abordar los problemas estructurales de las series de tiempo en momentos de incertidumbre. Además, trabaja con información posibilista que afecta la información en los modelos de volatilidad. Como resultado se pueden obtener pronósticos apegados a la realidad.

CAPÍTULO I:

PLANTEAMIENTO DEL PROBLEMA

Medir la variación de los precios de activos financieros, el pronóstico y la decisión de cobertura es una tarea importante para responsables políticos, investigadores, empresas y profesionales. En las últimas décadas ha emergido una creciente literatura relacionada con la modelación y pronóstico del comportamiento de la volatilidad en mercados financieros (Brunetti et al., 2016; Gatheral et al., 2018; Heathcode y Perri, 2018; Arellano et al., 2019). La figura 2 muestra la evolución del número de estudios sobre el tema durante más de cuatro décadas[1]. Note que se ha presentado una creciente atención en el análisis de la volatilidad en el nuevo siglo, lo cual se acentúa a partir de 2014.

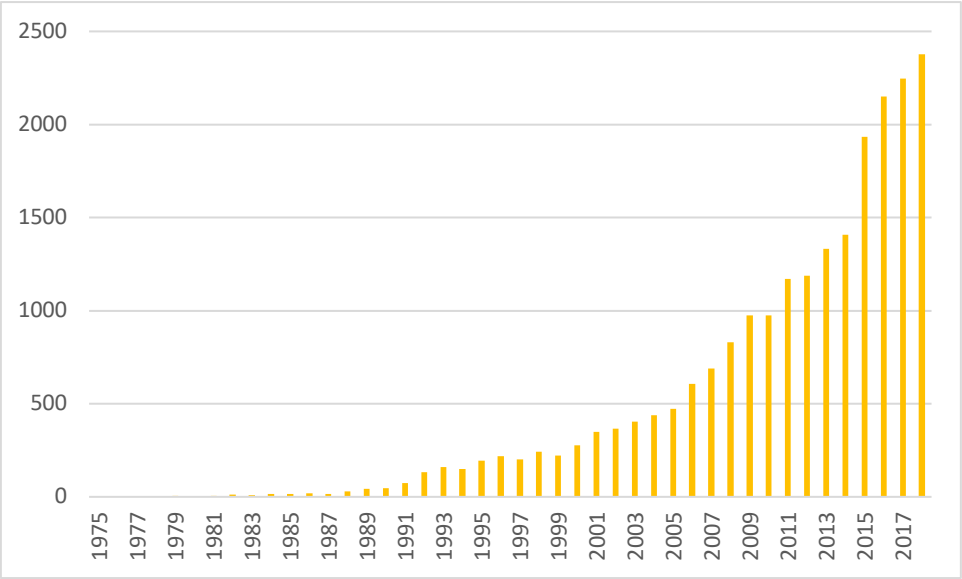

Figura 2. Evolución del estudio de la volatilidad (publicaciones por año) 1975-2018

Fuente: Elaboración propia a partir de datos de Web of Science 2019

[1] El estudio utilizó Web of Science Collection database con los índices SCI-EXPANDED, SSCI, A&HCI y ESCI. Las palabras clave fueron: "conditional heteroskedasticit* OR return volatility*OR conditional heteroscedasticit* OR stochastic volatility* OR volatility*. Se consideraron solo artículos, resúmenes, cartas y notas.

Estudiar el modelado y pronóstico de la volatilidad es una tarea que implica considerar numerosos factores de origen político-económico que impactan en el comportamiento de las series, como es el caso de anuncios en política monetaria y cambios de regímenes (Andersen et al., 2007; Ayadi et al., 2019). En primer lugar, la volatilidad no es directamente observable, por lo tanto, se estima a través de modelos paramétricos (Dash et al., 2015; Arellano et al., 2019). Estos modelos consideran diferentes formas de medir la volatilidad, de lo cual puede depender el éxito o fracaso en la estimación y el pronóstico.

Una técnica utilizada en la literatura para medir la volatilidad es la desviación estándar, la cual, representa la dispersión con respecto a la media (Hooper y Kohlhagen, 1978; Katsiampa, 2017). En investigación científica es común el utilizar la varianza logarítmica de los rendimientos llamada log-volatilidad (Kim et al., 1998; Abi Jaber y El Euch, 2019). Por otro lado, en las últimas dos décadas se ha propuesto la volatilidad realizada que incluye información intra-día con diferencias en precios máximos y mínimos (Andersen y Bollerslev, 1998; Gupta et al., 2018; Yang et al., 2020).

Adicionalmente a una medida de volatilidad es importante construir un modelo con capacidad para reproducir hechos estilizados en series de tiempo y exhibir propiedades estadísticas deseables. En la práctica es común encontrar variables vagas o imprecisas ligadas a la volatilidad como lo son las decisiones de inversión en el seguimiento a tendencias (Tsaur, 2012; Xing et al., 2019). De la misma forma eventos esporádicos y repentinos de origen económico, político, climático y de salud pública provocan una reacción en los tipos de cambio (Andersen et al., 2007; Ayadi et al., 2019; Njindan lyke, 2020). Lo anterior genera fenómenos estadísticos en el comportamiento de las series financieras, como lo es la asimetría, los saltos abruptos y los rendimientos no normales (Kristjanpoller et al., 2013; Patton y Sheppard, 2015; Yang y Chen, 2014; Wang et al., 2019; Han et al., 2020).

Algunos de los modelos paramétricos utilizados para estimar la volatilidad son aquellos que incluyen la familia de modelos de heteroscedasticidad condicional ARCH-GARCH propuestos por Engle (1982) y generalizados por Bollerslev (1986). Los modelos ARCH-GARCH consideran la volatilidad como un parámetro clave utilizando heteroscedasticidad condicional dependiente del tiempo. La principal ventaja de estos consiste en recoger información de los periodos anteriores para adaptarse al comportamiento de la serie conforme

se avanza en la estimación y pronóstico. Los modelos ARCH-GARCH y sus extensiones han sido utilizados en diversas áreas con resultados favorables como lo es en el mercado de valores (Chou, 1998; Fang et al., 2020), mercados derivados (Duan, 1995; Badescu et al., 2019), precio del petróleo (Mohammadi y Su, 2010; Zhang et al., 2019), y tipo de cambio (Caporale y Zekokh, 2019; Zhou et al., 2020).

Sin embargo, algunos trabajos demuestran como los modelos de heteroscedaticidad condicional tienden a fallar. Algunos hechos estilizados estadísticos, presentados de forma pronunciada afectan directamente la estimación y el pronóstico. Hajizadeh et al. (2012) y Lahmiri (2017) demuestran como los modelos ARCH-GARCH asumen correlación de estructura lineales entre datos de series de tiempo en épocas donde la no linealidad es persistente. Por su parte, Kim y Hwang (2019) comprueban los postulados de Kim et al. (1998) al demostrar que los modelos GARCH no consideran los ruidos o imprecisiones que las series de tiempo pueden presentar en algunos periodos.

En suma, la información utilizada en pronósticos de volatilidad suele verse afectada por una amplia gama de imprecisiones e incertidumbres que se traducen en puntuales comportamientos estadísticos. Por lo tanto, la estimación y pronóstico en modelos ARCH-GARCH pueden verse deficiente cuando una o más de estas características aparece.

DESCRIPCIÓN DEL PROBLEMA OBJETO DE INVESTIGACIÓN

La volatilidad puede ser observada en la mayoría de los productos financieros alrededor del mundo. Su estudio desde la década de los 80 ha proliferado en indicadores como mercado de valores (Chou, 1998; Wei et al., 2018), precio del petróleo (Mohammadi y Su, 2010; Pan y Yin, 2017), mercado de derivados (Duan, 1995; Badescu et al, 2019) y tipo de cambio (Diebold y Nerlove, 1989; Lahmiri, 2017).

El análisis de la volatilidad de tipo de cambio surge debido a las repercusiones que puede tener en decisiones financieras y económicas en cualquier horizonte de tiempo (Kholer,

2019). La figura 3 muestra los temas más citados en el estudio de la volatilidad de tipo de cambio basado en las 20 publicaciones más citadas por periodo[2].

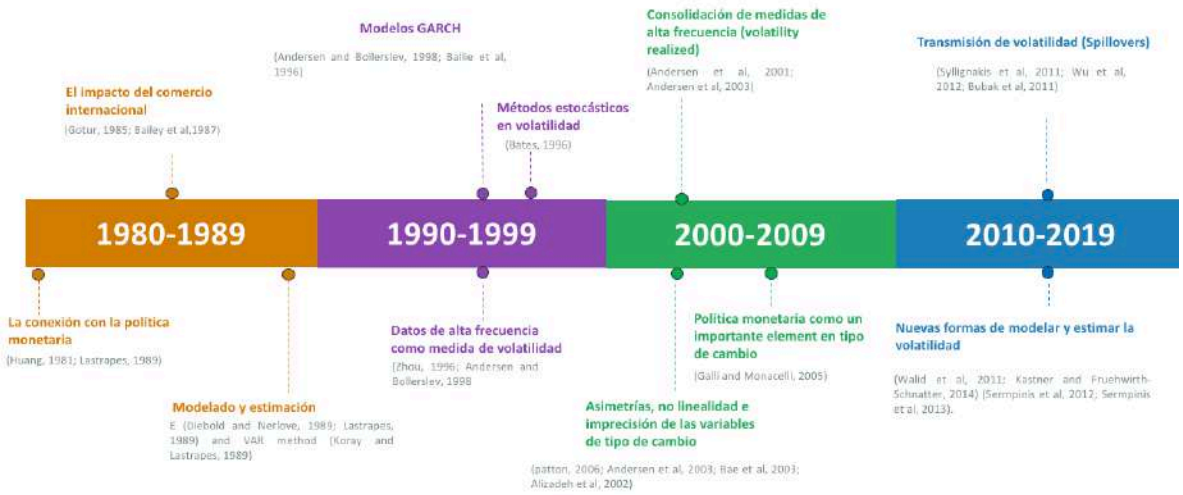

Figura 3. Temas más citados de la volatilidad de tipo de cambio 1980-2019

Fuente: Elaboración propia a partir de datos de Web of Science 2019

En la primera década, se observa, como los documentos más citados se centran en la relación de la volatilidad y las decisiones referentes a política monetaria, además del impacto a actividades en el comercio internacional, en este periodo resaltan las publicaciones en el estudio de modelación y estimación en la variabilidad de tipo de cambio con los modelos ARCH. En la segunda década, el análisis de modelación es líder en las principales publicaciones de volatilidad, destacan entre estas la utilización de modelos GARCH y metodologías estocásticas alternas, además nuevas formas de medir la volatilidad utilizando datos de alta frecuencia aparecen en las primeras posiciones. A principios del milenio, las nuevas medidas se consolidan entre las publicaciones más citadas y el estudio del comportamiento de la volatilidad y sus características se vuelven de gran interés. En la última

[2] La búsqueda se ha hecho en Web of Science Collection database con los índices SCI-EXPANDED, SSCI, A&HCI y ESCI, las palabras clave son “exchange rate” AND volatility*.

década, el estudio se ha centrado en la transmisión de volatilidad desde otros mercados financieros o economías, ante esto, nuevas metodologías para la modelación y estimación son propuestas, considerando combinación de métodos en la mayoría de los casos.

Con el objetivo de conocer los trabajos que han captado la atención en el tema durante las últimas décadas, la tabla 2 muestra las publicaciones más citadas en el estudio de la volatilidad de tipo de cambio para el periodo 1980-2019[3], los resultados son en las bases de datos Web of Science (WoS) y Scopus. La WoS es una plataforma bibliométrica digital reconocida entre investigadores internacionales por tener altos estándares de calidad (Merigó et al., 2015). Por su parte, Scopus es una alternativa a la WoS, además de su función en la búsqueda de literatura científica, es una herramienta utilizada para el análisis de citas (Vieira y Gomes, 2009).

Se puede observar que el trabajo más citado corresponde a "Modeling and forecasting realized volatility" propuesto por Andersen, Bollerslev, Diebold y Labys. Esta publicación analiza la modelación GARCH con datos de alta frecuencia, note que aparece en primer lugar tanto en la base de datos WoS como en Scopus. El segundo lugar en WoS es para Andersen y Bollerslev con "Answering the skeptics: Yes, standard volatility models do provide accurate forecasts" el cual no aparece en Scopus. El tercer artículo más citado el WoS es para Baillie, Bollerslev and Mikkelsen el cual aparece en segundo lugar en Scopus. Es importante hacer notar que entre las primeras publicaciones se encuentran una gran cantidad de estudios sobre modelación y pronóstico de volatilidad de tipo de cambio, mientras que una pequeña parte corresponde a estudios en la relación de esta con política monetaria.

Para el caso del tipo de cambio peso mexicano- dólar estadounidense las publicaciones en volatilidad alcanzan 76 en Wos y 72 en Scopus. La tabla 3 muestra los diez artículos más citados en el tema[4]. Se puede observar que las publicaciones más citadas de volatilidad de tipo de cambio en México son en su mayoría trabajos publicados a partir de inicios del milenio, donde años recientes son mencionados como 2016 y 2019. En este sentido, Diamandis y Drakos tienen la primera posición en WoS con el artículo "Financial

[3] Se utilizaron las palabras clave mencionadas en la nota 2.
[4] Se utilizaron las palabras clave "exchange rate" AND volatility* AND mexic* OR (usd/mxn).

liberalization, exchange rates and stock prices: Exogenous shocks in four Latin America countries". Por suparte, Bahmani-Oskooee y Hegerty de la universidad de Wisconsin tienen el primer lugar en Scopus y el segundo en WoS con el artículo "The Effects of Exchange-Rate Volatility on Commodity Trade between the United States and Mexico", note que estos autores aparecen en posiciones subsecuentes de la lista.

Tabla 2. Documentos más citados en volatilidad de tipo de cambio 1980-2019.

Web of Science

R	TC	Titulo	Autor/es	año	C/A
1	1160	Modeling and forecasting realized volatility	Andersen, TG; Bollerslev, T; Diebold, FX; Labys, P	2003	64.44
2	1041	Answering the skeptics: Yes, standard volatility models do provide accurate forecasts	Andersen, TG; Bollerslev, T	1998	45.26
3	854	Fractionally integrated generalized autoregressive conditional heteroskedasticity	Baillie, RT; Bollerslev, T; Mikkelsen, HO	1996	34.16
4	828	The distribution of realized exchange rate volatility	Andersen, TG; Bollerslev, T; Diebold, FX; Labys, P	2001	41.4
5	792	Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options	Bates, DS	1996	31.68
6	784	Modelling asymmetric exchange rate dependence	Patton, AJ	2006	52.27
7	646	Forecasting volatility in financial markets: A review	Poon, SH; Granger, CWJ	2003	35.89
8	526	A forecast comparison of volatility models: Does anything beat a GARCH(1,1)?	Hansen, PR; Lunde, A	2005	32.88
9	504	Monetary policy and exchange rate volatility in a small open economy	Gali, J; Monacelli, T	2005	31.5
10	442	Micro effects of macro announcements: Real-time price discovery in foreign exchange	Andersen, TG; Bollerslev, T; Diebold, FX; Vega, C	2003	24.56

Scopus

R	TC	Titulo	Autor	Año	C/A
1	1343	Modeling and forecasting realized volatility	Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P.	2003	79.00
2	1008	Fractionally integrated generalized autoregressive conditional heteroskedasticity	Baillie, R.T., Bollerslev, T., Mikkelsen, H.O.	1996	42.00
3	932	Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options	Bates, D.S.	1996	38.83
4	922	The distribution of realized exchange rate volatility	Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P.	2001	48.53
5	688	Bayesian analysis of stochastic volatility models	Jacouier, E., Polson, N.G., Rossl, P.E.	1994	26.46
6	629	Asymmetric dynamics in the correlations of global equity and bond returns	Cappiello, L., Engle, R.F., Sheppard, K.	2006	44.93
7	604	A forecast comparison of volatility models: Does anything beat a GARCH(1,1)?	Hansen, P.R., Lunde, A.	2005	40.27
8	604	Monetary policy and exchange rate volatility in a small open economy	Galí, J., Monacelli, T.	2005	40.27
9	594	Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility	Andersen, T.G., Bollerslev, T., Diebold, F.X.	2007	45.69
10	581	Multivariate stochastic variance models	Harvey, A.	1994	22.35

Fuente: elaboración propia a partir de datos de Web of Science y Scopus 2019. Abreviaciones: TC= total de citas, C/A= citas por año

Tabla 3. Documentos más citados en volatilidad de tipo de cambio peso mexicano-dólar estadounidense 1980-2019

WoS

R	TC	Titulo	Autores	Año
1	42	Financial liberalization, exchange rates and stock prices: Exogenous shocks in four Latin America countries	Diamandis, Panayiotis F.; Drakos, Anastassios A.	2011
2	40	The Effects of Exchange-Rate Volatility on Commodity Trade between the United States and Mexico	Bahmani-Oskooee, Mohsen; Hegerty, Scott W.	2009
3	35	Exchange rate volatility and international trade: International evidence from the MINT countries	Asteriou, Dimitrios; Masatci, Kaan; Pilbeam, Keith	2016
4	29	Cross-correlations between crude oil and exchange markets for selected oil rich economies	Li, Jianfeng; Lu, Xinsheng; Zhou, Ying	2016
5	20	Revisiting the interest rate-exchange rate nexus: a Markov-switching approach	Chen, SS	2006
6	18	Depositary Receipts, country funds, and the Peso crash: The intraday evidence	Bailey, W; Chan, KL; Chung, YP	2000
7	16	Exchange rate USD/MXN forecast through econometric models, time series and HOWMA operators	Leon Castro, Ernesto; Aviles Ochoa, Ezequiel; Gil Lafuente, Anna Maria	2016
8	14	Oil price shocks and exchange rate movements	Volkov, Nikanor I.; Yuhn, Ky-hyang	2016
9	13	Exchange-rate volatility and industry trade between Canada and Mexico	Bahmani-Oskooee, Mohsen; Bolhassani, Marzieh; Hegerty, Scott	2012
10	13	Does the introduction of futures on emerging market currencies destabilize the underlying currencies?	Jochum, C; Kodres, L	1998

Scopus

R	TC	Titulo	Autores	Año
1	56	The effects of exchange-rate volatility on commodity trade between the united states and Mexico	Bahmani-Oskooee, M., Hegerty, S.W.	2009
2	42	Mexico's integration into the North American capital market	Adler, M., Qi, R.	2003
3	34	Exchange rate volatility and international trade: International evidence from the MINT countries	Asteriou, D., Masatci, K., Pilbeam, K.	2016
4	31	Cross-correlations between crude oil and exchange markets for selected oil rich economies	Li, J., Lu, X., Zhou, Y.	2016
5	22	Revisiting the interest rate-exchange rate nexus: A Markov-switching approach	Chen, S.-S.	2006
6	22	Debt composition and balance sheet effects of exchange rate volatility in Mexico: A firm level analysis	Pratap, S., Lobato, I., Somuano, A.	2003
7	21	Monetary policy and macroeconomic stability in Latin America: The cases of Brazil, Chile, Colombia and Mexico	De Mello, L., Moccero, D.	2011
8	19	Oil price shocks and exchange rate movements	Volkov, N.I., Yuhn, K.-H.	2016
9	16	Return and volatility linkages among International crude oil price, gold price, exchange rate and stock markets: Evidence from Mexico	Singhal, S., Choudhary, S., Biswal, P.C.	2019
10	13	Exchange-rate volatility and industry trade between Canada and Mexico	Bahmani-Oskooee, M., Bolhassani, M., Hegerty, S.	2012

Fuente: elaboración propia a partir de datos de Web of Science y Scopus 2019. Abreviaciones: explicadas en tabla 2

Entre los artículos más citados, el estudio se orienta principalmente a la relación de la volatilidad de tipo de cambio con otros indicadores y/o precios. Solo León-Castro, Avilés-Ochoa y Gil Lafuente abordan temas de modelación y pronóstico.

El estudio de volatilidad de tipo de cambio ha tenido importantes antecedentes desde la década de los 80, autores como Andersen y Bollerslev precursores de los modelos ARCH-GARCH, han optado por aplicar sus ideas en el precio de las monedas. Además, muchos de los trabajos más citados en el tema se han orientado al estudio del modelado, la estimación y

el pronóstico. Lo anterior con el objetivo de construir herramientas que puedan contrarrestar los efectos negativos que la volatilidad de tipo de cambio puede generar en inversionistas, empresas y gobierno. En este sentido, el presente estudio se orienta hacia una propuesta para un estimador de parámetros ARCH-GARCH que permita mejorar el pronóstico en el tipo de cambio peso mexicano dólar estadounidense.

FORMULACIÓN DEL PROBLEMA DE INVESTIGACIÓN

Los modelos de pronóstico de volatilidad ARCH-GARCH son cuestionados en cuanto a su capacidad de adaptarse a las características estadísticas de las series de tiempo financieras. Algunas de estas características derivan de situaciones como la no linealidad[5] (Dash et al., 2015; Wang et al., 2019), la no normalidad y comportamientos atípicos[6] (Raggi y Bordignon, 2006; Pinho et al., 2016;), y la imprecisión de variables de decisión subjetivas[7] (Tsaur, 2012; Xing et al., 2019).

La búsqueda de nuevas alternativas ha permitido mejorar y adaptar los modelos existentes utilizando procedimientos alternos. Un proceso utilizado para contrarrestar las deficiencias propias de una metodología es la combinación de dos o más metodologías con la finalidad de optimizar los resultados (Jick, 1979; Lahmiri, 2017; Xing et al., 2019). Para el caso de los modelos ARCH-GARCH existen numerosas propuestas comprobadas en la practica. Thavaneswaran et al. (2009) utilizan números difusos para moderar la complejidad de los datos y posteriormente aplicar modelos GARCH. D`Urso et al. (2016) proponen diferentes modelos de agrupaciones difusas para clasificar la heteroscedasticidad en series de tiempo y utilizarlo en el análisis de volatilidad. Por su parte Hafner (2008) propone una agregación temporal en los modelos GARCH y Cheng y Chen (2013) usan operadores de agregación OWA para agregar información según diferentes características de las series utilizadas en modelos GARCH.

[5] Hace referencia al hecho de que en el gráfico cartesiano los datos no se aproximan a una linea.

[6] Las distribuciones de frecuencia no forman una campana de Gauss

[7] Las variables que le afectan no se pueden definir fácilmente.

En este sentido, utilizar operadores propone la agregación de datos en el análisis de una regresión. Estos datos pueden ser las series históricas como tal o información proveniente del exterior que la serie no recoge. La presente investigación propone utilizar operadores de agregación OWA en el proceso de estimación de parámetros para un modelo de volatilidad ARCH-GARCH. El uso del operador (OWA) refleja el carácter y actitud de los tomadores de decisión, todo esto bajo situación de incertidumbre (Merigó, 2012), esto es, tiene la capacidad de recoger información relevante para la estimación proveniente de entornos de incertidumbre. Lo anterior mediante su proceso de re-orden y asignación de pesos que permite analizar resultados de mínimo a máximo. Esta característica convierte al operador OWA en una herramienta funcional en el análisis de mercados financieros, tal es el caso de selección de portafolio (Laengle et al., 2017; Li et al., 2019) y pronóstico (Kaur et al., 2016; León-Catro et al., 2018).

Desde la introducción del operador OWA, una serie de extensiones han sido propuestas con el objetivo de cubrir diferentes campos en la teoría de decisión. Estas extensiones pueden auxiliar en el proceso de estimación de la volatilidad debido a sus características que consideran diferentes tipos de información. En este sentido, para tratar relación entre variables Yager y Filev (1999) proponen el operador OWA inducido (IOWA). Con el objetivo de generalizar el operador OWA, Yager (2004) introduce el operador generalizado (GOWA). El operador de agregación que combina información probabilística con la familia parametrizada OWA es el operador probabilístico (POWA) (Merigó, 2012), el cual añade un vector adicional de probabilidad que es ponderado con el vector de pesos existente. Por su parte, el operador ponderado OWA (OWAWA) (Merigó, 2009) propone el uso de un vector de pesos adicionales para introducir información posibilista que puede ser utilizado en escenarios de incertidumbre.

Tanto el operador OWA como sus extensiones pueden ser utilizadas en el proceso de estimación de modelos. Este trabajo utiliza el procedimiento de estimación de mínimos cuadrados ordinarios (MCO) donde los operadores sustituyen las medias aritméticas. La principal ventaja es crear un estimador que pueda recoger información no probabilística de los escenarios analizados. Lo anterior, con el objetivo de obtener pronósticos adaptados a los objetivos del tomador de decisión esperando prevenir las tendencias de las series (no

linealidad, no normalidad). En base a esto, se plantea la siguiente pregunta central de investigación:

¿Cuál es la efectividad del estimador MCO con operadores de agregación OWA en el pronóstico de modelos de volatilidad ARCH-GARCH, cuyas series de tiempo presentan no linealidad, no normalidad y comportamientos atípicos causados?

Siguiendo este camino, se desprende la siguiente interrogante secundaria:

¿Cuál es la medida de volatilidad que mejora el porcentaje de error de pronóstico de la metodología MCO con el operador OWA en modelos de volatilidad ARCH-GARCH?

JUSTIFICACIÓN

Las fluctuaciones del tipo de cambio presentan complejidad en su análisis debido a patrones complicados en las series de tiempo (no linealidad, cambios abruptos y no normalidad). En este sentido, a corto plazo, los tipos de cambio son muy difíciles de predecir utilizando modelos económicos (Engel et al., 2012; Rossi, 2013). En particular, para situaciones de incertidumbre, se encuentra que un modelo teórico simple, como el recorrido aleatorio, genera mejores pronósticos en tipo de cambio (Meese y Rogoff, 1983; Zorzi et al., 2017; Ren et al., 2019).

Entre los modelos populares que recogen recorridos aleatorios se encuentran los modelos de volatilidad ARCH-GARCH. Las variables en estos modelos teóricamente pueden contener muchos de los hechos estilizados de las series financieras. Sin embargo, estos modelos han presentado deficiencias en momentos de gran incertidumbre (Yu, 2002; Chan y Grant, 2016).

Cuando el responsable de la toma de decisiones no tiene información probabilística completa estará en un problema de indecisión bajo un entorno de riesgo e incertidumbre (Merigó, 2012). Los métodos de regresión difusa ofrecen una herramienta adecuada para manejar la imprecisión de las mediciones, la problemática de factores insuficientes y la confusión de la relación entre variables (Thavaneswaran et al., 2007; Tsaur, 2012; Bose y Mali, 2019).

Los sistemas difusos pueden ser una herramienta útil en la modelación y el pronóstico. Al respecto Muzzioli y Baets (2013) describen los casos en los que la modelación difusa puede ser de gran utilidad:

- La regresión difusa puede adaptarse tanto a datos nítidos como a datos difusos siendo estos cualitativos como cuantitativos.
- Cuando solo se dispone de un número limitado de observaciones y no es posible llevar a cabo suficientes experimentos para obtener resultados favorables
- Cuando los datos son imprecisos
- Cuando hay dificultad en verificar los supuestos distributivos
- Cuando la relación lineal entre las variables es inapropiada

Se debe tener en cuenta que las metodologías difusas no son de mucha utilidad si el modelo matemático de un proceso es claro en precisión y comportamiento de las variables. Sin embargo, es seguro que, cuando los factores que conducen a la incertidumbre aumentan, amplificando la complejidad del problema, los modelos analíticos comienzan a mostrar sus límites y los sistemas difusos son más adecuados (Maturo y Hosková-Mayerová, 2016).

En el caso de los operadores OWA, estos permiten subestimar o sobreestimar resultados acordes a las preferencias del tomador de decisión (Yager y Filev, 1999). Cuando se utilizan operadores OWA en escenarios financieros, sus características pueden recoger expectativas del mercado o situaciones conocidas por el tomador de decisión mediante su sistema de re orden y asignación de pesos. Esto permite que podamos analizar la información de mínimo a máximo considerando gran número de escenarios diferentes entre los que se encuentran situaciones donde las series de tiempo presentan irregularidades.

En este sentido, el estimador MCO que utiliza operadores de agregación OWA calcula parámetros para una regresión tipo ARCH-GARCH que agrega información adicional no probabilística, lo cual puede ajustar la estimación a comportamiento de las series financieras como lo es la no linealidad, cambios abruptos y no normalidad.

OBJETIVOS

Una mejor previsión es un elemento importante para una mejor toma de decisiones, sobre todo en la creciente volatilidad de los mercados financieros y en los flujos de capital internacionalizados (Gleason y Lee, 2003; Bustos, 2020). Desde su concepción, los modelos ARCH-GARCH han sido utilizados para el pronóstico con resultados favorables (Badescu et al., 2019; Fang et al., 2020; Zhou et al., 2020). Sin embargo, en diversos periodos marcados por inestabilidad estos modelos han presentado ineficiencias (Hajizadeh et al., 2012; Kim y Hwang., 2019). En este sentido, este trabajo propone un estimador de modelos ARCH-GARCH compuesto de MCO y operadores de agregación OWA, el cual, promete contrarrestar las ineficiencias en el pronóstico de estos modelos.

El estimador MCO es por naturaleza un estimador lineal (Bettis y Fairlie, 2001; Kilmer y Rodríguez, 2017). Por su parte una característica importante en el reordenamiento de los operadores OWA es que lo convierte en un operador no lineal. Lo anterior permite insertar a la metodología MCO una herramienta no lineal, la cual adicionalmente puede ser sobreestimada o subestimada con la asignación de pesos de acurdo a la información disponible del tomador de decisión. Por consiguiente, se tiene un estimador con resultados que pueden ser manejados mediante información posibilista, lo cual puede auxiliar al pronóstico en casos de incertidumbre. Ante esto, la presente investigación tiene como base el siguiente objetivo.

Determinar la efectividad del estimador MCO con operadores de agregación OWA en el pronóstico de modelos de volatilidad ARCH-GARCH, cuyas series de tiempo presentan no linealidad, no normalidad y comportamientos atípicos.

Lo anterior basándose en el siguiente objetivo específico:

Identificar la medida de volatilidad que mejora el porcentaje de error de pronóstico de la metodología MCO con el operador OWA en modelos de volatilidad ARCH-GARCH.

HIPÓTESIS

En las últimas décadas, se ha prestado una creciente atención a la previsibilidad de la volatilidad de los datos financieros proponiendo modelos híbridos para superar las limitantes de los modelos tradicionales (Lahmiri, 2017; Trapero y Kourentzes, 2019). Entre estas propuestas, destacan el uso de metodologías difusas y sistemas de agregación como metodologías adaptables a los comportamientos de los datos financieros (Cheng y Chen, 2013; D`Urso et al., 2016). Los modelos de series de tiempo difusas pueden ser funcionales para pronosticar el valor futuro en una situación en la que no se visualiza una tendencia o patrón en la variabilidad de las series (Tsaur, 2012).

En este sentido, los sistemas de agregación actúan recogiendo información, la cual puede aproximar la estimación a la realidad. Este trabajo utiliza operadores de agregación OWA para lograr ese objetivo. Estos operadores pueden obtener buenos resultados para datos no certeros (Yager, 1990) debido a que cuenta con un proceso de re-orden y asignación de pesos que genera una media que va de mínimo a máximo. Entre este rango diversas situaciones pueden ser analizadas, destacando los periodos extremos donde muchos de los problemas de estimación y pronóstico se presentan. Para el caso de la volatilidad una situación de análisis importante es los periodos de alta volatilidad donde la no linealidad, cambios abruptos y no normalidad están presentes. En este caso el tomador de decisión podrá sobreestimar las medias OWA acorde a su conocimiento sobre la información disponible de un fenómeno que afecta la volatilidad.

El estimador MCO que utiliza operadores OWA en una misma formulación tiene como objetivo utilizar las bondades del operador para introducir información a los parámetros y obtener estimaciones adaptables a la realidad.

En este sentido, este trabajo desarrolla la siguiente hipótesis de investigación:

El estimador MCO que incluye operadores de agregación OWA en la misma formulación, puede minimizar el error de pronóstico en la volatilidad utilizando modelos ARCH-GARCH, para series de tiempo con características de no linealidad, no normalidad e imprecisas.

Lo anterior es reforzado con a la hipótesis que señala la efectividad del estimador de MCO y operadores OWA minimizando el error de pronóstico aun cuando la volatilidad es medida con diferentes metodologías.

CAPITULO II: CONSIDERACIONES TEÓRICAS DE LA VOLATILIDAD DE TIPO DE CAMBIO

Explicar y pronosticar los movimientos en los tipos de cambio nominales puede presentar dificultades prácticas (Meese y Rogoff, 1983). El comportamiento de series históricas generalmente recorre un camino aleatorio, es decir, las tasas de crecimiento son eventos independientes entre sí. Fama (1965) desarrolla la hipótesis de mercados eficientes donde justifica la imposibilidad de predecir los rendimientos de los activos financieros y sostiene que el proceso estocástico subyacente a los rendimientos es un proceso martingala[8].

Investigaciones posteriores han demostrado que las series financieras además de obedecer los supuestos éticos de estas dos propuestas pueden presentar algunos datos estilizados como no linealidad, no normalidad, agrupamiento de volatilidad, asimetría y colas pesadas (Yang y Chen, 2014; Patton y Sheppard, 2015; Shen et al., 2015; Pinho et al., 2016; Byrne et al., 2016). Por lo tanto, es importante minimizar el riesgo cambiario ante las repentinas apariciones de cualquiera de estas características.

La volatilidad es hasta cierto punto predecible (Martens y Dijk, 2017), numerosos estudios han tratado de demostrar la factibilidad del pronóstico tomando como base la problemática de la incertidumbre cambiaria (Wang, 2009; Ghysels y Sinko, 2011; Andersen et al., 2011; Valeyre et al., 2013; Pinho et al., 2016).

A continuación, se presenta un repaso de las características de la volatilidad en las series financieras. Adicionalmente, se analizan brevemente los modelos mayormente utilizados en la estimación y pronóstico, y se mencionan las metodologías de estimación utilizadas en diferentes contextos.

[8] Un proceso martingala demuestra la inexistencia de una estrategia ganadora cuando existe un juego justo.

HECHOS ESTILIZADOS DE LAS SERIES FINANCIERAS: EL CASO DEL TIPO DE CAMBIO

Los movimientos en los precios de los activos financieros se originan teniendo en cuenta un enfoque basado en eventos (Andersen et al, 2007). En este sentido, la dinámica de los precios difiere entre sí, pues cada activo responde a fenómenos y noticias diferentes. Sin embargo, las variaciones que aparentemente pueden ser al azar, comparten y exhiben algunos rasgos estadísticos característicos. Estas propiedades son comunes a través de una amplia gama de instrumentos y de mercados a lo largo del tiempo y se denominan hechos estilizados.

De esta forma, conocer los hechos estilizados del tipo de cambio puede ser propio de la serie que estemos utilizando, ya que, mucho tiene que ver con las características particulares en cada caso. En las siguientes secciones se hace un breve repaso de algunos de los principales hechos estilizados comunes en las series financieras y se analiza su presencia en el tipo de cambio peso mexicano-dólar estadounidense.

Proceso Martingala en los rendimientos

Una martingala (Harnesses, 1965) puede ser definida como un proceso estocástico $(x_n)_{n\epsilon N}$ tal que x_n representa la ganancia al tiempo n de un jugador involucrado en un juego justo respecto a la información dada. Los procesos martingala tienen aplicaciones a varios problemas estocásticos y analíticos que tienen que ver con modelación, elección de portafolio y pronóstico (Schachermayer, 2003; Zheng et al, 2016; Najafi y Mehrdoust, 2017).

En este sentido, una martingala sigue una secuencia de variables aleatorias en la que, en un tiempo dado, la esperanza condicional del siguiente valor de la secuencia es igual al valor presente. Lo anterior está dado por las siguientes características:

- Sea $(\Omega, \mathcal{F}, \mathbb{Q})$ un espacio de probabilidad y $(\mathcal{F}_n)_{n\epsilon N}$ una colección de $\sigma - álgebras$ contenidas cada una en $\mathcal{F}$. Se dice que esa colección es una filtración si $\mathcal{F}_n \subset \mathcal{F}_m$ cuando $n \leq m$. Donde f_n es la información acumulada al tiempo n.

- De esta forma una colección de variables aleatorias reales $(X_n)_{n\epsilon N}$ es una martingala respecto de la filtración si considera lo siguiente:

1. $X_n\ es\ \mathcal{F}_n - medible$
2. $X_n \epsilon L_1$
3. $\mathrm{E}(X_{n+1}\ /\ \mathcal{F}_n) = X_n$ para cualquier $n\ \epsilon\ N$

Esto es, se debe tener una sucesión de variables aleatorias adaptables a la filtración, una esperanza condicional como un operador lineal y un juego justo con respecto a la información proporcionada. Los procesos martingala son fundamentales para la teoría moderna de la probabilidad, pues la herramienta teórica sobre la cual se construye la teoría moderna de las finanzas matemáticas y el cálculo estocástico es una idea basada en las martingalas.

Mercados eficientes y correlación

Un mercado en el cual los precios reflejan completamente la información disponible es un mercado eficiente (Fama, 1970). En este sentido, Lo and Mackinlay (1988) prueban que los precios de un activo financiero cuyo mercado es eficiente sigue un comportamiento de caminata aleatoria o martingala, en el cual el pronóstico del día de mañana puede calcularse con la información del día de hoy. Sin embargo, si la hipótesis del mercado eficiente se cumple en su totalidad el juego será justo y los precios no pueden ser completamente predecibles.

Por otro lado, la Psicología y la economía del comportamiento argumentan que la hipótesis de los mercados eficientes se basa en suposiciones no factuales con respecto al comportamiento humano, esto es, la irracionalidad (Lo, 2008). Al respecto Thaler (1994) explica que existen comportamientos cuasi racionales importantes en algunas situaciones definidas, las personas toman decisiones que son sustancialmente diferentes de las predichas por los modelos económicos estándar. En ese sentido, en los mercados puede existir numerosos factores por los cuales no son completamente eficientes.

Retornos TC Fix Retornos al cuadrado TC Fix

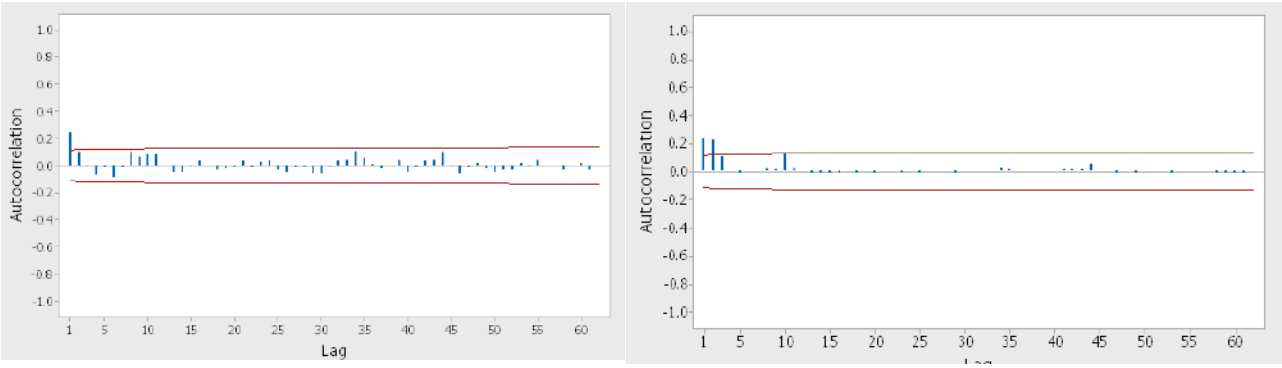

Figura 4. Correlogramas

tipo de cambio peso-dólar.

Para conocer la eficiencia de un mercado se han desarrollado diferentes metodologías. En el caso de los correlogramas, si el mercado del activo financiero es eficiente las correlaciones de los rendimientos no son significativas, mientras que las correlaciones de los rendimientos al cuadrado si lo son (Hamid et al, 2010).

En general, todos los retornos de activos financieros no presentan correlación. En el caso del tipo de cambio peso mexicano-dólar estadounidense, la figura 4 muestra una correlación de orden 1. Por su parte, los retornos al cuadrado presentan una estructura de correlación que sugieren una débil significancia, esto es, que la varianza condicional no es constante en los primeros ordenes, pero a partir del 2-3 si lo es. Lo anterior pone en duda la eficiencia del mercado cambiario en México. Los modelos de heteroscedasticidad condicional presentan una estructura de correlación de volatilidad que decae exponencialmente, tal y como sucede empíricamente en los retornos al cuadrado del tipo de cambio.

La no normalidad de los rendimientos

En el estudio estadístico de las series financieras es conveniente asumir que estas presentan una distribución normal. La normalidad involucra que los datos se agrupan alrededor de la media, lo cual facilita el análisis del pronóstico al considerar probabilísticamente que el

resultado se debe encontrar en un rango conocido. Sin embargo, estudios como los de Fama (1965), Taylor (2005), y Franses y Dijk (2003) muestran que la distribución empírica en rendimientos financieros presentan colas pesadas (curtosis mayor a tres), apalancamiento y en algunos casos asimetrías.

La existencia de estos escenarios dificulta el estudio de las series de tiempo, en tanto que debe existir un análisis mucho más profundo para conocer las características propias de cada serie. En ese sentido, han surgido una serie de modelos y métodos de estimación adaptables a las características de no normalidad persistentes en los datos (Trucíos y Hotta, 2016; Kastner et al., 2017).

Para el caso del tipo de cambio peso mexicano-dólar estadounidense, la figura 5 muestra como los rendimientos no llevan un comportamiento normal, presentando contrariamente una alta curtosis y un sesgo hacia la derecha. Esto resalta los grandes saltos que han existido en la serie durante diferentes periodos.

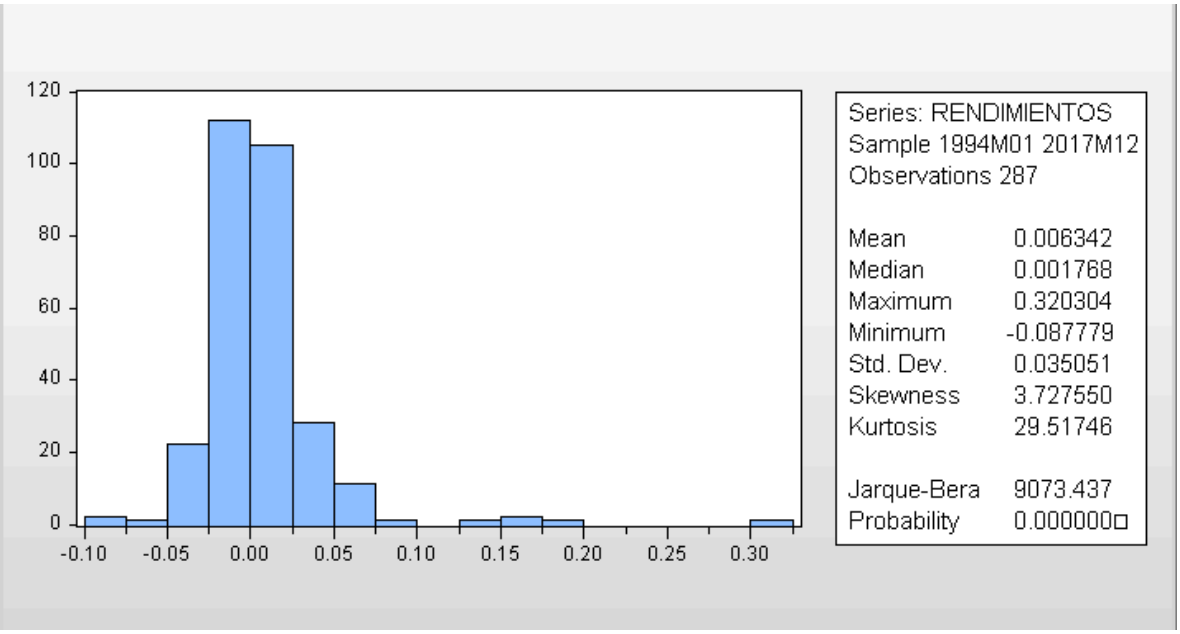

Figura 5. Distribución de frecuencias

Fuente: Elaboración propia a partir del tipo de cambio FIX peso mexicano-dólar estadounidense.

Agrupamiento de volatilidad y su asimetría

Según las actividades comerciales de los inversionistas fundamentales[9] y los movimientos de los seguidores de las tendencias, el precio del mercado fluctúa alrededor de un valor establecido de los activos con una baja o alta volatilidad dadas unas condiciones del entorno financiero (He et al., 2016). Esto es, los grandes movimientos en compra y venta de productos financieros generan tendencias de alta o baja volatilidad.

Con la existencia de ruido en los mercados financieros se genera un cambio irregular entre los dos regímenes de volatilidad (baja y alta volatilidad), por lo tanto, conduce a la agrupación de volatilidad (Gaunersdorfer et al., 2008; Ning et al., 2014; Hainaut, 2016). Grandes movimientos de volatilidad son seguidos por grandes movimientos de volatilidad caracterizados por la existencia de inestabilidades en los mercados, a su vez, pequeños movimientos de volatilidad son seguidos de pequeños movimientos de volatilidad caracterizados por periodos de relativa tranquilidad en los mercados.

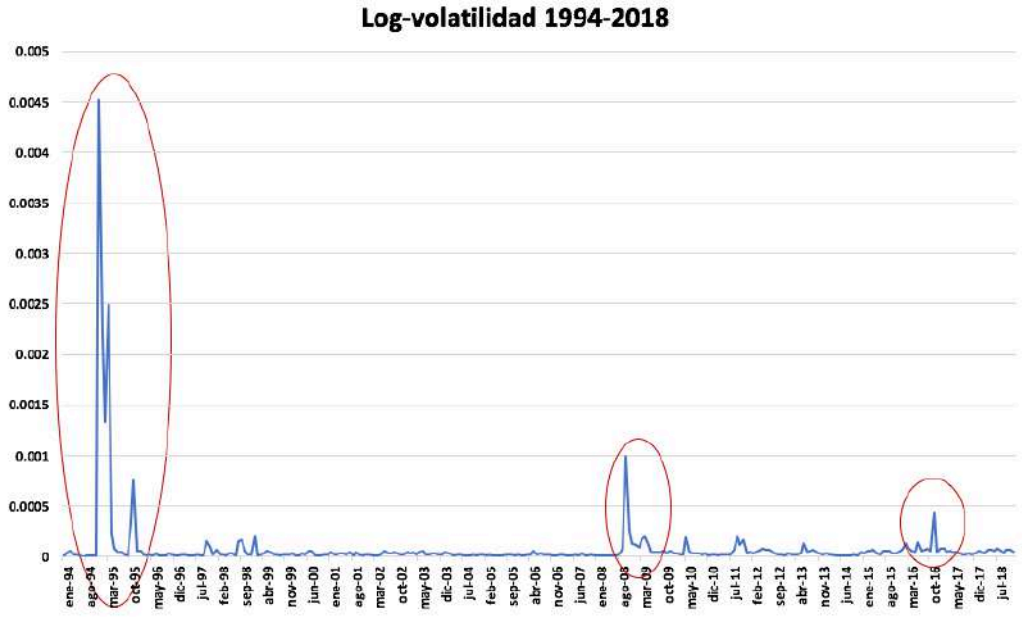

Figura 6. Agrupamiento de log-volatilidad.

Fuente: tipo de cambio peso mexicano-dólar estadounidense.

[9] Aquellos inversionistas que por sus grandes transacciones o gran influencia tienen un alto impacto en el mercado

La alta volatilidad de los rendimientos tiende a agruparse más a menudo que las bajas volatilidades de los rendimientos, esta característica es comúnmente denominada asimetría de la volatilidad (Ning et al., 2014). En épocas de volatilidad al alza es muy común encontrar movimientos mucho mayores que en épocas de volatilidad a la baja. En este sentido, trabajos como los de Maciel et al. (2016), Patton y Sheppard (2015) y Chimrani et al. (2018) consideran la asimetría en la volatilidad como un componente importante en los modelos y métodos de pronóstico.

Como lo muestra la figura 6, el agrupamiento de la volatilidad tiende a marcarse en periodos de alta turbulencia. Después de las grandes inestabilidades quedan secuelas marcadas con inestabilidades notorias. Lo anterior, provoca que los modelos lineales de estimación y pronóstico tiendan a fallar en los periodos señalados.

MODELOS DE HETEROSCEDASTICIDAD CONDICIONAL

La mayoría de los resultados empíricos de la literatura de volatilidad se basan en el uso de datos de frecuencia diarias junto con formulaciones de modelos de heteroscedasticidad condicional para estimar y pronosticar la variabilidad (Andersen et al., 2011). Engle (1982) desarrolló un método estocástico para estimar la volatilidad llamado modelo autorregresivo de heteroscedasticidad condicional (ARCH). En este modelo la varianza condicional depende del cuadrado de los rezagos de los términos aleatorios llamados errores o choques. Para este tipo de procesos, el pasado reciente proporciona información sobre la varianza de un periodo. Por lo tanto, la previsión de un valor de hoy se basa en la información pasada, todo ello bajo ciertos supuestos estándar (Engle, 1982). El modelo general queda como sigue:

$$y_t = \epsilon_t \quad donde\ \epsilon_t\ /\ \Psi_{t-1} \sim N(0, h_t) \tag{1}$$

$$u_t = \frac{\epsilon_t}{\sqrt{h_t}} \tag{2}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2, \tag{3}$$

donde u_t es el residual estandarizado, y h_t el componente de volatilidad, en el cual α_1 representa la magnitud en la que se considera el error anterior en el periodo actual. Los modelos ARCH han tenido éxito en la captura de la dinámica en la varianza utilizando modelos paramétricos simples (Ghysels et al., 2006). Sin embargo, en las aplicaciones empíricas se suele requerir un retraso relativamente largo en la ecuación de la varianza condicional e imponer una estructura de retardo fijo para conservar su funcionalidad.

Bollerslev (1986), amplió la clase ARCH para permitir tanto una memoria más larga, como una estructura de retraso flexible y propuso el modelo autorregresivo de heteroscedasticidad condicional generalizado (GARCH). En este la varianza condicional además de depender de los cuadrados de las perturbaciones pasadas, ahora también está en función de las varianzas condicionales de períodos anteriores. Se representa como sigue:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \tag{4}$$

donde β_1 mide la memoria que tiene la serie, esto es, la cantidad de volatilidad del día anterior conservada hoy. Para asegurar que el proceso de varianza es siempre positiva y estacionaria se debe cumplir que $\alpha_0 > 0, \beta_1 \geq 0, \beta_2 \geq 0$ and $\sum_{i=1}^{\max(p,q)} (\beta_1 + \beta_2) < 1$. Los modelos GARCH reflejan la dependencia no lineal de la varianza condicional de la serie de tiempo estimando una media y varianza condicionales, combinando ARCH y modelos de promedios móviles autoregresivos.

En años recientes trabajos como los de Valeyre et al. (2013), Pinho et al. (2016), continúan utilizando las bases de los modelos autorregresivos desarrollados por Engle y Bollerlev, a los cuales se le añaden nuevas metodologías para mejorar su eficiencia.

MÉTODOS DE ESTIMACIÓN DE PARÁMETROS

Método de mínimos cuadrados ordinarios

La regresión lineal estadística (LR) es una herramienta para el análisis y la suma de información utilizada en desarrollar modelos matemáticos en negocios (Maturo y Hoskova-mayerova, 2017), pronóstico económico (Chudik et al., 2018), ciencias naturales (Kilmer y Rodríguez; 2016), ingeniería (Braun et al., 2014), entre otros. La LR es un modelo que cuenta con ventajas frente a otros modelos tales como la simplicidad en su aplicación y la parsimonia (Dimson y Marsh, 1990; Raposo; 2016).

El método pronostica bajo el supuesto de continuar la correlación entre las variables en el futuro (Song et al, 2005). Su funcionamiento se establece con una variable dependiente y acompañada con sus subsecuentes variables independientes $x_1, x_2, x_3 \ldots\ldots x_n$. El modelo de regresión lineal en su forma simple queda representado como sigue:

$$\hat{y} = \beta_0 + \sum_{j=1}^{p} \beta_j x_j, \tag{5}$$

donde y representa la variable dependiente y los parámetros beta ($\beta_0 \ldots\ldots \beta_j$) son los coeficientes a estimar en la regresión lineal. Un método popular para estimar los parámetros es el de mínimos cuadrados ordinarios (MCO) cuya idea fue introducida por Cavalli-Sforza y Edwars (1967) en el área genética y subsecuentemente desarrollado para el área de las ciencias económico financieras por Cox (1975).

El MCO utiliza una línea media para ajustar datos, por lo tanto, la distancia vertical desde cada punto de datos a la línea se minimiza en todos los puntos (Heshmaty y Kandel, 1985). El objetivo es encontrar la mejor curva que minimiza la suma residual de cuadrados (Raposo, 2016). La forma general de MCO se denota como sigue:

$$min \sum (\sigma_i^2 - \hat{\sigma}_i^2)^2, \tag{6}$$

donde σ_i^2 representa la variabilidad observada, y $\hat{\sigma}_i^2$ la variabilidad estimada o pronosticada. El MCO facilita el trabajo en muestras grandes con tendencia lineal (Bettis y Fairlie, 2001; Kilmer y Rodriguez, 2017). Sin embargo, esta linealidad puede afectar su desempeño en datos con comportamientos atípicos (Horrace y Oaxaca, 2006; Romano y Wolf, 2016).

El MCO puede ser aplicado tanto en regresiones simples como en regresiones múltiples. Para el caso de regresión simple, encontrar los valores de α y β que minimizan la expresión anterior se calcula como sigue:

$$\beta = \frac{Cov(x,y)}{var(x)} = \frac{\sum_{k=1}^{k}(x_k - \bar{x})(y_k - \bar{y})}{\sum_{k=1}^{k}(x_k - \bar{x})^2}, \tag{7}$$

$$\alpha = \bar{y} - \beta\bar{x}, \tag{8}$$

donde $\bar{x}$ y $\bar{y}$ son las medias en los sets x_k, y_k respectivamente.

En el caso de la regresión múltiple los cálculos se extienden según el número de variables independientes que se analicen en un modelo. Considerando una regresión múltiple con dos variables independientes $y_j = \alpha + \beta_1 x_j + \beta_2 z_j$, los parametros estan dados por las siguientes ecuaciones:

$$\alpha = \bar{y} - \beta_1\bar{x} - \beta_2\bar{z} \tag{9}$$

$$\beta_1 = \frac{(\sum y_i x_i)(\sum z_i^2) - (\sum y_i z_i)(\sum x_i z_i)}{(\sum x_i^2)(\sum z_i^2) - (\sum x_i z_i)^2} = \frac{[Cov(y,x)][var(z)] - [Cov(y,z)][Cov(x,z)]}{[var(x)][var(z)] - [Cov(x,z)]^2} \quad (10)$$

$$\beta_2 = \frac{(\sum y_i z_i)(\sum x_i^2) - (\sum y_i x_i)(\sum x_i z_i)}{(\sum x_i^2)(\sum z_i^2) - (\sum x_i z_i)^2} = \frac{[Cov(y,z)][var(x)] - [Cov(y,x)][Cov(x,z)]}{[var(x)][var(z)] - [Cov(x,z)]^2}, \quad (11)$$

donde *cov* es la covarianza [(*z,y)(x,y)(x,z*)], entonces $cov = \sum_{k=1}^{k}(a_k - \bar{a})(b_k - \bar{b})$, y *var* es la varianza (*x;z*), donde $var = \sum_{k=1}^{k}(a_k - \bar{a})^2$. Las varianzas y los valores estándar del estimador MCO son estudiadas en (Gujarati & Porter, 2009).

Método de máxima verosimilitud

Desde que Fisher (1992; 1925) analizó el método de máxima verosimilitud, este se ha convertido en una de las herramientas comúnmente utilizadas para la estimación y pronóstico estadístico. El método de máxima verosimilitud (MLE) se utiliza para estimar una variedad de modelos, como lo son los modelos lineales, análisis factorial, análisis de componentes factoriales y modelos de ecuaciones estructurales (Snoke et al, 2018).

Una suposición fundamental en el MLE es que se conoce el comportamiento estocástico que determina un fenómeno de investigación, mediante una función de probabilidad (White, 1982; Jiao et al., 2017). Esta función probabilística llamada comúnmente función de

verosimilitud se construye mediante una función de densidad de probabilidad conjunta de los datos dados en términos de un conjunto de parámetros, de modo que:

$$\left(f(y_1, y_2, \dots., y_t \setminus \emptyset)\right), \tag{12}$$

donde $y_1, y_2, \dots, y_t$ son los rendimientos del activo financiero de 1 a t, y ϕ es el vector de parámetros que se desea estimar. El objetivo del MLE es maximizar la función de verosimilitud mediante la derivada de esta. Sin embargo, la estructuración de la función de verosimilitud en algunos casos puede ser compleja y no se puede llegar a una solución en la forma simple de MLE (Allman et al, 2018).

El MLE es usualmente utilizado para estimar los parámetros en los modelos tipo GARCH (Pedersen y Rahbek, 2016; Hansen y Huang, 2016; Augustyniak et al., 2018). En estos modelos se busca la función de los rendimientos de los precios y el resultado es aquel que maximiza el comportamiento de esta. El objetivo es conocer el comportamiento adecuado de la función para hacer una estimación y un buen pronóstico en cualquier variable de serie de tiempo.

CAPITULO III: LOS OPERADORES DE AGREGACIÓN OWA

Las series temporales financieras presentan patrones como lo son la no linealidad, los cambios abruptos, la no estacionalidad y agrupamientos de volatilidad, donde las variables que intervienen no tienen una clara tendencia a moverse a un valor fijo. Esto provoca cierto nivel de vaguedad y confusión en su estudio (Fuller y Maijlender, 2003; Chang y Liu 2008; Korol 2014).

En un entorno de información imperfecta, la lógica difusa puede verse como un intento de formalización de la capacidad para tomar decisiones racionales (Zadeh, 2008). Después de que Zadeh (1965) introdujo conectivos de conjuntos borrosos de múltiples valores y Sugeno (1974) aplica las llamadas medidas difusas e integrales, se rompen las fronteras entre máximos, mínimos y medias aritméticas en el estudio de series de datos complejos, trayendo consigo el abordaje del estudio de la familia de operadores de promedio.

Cuando se analiza información, es muy común revisar una gran cantidad de aspectos que deben recopilarse y mostrarse en resultados simples. Los operadores de agregación pueden seleccionar información diferente y proporcionar un resultado sencillo o práctico (Merigó et al., 2016). Muchos operadores de agregación se han desarrollado con el objetivo de incluir información. En el caso de operadores de máximos y mínimos podemos encontrar el Operador de promedio aritmético (AA) (Mamdani, 1974; Marichal, 2001), el Operador de promediado ponderado difuso (FWA) (Aczel, 1984; Poyhonem y Hamalainen, 2001), el Operador de promedio geométrico (GA) (Dong y Wong, 1987; Kao y Liu, 2001) y el operador de promedio ponderado ordenado (OWA) (Yager, 1992; Liu y Wang, 2016).

En este apartado se describe el funcionamiento del operador de agregación OWA, sus características y algunas extensiones que se han propuesto desde su concepción.

BREVE RESEÑA DEL OPERADOR OWA

El operador de promedio ponderado ordenado (OWA) introducido por Yager (1988) es un operador de agregación muy popular en entornos de incertidumbre. Este proporciona una

familia parametrizada de operadores de agregación entre el máximo, el mínimo y el promedio. Por lo tanto, el operador OWA es capaz de subestimar o sobreestimar los resultados iniciales de la persona que toma las decisiones de acuerdo con un grado de optimismo o pesimismo (Merigó et al., 2012). De esta forma el operador de agregación OWA trabaja con múltiples criterios que consisten en adoptar una postura determinada por parte de los decisores, estos criterios son los siguientes:

- Criterio optimista. Supone que se presentara la situación más favorable. Por tanto, se busca elegir el resultado más favorable posible, este método también es llamado maximax pues se elige el máximo de los máximos. Se presenta como sigue:

$$Decisión = Max\{E_i\} = Max\left[Max\{a_j\}\right]. \tag{13}$$

- Criterio pesimista o de Wald (1950). Supone que se presenta la situación más desfavorable posible. Por lo tanto, el resultado consiste en elegir el mejor resultado de entre los peores. Este método se conoce como método maximin, esto es, elegir de los mínimos beneficios el mayor de todos:

$$Decisión = Max\{E_i\} = Max\left[Min\{a_j\}\right]. \tag{14}$$

- Criterio de Hurwicz (1951). Representa una combinación entre el criterio optimista y e criterio pesimista. Realiza una ponderación con un coeficiente de optimismo y otro de pesimismo, para posteriormente sumarlo y elegir la alternativa que proporcione un mejor resultado.

$$Decisión = Max\{E_i\} = Max\left[\alpha Max\{a_j\} + (1-\alpha)Min\{a_j\}\right]. \tag{15}$$

donde $\alpha + (1-\alpha) = 1$

- El criterio de Laplace (1951). Implica asociar un mismo grado de probabilidad a las distintas situaciones o estados de la naturaleza, mientras no haya algo que indique lo contrario:

$$Decisión = Max\{E_I\} = Max\left[\left({}^{1}/_{n}\right)\sum_{j=1}^{n} a_j\right]. \quad (16)$$

- Criterio de Savage (1951). Considera que para cada situación o estado existe una estrategia óptima, para ello se le asigna un valor de costo de oportunidad a cada estrategia. Posteriormente se elige el mayor costo de oportunidad para cada alternativa y de entre estos se elige el menor. Este método es conocido como minimax:

$$Decisión = Min\{E_j\} = Min[Max\{s_j\}], \quad (17)$$

donde: $s_j = Max\{a_j\} - a_j$

La idea básica de los operadores OWA es colocar pesos en los componentes de los vectores de clasificación después de una separación preliminar de las calificaciones individuales. La otra contribución crucial de este operador es conectar estos pesos con cuantificadores difusos (Yager et al., 2011). Se define como sigue.

Un operador OWA de dimensiones n es un trazo $f: R^n \rightarrow R$ que tiene un vector asociado n $W = (w_1, w_2, \dots., w_t)^t$ tal que $W_j \in [0,1], \sum_{j=1}^{n} w_j = 1$. De esta forma su construcción general queda como sigue:

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad (18)$$

donde b_j es el largo del vector de las observaciones a_i, mientras que w_j representa el vector de los pesos asignados a cada observación.

Debe de tomarse en cuenta que, si $W = (1,0,\dots,0)^T$, entonces f es reducido al máximo operador; si $W = (0,0,000,1)^T$, en este caso f es reducido al operador mínimo; si $W = (1/n, 1/n, \dots, 1/n)^T$, entonces es reducido al operador de promedio aritmético.

El aspecto fundamental del operador OWA es reordenar los pesos, esto es, un argumento a_i no está asociado con un peso en particular w_i , de esta forma, un peso w_i está asociado con una posición particular de ordenamiento *i* de las observaciones (Xu y Da, 2003).

El operador OWA es conmutativo, monótono, cerrado e idempotente (Yager, 1988).

- Conmutativo, pues $OWA(a_1, a_2, \dots, a_n) = f(e_1, e_2, \dots, e_n)$ donde $(a_1, a_2, \dots, a_n)$ es cualquier permutación de argumentos $(e_1, e_2, \dots, a_n)$.
- Monótono pues $a_i \geq u_i$, para todos a_i, entonces, $OWA(a_1, a_2, \dots, a_n) \geq OWA(u_1, u_2, \dots, u_n)$.
- Cerrado porque la agregación es delimitada por el mínimo y el máximo, esto es, $Min\{a_i\} \leq OWA(a_1, a_2, \dots, a_n) = a$.
- Idempotente pues si $a_i = a$, para todos los a_i, entonces, $OWA(a_1, a_2, \dots a_n) = a$.

Las características implícitas en los operadores de agregación OWA lo convierten en una familia de modelos que refleja el carácter actitudinal (grado de optimismo) de las preferencias en los mercados (Merigó, 2012), esto ante una situación de toma de decisiones bajo incertidumbre. La familia de operadores OWA ha sido utilizada en un rango de aplicaciones como toma de decisiones (Xu, 2006; Liu y Wang, 2016), selección de portafolio (Zhang y Zhang, 2004; Singh et al., 2010) y pronóstico (Cheng et al., 2013; Leon-Catro, 2018).

EXTENSIONES DEL OPERADOR DE AGREGACIÓN OWA

Desde su introducción, el operador OWA ha sido extendido en diversos caminos, esto crea una familia de operadores de agregación con base en la agregación OWA. Este apartado presenta una breve reseña de las características y funcionamientos de los operadores IOWA, GOWA, OWAWA, IGOWA, IPOWA, IOWAWA e IPOWAWA.

Una de las extensiones más popular del operador OWA es el operador inducido OWA (IOWA) introducido por Yager y Filev (1999). El operador IOWA es utilizado para agregar duplas como (v_i, a_i), donde v_i es llamado valor de orden inducido y a_i es el valor del argumento. El uso de un valor inducido provee una herramienta para modelar en diferentes situaciones (Yager, 2003). Este es definido como sigue:

Un operador IOWA de dimensiones n es un mapeo $IOWA: R^n \rightarrow R$, tal que tiene un vector de pesos asociado $W = [w_1, w_2, .., w_n]^T$, donde $w_i =\in [0,1]$ y $\sum_{i=1}^{n} w_i = 1$, se tiene:

$$IOWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j \ , \tag{19}$$

donde b_j es el valor a_i en la tupla IOWA $\langle u_i, a_i\rangle$, la cual tiene el más grande u_i. El operador IOWA también incluye los casos particulares de máximo, mínimo y promedio.

Otra extensión OWA ocurre cuando los argumentos se extraen del intervalo unitario. Yager (2004) introduce el operador OWA generalizado (GOWA), el cual combina el operador OWA con el operador de medias generalizadas en una misma formulación. Este es definido como sigue:

Es un operador GOWA con mapeo $GOWA: R^n \rightarrow R$, tal que tiene un vector de pesos asociado $W = [w_1, w_2, .., w_n]^T$, donde $w_i =\in [0,1]$ y $\sum_{i=1}^{n} w_i = 1$, adicionalmente se considera un parámetro lambda $\lambda \in [-\infty, \infty]$. Se tiene:

$$GOWA(a_1, a_2, \dots, a_n) = \left(\sum_{j=1}^{n} w_j {b_j}^{\lambda}\right)^{1/\lambda}, \tag{20}$$

donde b_j es el valor más grande en a_i. El operador GOWA tiene algunos casos especiales; cuando $\lambda = 1$ el operador GOWA es el operador OWA. Si $\lambda = 0$ el operador GOWA se vuelve el operador OWG (Chiclana et al, 2000). Cuando $\lambda = -1$ la agregación es el operador OWHA (Chen et al, 2004). Si $\lambda = 2$ se forma el operador OWQA (Dyckhoff y Pedrycz, 1984).

También es posible añadir probabilidades al cálculo del operador OWA. El OWA probabilístico (POWA) (Merigó, 2009; Merigó, 2012) provee una completa representación de la realidad unificando las probabilidades y el reordenamiento OWA. Este considera el grado de importancia en cada proceso de agregación. La formulación es como sigue:

$$POWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \widehat{p_j}\, b_j, \tag{21}$$

donde b_j es el más grande a_i, el cual esta asociado a una probabilidad p_i. Donde existe tal relación que $\hat{p}_j = \beta w_j + (1-\beta)p_j$ con $\beta \in [0,1]$. Entonces, si $\beta = 0$ es operador probabilístico PA aparece, y si $\beta = 1$ se obtiene el operador OWA. El operador POWA también es monótono, conmutativo e idempotente.

Entre las extensiones que se enfocan en el vector de pesos ponderado, está el promedio ponderado OWA (OWAWA) (Merigó, 2009). Este unifica el operador OWA con el promedio ponderado (WA) en la misma formulación considerando un grado de importancia para cada análisis. Su formula es la siguiente:

Un operador OWAWA es un mapeo $OWAWA: R^n \rightarrow R$ asociado con vector de pesos W tal que $w_i =\in [0,1]$ y $\sum_{i=1}^{n} w_i = 1$, de acuerdo a la siguiente ecuación:

$$OWAWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \widehat{v_j}\, b_j, \tag{22}$$

donde b_j el elemento más grande a_i, el cual tiene un peso asociado v_i. Donde $\sum_{i=1}^{n} v_i = 1$ y $v_i \in [0,1]$. El operador OWAWA cuenta con propiedades similares a las del operador de agregación OWA.

Adicionalmente, es posible unir los atributos del operador GOWA y el operador IOWA en una formulación. El operador IGOWA fue propuesto por Merigó y Gil-Lafuente (2009), este operador utiliza medias generalizadas y variables ordenadas inducidas en el proceso de reordenamiento. Se define como sigue:

Es un operador IGOWA de dimensiones n si tenemos un mapeo $IGOWA: R^n \rightarrow R$, con un vector de pesos asignado W con $w_i \in [0,1]$ y $\sum_{i=1}^{n} w_i = 1$, un set de variables inducidas u_i, y un parámetro lambda $\lambda \in [-\infty, \infty]$ acorde a la siguiente formula:

$$IGOWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = \left(\sum_{j=1}^{n} w_j {b_j}^{\lambda}\right)^{1/\lambda}, \qquad (23)$$

donde $(b_1, \dots, b_n)$ es $(a_1, \dots, a_n)$ reordenado en forma descendente según los argumentos u_1. El operador OWA comparte las propiedades del operador OWA; monótono, conmutativo e idempotente.

Si el propósito es no utilizar el proceso de re-orden del OWA tradicional, el operador POWA y el operador OWAWA pueden ser modificados con el proceso de orden inducido. El operador IPOWA (Yager et al, 1995; Merigó, 2010) unifica las probabilidades y la complejidad de variables en algunos casos particulares. En este sentido, IOWAWA (Merigó, 2011) también puede realizar unificaciones en tomas de decisiones bajo incertidumbre y riesgo. Ambas definiciones se desarrollan como sigue:

Un operador IPOWA de dimensiones n es un mapeo $IPOWA: R^n \rightarrow R$ que tiene un vector asociado de pesos $W = [w_1, w_2, .., w_n]^T$ tal que $w_i =\in [0,1]$ y $w_i + \cdots + w_n = 1$, entonces un argumento inducido IOWA u_i tiene un vector de probabilidad asociada p con $\sum_{i=1}^{n} p_i = 1$ y $p_i \in [0,1]$, entonces:

$$IPOWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} \widehat{p_j}\, b_j \qquad (24)$$

donde b_j es el más grande de los valores u_i.

Un operador IOWAWA es una aplicación $IOWAWA: R^n \rightarrow R$ de dimensiones N, si este tiene un vector de pesos asociado W tal que $w_i =\in [0,1]$ y $w_i + \cdots + w_n = 1$, entonces los

argumentos inducidos IOWA tienen un adicional vector de pesos asociados V, donde $\sum_{i=1}^{n} v_i = 1$ y $v_i \in [0,1]$, acorde a la siguiente formula:

$$IOWAWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} \widehat{v_j}\, b_j, \tag{25}$$

donde b_j es el más grande de los valores u_i.

Es también posible adherir los atributos del operador IOWA, el operador POWA y el operador OWAWA en una formulación, esta no solo usa variables inducidas de orden sino también representa complejidad en el proceso de reordenamiento de la agregación. El operador IPOWAWA (Merigó et al, 2012) unifica el operador IOWA, el promedio ponderado y la agregación probabilística en la misma formulación. Es definido como sigue:

Un operador IPOWAWA de dimensiones *n* es una aplicación $IPOWAWA: R^n \to R$, si tiene un vector de pesos adociado *W*, donde la suma de los pesos es 1 y $w_j \in [0,1]$, un vector de pesos *V* correspondiente a los promedios ponderados, con $\sum_{i=1}^{n} v_i = 1$ y $v_i \in [0,1]$ y un vector P que afecta la probabilidad, donde $\sum_{i=1}^{n} p_i = 1$ y $p_i \in [0,1]$. Adicionalmente un IOWA par inducido $\langle u_i, a_i\rangle$ es considerado, entonces:

$$\begin{aligned} &IPOWAWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = C_1 \sum_{j=1}^{n} w_j b_j + \\ &C_2 \sum_{j=1}^{n} v_i a_i + C_3 \sum_{j=1}^{n} p_i a_i, \end{aligned} \tag{26}$$

donde b_j es el valor a_i en el IOWA con el elemento más grande u_i, y C_1, C_2 y $C_3 \in [0,1]$, con $C_1 + C_2 + C_3 = 1$. Note que los casos especiales son obtenidos acorde al uso de C; si $C_1 = 1$ se obtiene el operador IOWA, si $C_2 = 1$ el promedio ponderado es obtenido, cuando $C_3 = 1$ se obtiene el promedio probabilístico, si $C_1 = O$ se forma el promedio ponderado probabilístico.

CAPITULO IV: METODOLOGÍA

Con el objetivo de mejorar la eficiencia de los modelos GARCH en el pronóstico, se ha propuesto la unión de estos con otras metodologías. El uso de sets difusos (Thavaneswaran et al., 2009; Chen y Chen, 2015; Garcia y Kristjanpoller, 2019) y procesos de agregación (Hafner, 2008; Cheng y Chen, 2013) son algunas de los trabajos que han mejorado el funcionamiento de los modelos GARCH.

La presente investigación propone la utilización de operadores de agregación OWA en el proceso de MCO para la estimación de los parámetros en modelos GARCH. El objetivo es minimizar el error de pronóstico alternando diferentes formas de medir la volatilidad.

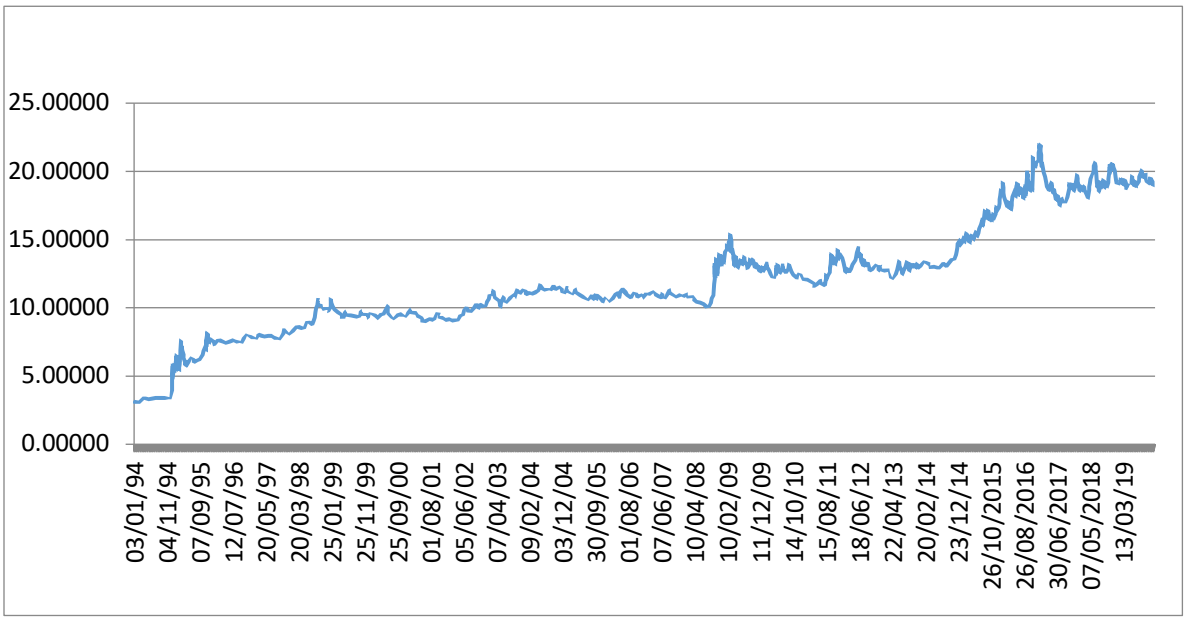

Figura 7. Evolución del tipo de cambio FIX 1994-2019.

Fuente: BANXICO 2020.

El proceso metodológico que se lleva a cabo para el cumplimiento del objetivo esta enumerado en los siguientes pasos:

Paso 1. El estudio es realizado en el tipo de cambio, donde se utiliza el tipo de cambio peso mexicano-dólar estadounidense. Para lo anterior, se elaboró una base de datos histórica del tipo de cambio Fix de 1994-2019 en forma diaria y mensual. Adicionalmente, se construyó una base de datos del tipo de cambio máximos y mínimos de demanda (Bid) en FOREX 2011-2019[10].

La figura 7 muestra la evolución del tipo de cambio diario FIX de 1994 a 2019, donde se observa la tendencia al alza de la moneda con saltos marcados en épocas de inestabilidad. Por su parte, la figura 8 muestra una comparación entre el tipo de cambio FOREX mensual y el tipo de cambio FIX para el periodo 2011-2019. En el caso del tipo de cambio FOREX se ha tomado los precios máximos y mínimos diarios para posteriormente calcular su promedio mensual. Note el comportamiento similar entre ambos indicadores.

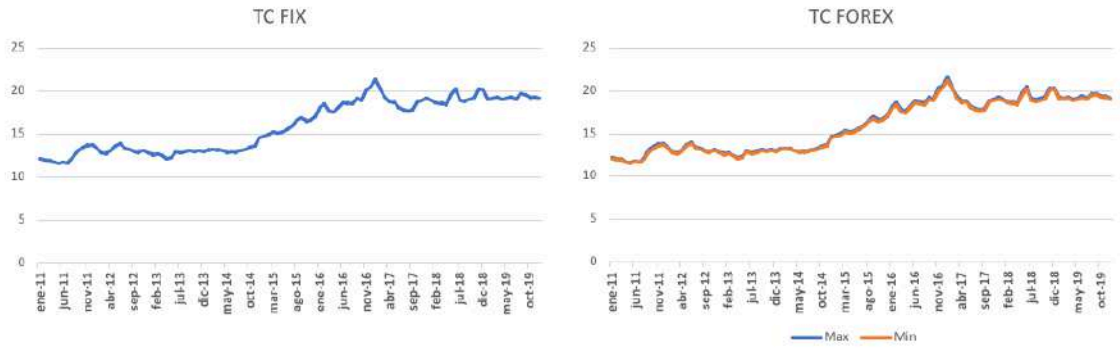

Figura 8. Comparación de tipo de cambio FIX y FOREX en el periodo 2011-2019.

Fuente: BANXICO 2020, FOREX 2020.

[10] La información del tipo de cambio peso mexicano- dólar estadounidense en la plataforma FOREX a partir de 2011.

Paso 2. La volatilidad no es una variable observada, por lo tanto, se tiene que calcular (Andersen et al, 2001; Becker et al, 2015). La literatura introduce varias formas de cálculo con el objetivo de obtener una mejor estimación y pronóstico. En este sentido, se tiene:

Desviación estándar. La volatilidad es comúnmente calculada mediante desviación estándar (SD) (Daly, 2008). Esta es una medida de dispersión con respecto a la media que se desarrolla como sigue:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2}, \tag{27}$$

donde n es el número de datos y μ es la media aritmética.

Log-volatilidad. Una propuesta frecuente en investigación científica es el cálculo de volatilidad mediante varianzas de los logaritmos de los retornos (Kim et al, 1998; Chan y Grant, 2016). Log-volatilidad se calcula mediante los rendimientos (R) del precio de la moneda, esto es como la diferencia entre el precio de hoy y el logaritmo de precios de ayer, como sigue:

$$R = \ln\left(\frac{p_t}{p_{t-1}}\right) \tag{28}$$

$$h_t^2 = (R_i - \bar{R})^2, \tag{29}$$

donde p_t es el periodo actual del precio y h_t^2 es la log-volatilidad analizada. El objetivo de este cálculo es poder eliminar la tendencia creando una serie estacionaria, la cual se pueda adaptar mucho mejor a los modelos de volatilidad (Seyyed y Al-Hajji, 2005).

La volatilidad realizada definida por Andersen y Bollerslev (1998) como la suma de los cuadrados en los retornos intra-día de alta frecuencia en un intervalo dado. Esto es, la suma de los cuadrados de las diferencias entre el precio más alto y el precio más bajo en las

cotizaciones de un periodo corto de tiempo (horas, días). Considerando un periodo de un día, el cálculo se representa de la siguiente forma:

$$R = lnH_t - lnL_t \tag{30}$$

$$h_t^2 = \sum R^2, \tag{31}$$

donde H es el precio más alto del día y L es el precio más bajo del día. La volatilidad realizada considera la información sucedida durante periodos de tiempo cortos a diferencia de otros cálculos que solo consideran un precio de cierre (Engle y Galo, 2006). Por lo tanto, considerar la volatilidad en esta forma promete mejorar la eficiencia de los modelos de pronóstico (Wang et al., 2016; Audrino y Knaus, 2016; Shin y Zhong, 2017).

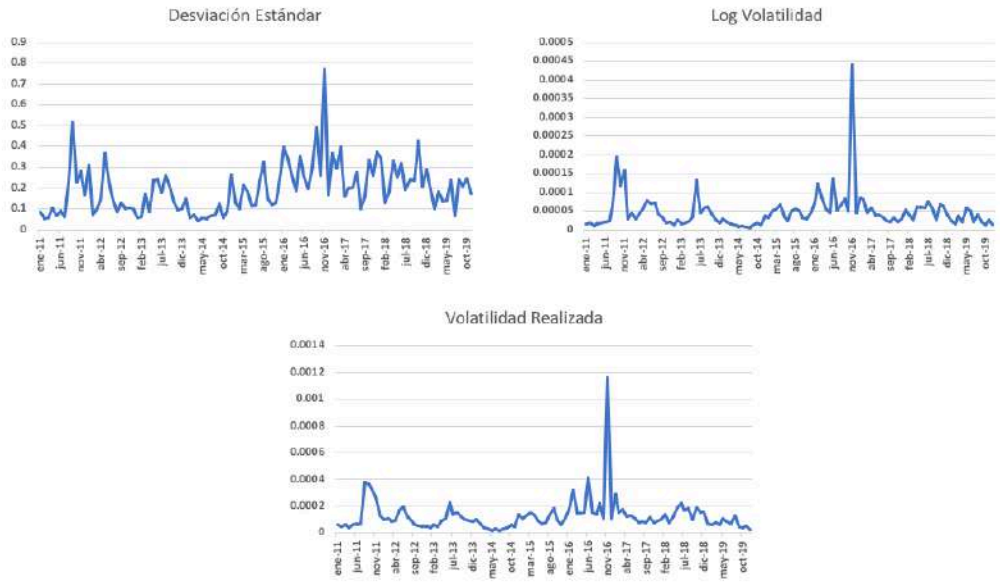

Figura 9. Comparación de diferentes cálculos de volatilidad periodo 2011-2019.

Elaboración propia a partir de datos de BANXICO 2020, FOREX 2020.

En la figura 9 se puede observar la volatilidad calculada para el tipo de cambio peso mexicano-dólar estadounidense con las tres diferentes formas que se han mencionado en este apartado. Tanto la desviación estándar como la log-volatilidad han sido calculadas con el tipo de cambio FIX, mientras que la volatilidad realizada es conforme a datos intra-día de FOREX. Se observa como la log-volatilidad y la volatilidad realizada tienen gran similitud, mientras que la desviación estándar es la medida con mayor inestabilidad.

Paso 3. Con la finalidad de hacer un comparativo entre modelos se formulan 2 diferentes modelos de volatilidad propuestos por la literatura.

- Modelo autorregresivo de heteroscedasticidad condicional ARCH (1):

$$h_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2. \tag{32}$$

- Modelo general autorregresivo de heteroscedasticidad condicional GARCH (1,1):

$$h_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 h_{t-1}^2. \tag{33}$$

Cada uno de estos modelos es transformado acorde al estimador utilizado en la cantidad de estimadores propuestos (según el operador OWA utilizado).

Paso 4. Para la estimación se utiliza el método propuesto, el cual es explicado en el apartado 4.1 y 4.2. Los resultados se comparan con estimaciones realizadas con MLE y OLS.

Paso 5. Para probar la eficiencia de las medidas, modelos y métodos de estimación se calcula el error de pronóstico de los modelos estimados con diferentes técnicas:

- Error porcentual absoluto medio (MAPE):

$$MAPE = \left(\frac{1}{n}\sum\frac{|actual - pronóstico|}{actual}\right) * 100. \quad (34)$$

- Desviación media absoluta (MAD):

$$MAD = \frac{1}{n}\sum|actual - pronóstico|. \quad (35)$$

- Error medio cuadrado (MSE):

$$MSE = \frac{1}{n}\sum(actual - pronóstico)^2. \quad (36)$$

El resultado obtenido es desarrollado en la siguiente sección y en los diferentes artículos presentados en los anexos.

REGRESIÓN LINEAL CON OPERADORES OWA

Utilizar operadores OWA habilita la posibilidad de considerar un proceso de reordenamiento partiendo de muchos factores como lo es el grado de optimismo y aspectos psicológicos (Merigó, 2011). En la estimación de regresión lineal simple LR-OWA los parámetros son calculados con medias OWA, donde un componente es utilizado para ordenar los argumentos. El estimador LR-OWA se define como sigue:

Una regresión simple OWA (LR-OWA) de dimensiones n en un modelo $OWA: R^n \rightarrow R$ tal que, tiene un vector de pesos asociado $W = [w_1, w_2, .., w_n]^T$, donde $w_i =\in [0,1]$ y $\sum_{i=1}^{n} w_i = 1$, entonces se tiene:

$$y_{Iowa} = \alpha_{IOWA} + \beta_{IOWA} x_j, \tag{37}$$

donde los parámetros se definen como sigue:

$$\beta_{IOWA} = \frac{Cov_{IOWA}(x, y)}{var_{IOWA}(x)} = \frac{\sum_{k=1}^{k} w_j (x_j - \mu)(y_j - \upsilon)}{\sum_{k=1}^{k} w_j (x_j - \mu)^2} \tag{38}$$

$$\alpha_{IOWA} = \upsilon - \beta_{IOWA} \mu, \tag{39}$$

donde μ y ν son las medias (OWA operador x,y), y los argumentos $(a_i - \mu)^2$ y $(x_j - \mu)(y_j - \upsilon)$ tienen un vector asociado (WA) w_j con $\sum_{i=1}^{n} w_j = 1$ y $w_j \in [0,1]$.

Se puede observar que la estimación de los parámetros puede ser calculada de diferentes formas dependiendo del lugar en la ecuación donde se utilice la media OWA.

Cuando existen dos variables independientes se tiene una regresión lineal múltiple, y su proceso de estimación se realiza mediante regresión múltiple OWA (MLR-OWA), se define como sigue:

Una regresión lineal múltiple OWA (MLR-OWA) con dos variables independientes de dimensiones *n*, es un modelo $OWA: R^n \rightarrow R$, dadas las variables $x_k \in U^n$, $y_k \in U$ y $z_k \in U^n$, tal que tiene un vector de pesos $W = [w_1, w_2, .., w_n]^T$ con $w_i =\in [0,1]$ y $\sum_{i=1}^{n} w_i = 1$, entonces se tiene:

$$y_{OWA} = \alpha_{OWA} + \beta_{1_{OWA}} x_j + \beta_{2_{OWA}} z_{j,} \tag{40}$$

donde α_{OWA}, $\beta_{1_{OWA}}$ y $\beta_{2_{OWA}}$ son estimados mediante el metodo de minimos cuadrados ordinarios con varianzas y covarianzas OWA como sigue:

$$\beta_{1OWA} = \frac{[Cov_{OWA}(y,x)][var_{OWA}(z)] - [Cov_{OWA}(y,z)][Cov_{OWA}(x,z)]}{[var_{OWA}(x)][var_{OWA}(z)] - [Cov_{OWA}(x,z)]^2} \tag{41}$$

$$\beta_{1OWA} = \frac{\left[\sum_{k=1}^{k} w_j(y_j-\upsilon)(x_j-\mu)\right]\left[\sum_{k=1}^{k} w_j(z_j-\nu)^2\right] - \left[\sum_{k=1}^{k} w_j(y_j-\upsilon)(z_j-\nu)\right]\left[\sum_{k=1}^{k} w_j(x_j-\mu)(z_j-\nu)\right]}{\left[\sum_{k=1}^{k} w_j(x_j-\mu)^2\right]\left[\sum_{k=1}^{k} w_j(z_j-\nu)^2\right] - \left[\sum_{k=1}^{k} w_j(x_j-\mu)(z_j-\nu)\right]^2}, \tag{42}$$

donde x_j, z_j y y_j son los argumentos más grandes en la variable *x*, *z* y *y* respectivamente, y μ, ν and υ son las medias calculadas con el operador OWA.

En el caso del estimador β_2 la formula se desarrolla similar a la anterior como sigue:

$$\beta_{2OWA} = \frac{[Cov_{OWA}(y,z)][var_{OWA}(x)] - [Cov_{OWA}(y,x)][Cov_{OWA}(x,z)]}{[var_{OWA}(x)][var_{OWA}(z)] - [Cov_{OWA}(x,z)]^2} \tag{43}$$

$$\beta_{2OWA} = \frac{\left[\sum_{k=1}^{k} w_j(y_j-\upsilon)(z_j-\nu)\right]\left[\sum_{k=1}^{k} w_j(x_j-\mu)^2\right] - \left[\sum_{k=1}^{k} w_j(y_j-\upsilon)(x_j-\mu)\right]\left[\sum_{k=1}^{k} w_j(x_j-\mu)(z_j-\nu)\right]}{\left[\sum_{k=1}^{k} w_j(x_j-\mu)^2\right]\left[\sum_{k=1}^{k} w_j(z_j-\nu)^2\right] - \left[\sum_{k=1}^{k} w_j(x_j-\mu)(z_j-\nu)\right]^2} \tag{44}$$

Podemos estimar α usando β_1 y β_2 como sigue:

$$\alpha_{OWA} = \upsilon - \beta_{1OWA}\mu - \beta_{2OWA}\nu, \tag{45}$$

donde μ, ν y υ son medias OWA, β_{1OWA} y β_{2OWA} son calculadas con varianzas y covarianzas OWA.

MODELOS ARCH-GARCH CON OPERADORES OWA

Una aplicación del estimador MLR-OWA es utilizando modelos ARCH-GARCH para la estimación de la volatilidad. A continuación, se muestra su definición.

Es un OWA-ARCH de dimensiones n si tenemos un modelo con un vector de parámetros θ = α y β, donde la estimación está dada por medias OWA con MCO como OWA: $Rn \times Rn \rightarrow R$ tal que tiene un vector de pesos asociado $W = [w_1, w_2, .., w_n]^T$ donde $w_i \in [0,1]$ y $\sum_{i=1}^{n} w_i = 1$, entonces tenemos:

$$h_{IOWA} = \alpha_{IOWA} + \beta_{IOWA} a_{t-1}^2, \tag{46}$$

donde h es la volatilidad del precio y a_{t-1}^2 es el error al cuadrado de un periodo atrás. Entonces los parámetros son estimados con LR-OWA como sigue:

$$\beta_{IOWA} = \frac{\sum_{k=1}^{k} w_j (a_{t-1}^2 - \rho)(h - \omega)}{\sum_{k=1}^{k} w_j (a_{t-1}^2 - \rho)^2} = \frac{Cov_{IOWA}(a_{t-1}^2, h)}{Var_{IOWA}(a_{t-1}^2)} \tag{47}$$

$$\alpha_{IOWA} = \omega - \beta_{IOWA}\rho, \tag{48}$$

donde ω y ρ son medias OWA. Note que el OWA-ARCH comparte las propiedades de los operadores OWA monótono, conmutativo e idempotente. De la misma forma, al utilizar modelos GARCH con MLR-OWA tenemos que:

Es un OWA-GARCH de dimensiones n con un modelo parametrizado por $\theta = \alpha, \beta_1, \beta_2$. Estos parametros son estimados con medias MCO OWA, donde $OWA: Rn \times Rn \rightarrow R$ con un vector de pesos $W = [w_1, w_2, .., w_n]^T$ donde $w_i \in [0,1]$ y $\sum_{i=1}^{n} w_i = 1$. Entonces:

$$h_{IOWA} = \alpha_{IOWA} + \beta_{1_{IOWA}} a_{t-1}^2 + \beta_{2_{IOWA}} h_{t-1}^2, \tag{49}$$

donde h_{t-1}^2 es la volatilidad del periodo anterior. El vector de parámetros θ es estimado como sigue:

$$\beta_{1IOWA} = \frac{[Cov_{IOWA}(h, a_{t-1}^2)][var_{IOWA}(h_{t-1}^2)] - [Cov_{IOWA}(h, h_{t-1}^2)][Cov_{IOWA}(a_{t-1}^2, h_{t-1}^2)]}{[var_{IOWA}(a_{t-1}^2)][var_{IOWA}(h_{t-1}^2)] - [Cov_{IOWA}(a_{t-1}^2, h_{t-1}^2)]^2} \tag{50}$$

$$\beta_{1IOWA} = \frac{[\sum_{k=1}^{k} w_j(h_j - \omega)(a_{t-1}^2 - \rho)][\sum_{k=1}^{k} w_j(h_{t-1}^2 - \varphi)^2] - [\sum_{k=1}^{k} w_j(h_j - \omega)(h_{t-1}^2 - \varphi)][\sum_{k=1}^{k} w_j(a_{t-1}^2 - \rho)(h_{t-1}^2 - \varphi)]}{[\sum_{k=1}^{k} w_j(a_{t-1}^2 - \rho)^2][\sum_{k=1}^{k} w_j(h_{t-1}^2 - \varphi)^2] - [\sum_{k=1}^{k} w_j(a_{t-1}^2 - \rho)(h_{t-1}^2 - \varphi)]^2}, \tag{51}$$

donde ω, ρ y φ son medias OWA. El vector de parámetros. De la misma forma, β_2 es Estimado como sigue:

$$\beta_{2IOWA} = \frac{[Cov_{IOWA}(h, h_{t-1}^2)][var_{IOWA}(a_{t-1}^2)] - [Cov_{IOWA}(h, a_{t-1}^2)][Cov_{IOWA}(a_{t-1}^2, h_{t-1}^2)]}{[var_{IOWA}(a_{t-1}^2)][var_{IOWA}(h_{t-1}^2)] - [Cov_{IOWA}(a_{t-1}^2, h_{t-1}^2)]^2} \tag{52}$$

$$\beta_{2IOWA} = \frac{[\sum_{k=1}^{k} w_j(h_j - \omega)(h_{t-1}^2 - \varphi)][\sum_{k=1}^{k} w_j(a_{t-1}^2 - \rho)^2] - [\sum_{k=1}^{k} w_j(h_j - \omega)(a_{t-1}^2 - \rho)][\sum_{k=1}^{k} w_j(a_{t-1}^2 - \rho)(h_{t-1}^2 - \varphi)]}{[\sum_{k=1}^{k} w_j(a_{t-1}^2 - \rho)^2][\sum_{k=1}^{k} w_j(h_{t-1}^2 - \varphi)^2] - [\sum_{k=1}^{k} w_j(a_{t-1}^2 - \rho)(h_{t-1}^2 - \varphi)]^2}. \tag{53}$$

Para estimar α usamos:

$$\alpha_{IOWA} = \omega - \beta_{1IOWA}\rho - \beta_{2IOWA}\varphi. \tag{54}$$

Note que si $\beta_2 \approx 1$ el modelo GARCH se convierte en un modelo ARCH.

CAPITULO V: RESULTADOS

Con el objetivo de conocer la efectividad de las metodologías propuestas para pronóstico y estimación vistas en este trabajo, a continuación, se presentan los resultados obtenidos en su aplicación a diferentes periodos y con diferentes operadores de agregación. El propósito es conocer la utilidad de estas herramientas en cuanto a su capacidad de adaptarse a propiedades estadísticas y probar su funcionalidad minimizando errores de pronóstico.

En primer lugar, se realiza una comparación de modelos GARCH y modelos estocásticos, donde los resultados muestran la eficiencia de modelos estocásticos. Ante esto se hace una propuesta para mejorar la estimación y pronóstico de los modelos GARCH que es aplicada en el apartado dos. Por último, se calcula la volatilidad de formas diferentes y se prueba la eficiencia en diferentes escenarios.

MODELOS DE VOLATILIDAD GARCH Y ESTOCÁSTICOS

El primer análisis realiza una comparación entre modelos GARCH y modelos estocásticos. Para esto, se utilizan datos diarios del tipo de cambio FIX peso mexicano-dólar estadounidense durante el periodo 1994-2018. Los datos fueron calculados mensualmente, donde el primer dato corresponde a abril de 1994 y el último dato a octubre de 2018.

La volatilidad ha sido calculada como log-volatilidad mediante el retorno de los precios como en la ecuación (29). Basados en los modelos tradicionales GARCH y estocásticos se desarrollan cinco modelos:

$$\text{GARCH 1,1: } hR_t^2 = \alpha_0 + \alpha_1 aR_{t-1}^2 + \beta_1 hR_{t-1}^2 \tag{55}$$

$$\text{GARCH 1,2: } hR_t^2 = \alpha_0 + \alpha_1 aR_{t-1}^2 + \beta_1 hR_{t-1}^2 + \beta_2 hR_{t-2}^2 \tag{56}$$

$$\text{EGARCH: } hR_t^2 = \alpha_0 + \beta_1 \log\left(hR_{t-1}^2\right) + \gamma \frac{aR_{t-1}}{\sqrt{hR_{t-1}^2}} + \omega \left[\frac{|aR_{t-1}|}{\sqrt{hR_{t-1}^2}} - \sqrt{2/\pi}\right] \quad (57)$$

$$\text{SV 1: } hR_t^2 = \mu + \phi hR_{t-1} \quad (58)$$

$$\text{SV 2: } hR_t^2 = \mu + \phi hR_{t-1} + \phi hR_{t-2}, \quad (59)$$

donde la volatilidad estimada es hR_t^2 para todos los casos. Note que se abordad 3 modelos GARCH y dos modelos estocásticos.

En el caso de la estimación en los modelos GARCH se utiliza máxima verosimilitud con funciones gaussianas. La tabla 4 muestra los resultados.

Tabla 4. Análisis de modelos GARCH

Modelo	**Parámetros**	**Significancia de los parámetros**	**Criterio Akaike info**	**Criterio Schwarz**
GARCH (1.1)	$\alpha_0 = 0.000433$	0.0000	-4.314630	-4.264637
	$\alpha_1 = 0.465281$	0.0000		
	$\beta_1 = 0.128459$	0.0303		
GARCH (1,2)	$\alpha_0 = 0.000434$	0.0000	-4.314609	-4.252118
	$\alpha_1 = 0.459434$	0.0000		
	$\beta_1 = 0.208128$	0.0273		
	$\beta_2 = -0.073937$	0.1361		
EGARCH	$\alpha_0 = -3.428093$	0.0000	-4.403681	-4.341190
	$\beta_1 = 0.215611$	0.0005		
	$\gamma = 0.483839$	0.0000		
	$\omega = 0.553491$	0.0000		

Akaike y Schwarts criterio muestran que EGARCH es un mejor modelo. Sin embargo, el parámetro α_0 es negativo, lo cual indica que se rompe la asumcion de positividad. Se selecciona como mejor modelo el GARCH (1,1).

Para la estimación de los parámetros en los modelos estocásticos se utiliza Monte Carlo vía Cadenas de Markov (MCMC) mediante el algoritmo Metropolis Hasting. Los resultados son mostrados en la tabla 5.

Tabla 5. Análisis de modelos estocásticos

Modelo	**Parámetros**	**Montecarlo estándar error de los parámetros**	**Eficiencia máxima MCMC**
SV (1)	$\mu = 0.0009798$	0.00097	0.1342
	$\phi = 0.2209891$	0.00169	
SV (2)	$\mu = 0.0007921$	0.00001	0.1032
	$\phi = 0.183780$	0.00215	
	$\rho = 0.172050$	0.00209	

La eficiencia MCMC muestra que el modelo estocástico 1 es el mejor modelo con un error dentro de los parámetros de tolerancia.

Con el objetivo de conocer el modelo más funcional en el pronóstico, se presenta una comparación de los mejores modelos en cada tipo y se utiliza desviación media absoluta (MAD) error medio (RMSE) y porcentaje de error medio absoluto (MAPE) como medidas de error.

Los resultados en la tabla 6 muestran que el modelo estocástico minimiza el error de pronóstico para los siguientes 6 periodos. Note que la volatilidad en estos periodos presentó grandes ascensos.

Tabla 6. Análisis de pronóstico.

	Periodo	Valor	Volatilidad real	Error absoluto	Error al cuadrado	Porcentaje de error	MAD	RMSE	MAPE
GARCH (1,1)							0.00135	0.00422	13132.6
	1	0.00051	0.00232715	0.00182	3.313E-06	0.78			
	2	0.00158	0.00018830	0.00139	1.939E-06	7.40			
	3	0.00072	0.00340258	0.00268	7.176E-06	0.79			
	4	0.00211	0.00000002	0.00211	4.448E-06	91872.1			
	5	0.00070	0.00002223	0.00068	4.647E-07	30.67			
	6	0.00053	0.00041893	0.00011	1.319E-08	0.27			
	7	0.00070	0.00003959	0.00066	4.315E-07	16.59			
SV1							0.00133	0.000002	10787.4
	1	0.00098	0.00232715	0.00135	1.81E-06	0.578			
	2	0.00149	0.00018830	0.00131	1.71E-06	6.934			
	3	0.00102	0.00340258	0.00238	5.67E-06	0.700			
	4	0.00173	0.00000002	0.00173	3.00E-06	75433.1			
	5	0.00098	0.00002223	0.00096	9.17E-07	43.081			
	6	0.00098	0.00041893	0.00057	3.20E-07	1.351			
	7	0.00107	0.00003959	0.00103	1.07E-06	26.089			

PRONÓSTICO DE VOLATILIDAD MEDIANTE MODELOS GARCH Y OPERADORES DE AGREGACIÓN

En el apartado anterior se muestra como los modelos de volatilidad estocástica tienden a ser superiores en el pronóstico en épocas de alta volatilidad. Sin embargo, el uso de algoritmos complica su aplicación.

La propuesta de este trabajo utiliza operadores de agregación OWA como una forma simple de mejorar resultados en modelos de volatilidad con el uso de un número limitado de información.

En este sentido, se utiliza una regresión múltiple, la cual es estimada con mínimos cuadrados ordinarios que usan operadores de agregación OWA. Esta regresión es aplicada a modelos de volatilidad ARCH-GARCH. Los datos provienen de la volatilidad de tipo de cambio peso mexicano dólar estadounidense en el periodo comprendido de agosto de 2017 a julio de 2019. Los operadores de agregación utilizados son OWA, IOWA e IGOWA. Para la medición de

la volatilidad se usa desviación estándar como en la ecuación 27. Se utilizan doce modelos como se muestra a continuación:

OWA-ARCH $h_{OWA} = \alpha_{OWA} + \beta_{OWA} a_{t-1}^2,$ (60)

OWA-GARCH $h_{OWA} = \alpha_{OWA} + \beta_{1_{OWA}} a_{t-1}^2 + \beta_{2_{OWA}} h_{t-1}^2,$ (61)

IOWA-ARCH $h_{IOWA} = \alpha_{IOWA} + \beta_{IOWA} a_{t-1}^2,$ (62)

IOWA-GARCH $h_{IOWA} = \alpha_{IOWA} + \beta_{1_{IOWA}} a_{t-1}^2 + \beta_{2_{IOWA}} h_{t-1}^2,$ (63)

$IGOWA - ARCH_{\lambda=-1}$ $h_{IGOWA} = \alpha_{IGOWA} + \beta_{IGOWA} a_{t-1}^2,$ (64)

$IGOWA - GARCH_{\lambda=-1}$ $h_{IGOWA} = \alpha_{IGOWA} + \beta_{1_{IGOWA}} a_{t-1}^2 + \beta_{2_{IGOWA}}$ (65)

$IGOWA - ARCH_{\lambda=2}$ $h_{IGOWA} = \alpha_{IGOWA} + \beta_{IGOWA} a_{t-1}^2,$ (66)

$IGOWA - GARCH_{\lambda=2}$ $h_{IGOWA} = \alpha_{IGOWA} + \beta_{1_{IGOWA}} a_{t-1}^2 + \beta_{2_{IGOWA}}$ (67)

OLS-ARCH $h = \alpha + \beta a_{t-1}^2,$ (68)

OLS-GARCH $h = \alpha + \beta_1 a_{t-1}^2 + \beta_2 h_{t-1}^2,$ (69)

MLE-ARCH $h = \alpha + \beta a_{t-1}^2,$ (70)

MLE-GARCH $h = \alpha + \beta_1 a_{t-1}^2 + \beta_2 h_{t-1}^2,$ (71)

donde OLS-ARCH y OLS-GARCH son estimados con mínimos cuadrados simples. MLE-ARCH y MLE-GARCH son estimados con máxima verosimilitud. Los IGOWA-ARCH y IGOWA-GARCH son presentados en dos casos cuando $\lambda = -1$ y $\lambda = 2$. Los datos y vectores de peso utilizados en cada caso se observan en la tabla 7.

Tabla 7. Datos utilizados en la estimación

Fecha	h_t	a_{t-1}^2	h_{t-1}	OWA			IOWA			IGOWA		
				W_{h_t}	$W_{a_{t-1}^2}$	$W_{h_{t-1}}$	W_{h_t}	$W_{a_{t-1}^2}$	$W_{h_{t-1}}$	W_{h_t}	$W_{a_{t-1}^2}$	$W_{h_{t-1}}$
ago-17	0.0998	0.0005413	0.2751	0.15	0.17	0.17	0.04	0.05	0.05	0.08	0.06	0.06
sep-17	0.1576	0.0000568	0.0998	0.08	0.08	0.08	0.04	0.04	0.04	0.03	0.05	0.05
oct-17	0.3334	0.0000224	0.1576	0.08	0.08	0.08	0.04	0.04	0.04	0.04	0.05	0.05
nov-17	0.2623	0.0022251	0.3334	0.07	0.07	0.07	0.04	0.04	0.04	0.03	0.03	0.03
dic-17	0.3732	0.0000011	0.2623	0.06	0.06	0.06	0.05	0.04	0.04	0.08	0.03	0.03
ene-18	0.3409	0.0000576	0.3732	0.06	0.05	0.05	0.04	0.04	0.04	0.06	0.03	0.03
feb-18	0.1296	0.0004293	0.3409	0.04	0.05	0.05	0.04	0.04	0.04	0.02	0.04	0.04
mar-18	0.1819	0.0004130	0.1296	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.04	0.04
abr-18	0.3296	0.0000504	0.1819	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.04	0.04
may-18	0.2525	0.0003804	0.3296	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.04	0.04
jun-18	0.3186	0.0032574	0.2525	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.04	0.04
jul-18	0.1915	0.0008624	0.3186	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.04	0.04
ago-18	0.2421	0.0063650	0.1915	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.04	0.04
sep-18	0.2378	0.0000458	0.2421	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.04	0.04
oct-18	0.4284	0.0000040	0.2378	0.04	0.03	0.03	0.05	0.04	0.04	0.1	0.04	0.04
nov-18	0.2080	0.0000067	0.4284	0.03	0.03	0.03	0.04	0.04	0.04	0.03	0.04	0.04
dic-18	0.2893	0.0023223	0.2080	0.03	0.03	0.03	0.04	0.04	0.04	0.03	0.04	0.04
ene-19	0.1941	0.0001897	0.2893	0.02	0.02	0.02	0.04	0.04	0.04	0.03	0.04	0.04
feb-19	0.1005	0.0034085	0.1941	0.01	0.02	0.02	0.04	0.05	0.05	0.06	0.05	0.05
mar-19	0.1850	0.0000000	0.1005	0.01	0.01	0.01	0.04	0.04	0.04	0.03	0.03	0.03
abr-19	0.1384	0.0000227	0.1850	0.01	0.01	0.01	0.04	0.04	0.04	0.03	0.03	0.03
may-19	0.1399	0.0004210	0.1384	0.01	0.01	0.01	0.04	0.04	0.04	0.03	0.04	0.04
jun-19	0.2340	0.0000042	0.1399	0.01	0.00	0.00	0.05	0.04	0.04	0.03	0.06	0.06
jul-19	0.0645	0.0000001	0.2340	0.01	0.00	0.00	0.05	0.06	0.06	0.08	0.06	0.06

Posteriormente, son calculados los parámetros de la regresión como lo muestra el apartado 4.2. La estimación de los parámetros se resume en la tabla 8.

Tabla 8. Análisis de resultados en parámetros.

	OWA	$\boldsymbol{IOWA}$	$\boldsymbol{IGOWA_{\lambda=-1}}$	$\boldsymbol{IGOWA_{\lambda=2}}$	OLS
$\boldsymbol{\beta_1}$ ARCH	-0.0794142	2.347352628	15.29494404	7.876533386	0.291744918
$\boldsymbol{\beta_1}$ GARCH	-0.0733192	1.679709418	15.15065483	7.862485649	0.563433676
$\boldsymbol{\beta_2}$ GARCH	0.007406485	-0.13274278	0.032551045	-0.09331819	0.140515852
α ARCH	0.229047507	0.213103793	0.168879904	0.22297382	0.226116142
α GARCH	0.227204603	0.246464296	0.161740636	0.247201001	0.192835861

Tabla 9. Análisis de resultados

	h_{tc}	h_{tR}	MAD	MSE	MAPE
OWA-ARCH	0.2353 0.2351 0.2351	0.2436 0.2048 0.2447	0.0161	0.0004	0.0737
OWA-GARCH	0.2278 0.2342 0.2345		0.0185	0.0004	0.0834
IOWA-ARCH	0.2284 0.2286 0.2286		0.0183	0.0004	0.0814
IOWA-GARCH	0.2019 0.2232 0.2264		0.0261	0.0008	0.1118
IGOWA-ARCH $\lambda = -1$	0.1766 0.1929 0.1810		0.0475	0.0029	0.1978
IGOWA-GARCH $\lambda = -1$	0.1739 0.1851 0.1798		0.0257	0.0016	0.1079
IGOWA-ARCH $\lambda = 2$	0.2598 0.2605 0.2606		0.0293	0.0012	0.1346
IGOWA-GARCH $\lambda = 2$	0.2604 0.2610 0.2610		0.0149	0.0006	0.0684
OLS-ARCH	0.2263 0.2261 0.2261		0.0190	0.0004	0.0836
OLS-GARCH	0.2022 0.2216 0.2240		0.0263	0.0008	0.1120
MLE-ARCH	0.356 0.354 0.354		0.1233	0.0155	0.5440
MLE-GARCH	0.2031 0.4626 0.9108		0.3215	0.1706	1.3824

Abreviaciones: h_{tc} = volatilidad calculada, h_{tR} = volatilidad real

Después se pronosticar la volatilidad para los tres periodos siguientes y se contrastan con la volatilidad real experimentada. Para conocer los diferentes errores de pronóstico se utilizan desviación media absoluta (MAD) error medio al cuadrado (MSE) y porcentaje de error medio absoluto (MAPE).

La tabla 9 muestra que la mejor estimación que minimiza el pronóstico de error es el IGOWA-GARCH con λ = 2 seguido por el OWA-ARCH y el IGOWA-ARCH. Los resultados indican que el error mínimo se encuentra en la estimación más alta de volatilidad. Lo anterior muestra que, en este período de inestabilidad, el uso de un operador OWA con complejidad funciona mejor.

USO DE VOLATILIDAD REALIZADA EN MODELOS GARCH CON OPERADORES DE AGREGACIÓN OWA

En este apartado se realizan nuevos pronósticos para probar la metodología propuesta en este trabajo. Para lo anterior se utilizan datos intra-día y cuatro escenarios diferentes; dos para considerar alta volatilidad y dos de estabilidad. La aplicación utiliza los operadores OWA probabilístico (POWA), con promedio ponderado (OWAWA), inducido probabilístico (IPOWA), Inducido con promedio ponderado IOWAWA e inducido probabilístico con promedio ponderado. IPOWAWA.

Los datos fueron tomados de las bases de la plataforma FOREX del tipo de cambio peso mexicano dólar estadounidense. La volatilidad es calculada como volatilidad realizada explicada en la ecuación 31. Como muestra la figura 9 existen periodos de alta volatilidad y periodos de constante tranquilidad en la volatilidad de tipo de cambio peso mexicano-dólar estadounidense. En base a lo anterior, los periodos analizados son marzo de 2015 a noviembre de 2016 como periodo de alta volatilidad (escenario I), de igual forma julio de 2018 a marzo de 2020 (escenario II), para los periodos de tranquilidad se analizan marzo de 2017 a noviembre de 2018 (escenario III) y de agosto de 2013 a noviembre de 2015 (escenario IV). Se utilizan cinco modelos para cada periodo, estos son:

POWA-GARCH $$h_{POWA} = \alpha_{POWA} + \beta_{1_{POWA}} a_{t-1}^2 + \beta_{2_{POWA}} h_{t-1}^2, \quad (72)$$

OWAWA-GARCH $$h_{OWAWA} = \alpha_{OWAWA} + \beta_{1_{OWAWA}} a_{t-1}^2 + \beta_{2_{OWAWA}} h_{t-1}^2, \quad (73)$$

$$IPOWA - GARCH \quad h_{IPOWA} = \alpha_{IPOWA} + \beta_{1_{IPOWA}} a_{t-1}^2 + \beta_{2_{IPOWA}}, \tag{74}$$

$$IOWAWA - GARCH \quad h_{IOWAWA} = \alpha_{IOWAWA} + \beta_{1_{IOWAWA}} a_{t-1}^2 + \beta_{2_{IOWAWA}} \tag{75}$$

$$IPOWAWA - GARCH \quad h_{IPOWAWA} = \alpha_{IPOWAWA} + \beta_{1_{IPOWAWA}} a_{t-1}^2 + \beta_{2_{IPOWAWA}}, \tag{76}$$

donde cada parámetro es estimado como OWA-GARCH visto en el apartado 4.2. Las series utilizadas en cada periodo se analizan en la tabla 10.

Tabla 10. Datos de la estimación por escenario.

Escenario 1		Escenario 2		Escenario 3		Escenario 4	
Fecha	Volatilidad	Fecha	Volatilidad	Fecha	Volatilidad	Fecha	Volatilidad
mar-15	0.00015136	jul-18	0.00017067	mar-17	0.00017538	ago-13	0.00015159
abr-15	0.00013123	ago-18	0.00018691	abr-17	0.00012097	sep-13	0.00012438
may-15	9.6595E-05	sep-18	0.00010308	may-17	0.00013242	oct-13	0.00010483
jun-15	7.3694E-05	oct-18	0.00019491	jun-17	0.0001082	nov-13	9.1148E-05
jul-15	8.0367E-05	nov-18	0.00016031	jul-17	7.9837E-05	dic-13	8.3464E-05
ago-15	0.0001396	dic-18	0.00016236	ago-17	8.3009E-05	ene-14	9.7449E-05
sep-15	0.00018105	ene-19	7.3206E-05	sep-17	7.6264E-05	feb-14	7.0966E-05
oct-15	0.00010183	feb-19	6.3266E-05	oct-17	0.00011621	mar-14	3.7635E-05
nov-15	6.1719E-05	mar-19	7.5948E-05	nov-17	7.8999E-05	abr-14	2.9384E-05
dic-15	0.00010698	abr-19	6.0821E-05	dic-17	9.1143E-05	may-14	2.1463E-05
ene-16	0.00017764	may-19	0.00010832	ene-18	0.00010322	jun-14	3.0004E-05
feb-16	0.00031825	jun-19	8.7131E-05	feb-18	0.00013789	jul-14	2.0852E-05
mar-16	0.00014858	jul-19	6.9419E-05	mar-18	8.205E-05	ago-14	2.9485E-05
abr-16	0.00015253	ago-19	0.00013476	abr-18	0.00012177	sep-14	4.2663E-05
may-16	0.00015418	sep-19	4.9298E-05	may-18	0.00018826	oct-14	4.2663E-05
jun-16	0.00041406	oct-19	3.6931E-05	jun-18	0.00022125	nov-14	4.8248E-05
jul-16	0.00015311	nov-19	5.8895E-05	jul-18	0.00017067	dic-14	0.00013942
ago-16	0.00014158	dic-19	2.8177E-05	ago-18	0.00018691	ene-15	0.0001124

Las medias, varianzas y covarianzas OWA se forman con los vectores de peso mostrados en la tabla 11. Existe un criterio de colocar pesos más grandes en el operador OWA. El operador probabilístico utiliza un peso según su probabilidad de ocurrencia en cada serie elegida. El ordenamiento ponderado les da más peso a valores más grandes en los escenarios 1 y 2,

mientras que asigna más peso a valores más pequeños en el escenario 3 y 4. Los pesos inducidos están orientados a valores más grandes.

Tabla 11. Vectores de peso según el operador y el escenario de trabajo.

Escenario 1				Escenario 2				Escenario 3				Escenario 4			
P	O	WA	I	P	O	WA	I	P	O	WA	I	P	O	WA	I
0.04	0.04	0.06	0.03	0.05	0.04	0.12	0.03	0.04	0.04	0.02	0.03	0.03	0.04	0.01	0.03
0.08	0.04	0.04	0.05	0.05	0.04	0.14	0.05	0.06	0.04	0.03	0.05	0.03	0.04	0.01	0.05
0.08	0.04	0.04	0.1	0.03	0.04	0.05	0.1	0.04	0.04	0.03	0.1	0.05	0.04	0.02	0.1
0.08	0.05	0.04	0.15	0.05	0.05	0.16	0.15	0.07	0.05	0.05	0.15	0.05	0.05	0.03	0.15
0.08	0.06	0.04	0.1	0.05	0.06	0.1	0.1	0.07	0.06	0.12	0.1	0.05	0.06	0.03	0.1
0.07	0.07	0.04	0.05	0.05	0.07	0.1	0.05	0.07	0.07	0.1	0.05	0.05	0.07	0.02	0.05
0.03	0.07	0.08	0.02	0.07	0.07	0.03	0.02	0.07	0.07	0.16	0.02	0.05	0.07	0.03	0.02
0.08	0.06	0.04	0.1	0.07	0.06	0.02	0.1	0.07	0.06	0.03	0.1	0.07	0.06	0.07	0.1
0.08	0.05	0.04	0.15	0.07	0.05	0.03	0.15	0.07	0.05	0.14	0.15	0.07	0.05	0.12	0.15
0.08	0.05	0.04	0.05	0.07	0.05	0.02	0.05	0.07	0.05	0.07	0.05	0.07	0.05	0.14	0.05
0.03	0.05	0.08	0.02	0.03	0.05	0.05	0.02	0.07	0.05	0.05	0.02	0.07	0.05	0.1	0.02
0.01	0.05	0.08	0.01	0.03	0.05	0.03	0.01	0.04	0.05	0.02	0.01	0.07	0.05	0.16	0.01
0.07	0.05	0.06	0.03	0.07	0.05	0.03	0.03	0.07	0.05	0.1	0.03	0.07	0.05	0.1	0.03
0.04	0.05	0.06	0.03	0.03	0.05	0.07	0.03	0.06	0.05	0.03	0.03	0.07	0.05	0.05	0.03
0.03	0.06	0.08	0.02	0.07	0.06	0.01	0.02	0.03	0.06	0.01	0.02	0.07	0.06	0.05	0.02
0.01	0.06	0.08	0.01	0.07	0.06	0.01	0.01	0.03	0.06	0.01	0.01	0.07	0.06	0.03	0.01
0.04	0.05	0.06	0.03	0.07	0.05	0.02	0.03	0.04	0.05	0.02	0.03	0.03	0.05	0.01	0.03
0.07	0.1	0.04	0.05	0.07	0.1	0.01	0.05	0.03	0.1	0.01	0.05	0.03	0.1	0.02	0.05

Abreviaciones. P=vector de peso POWA, O=vector de peso OWA, WA=vector de peso OWAWA, I=vector de peso IOWA.

La estimación OWA se realiza en conjunto con la estimación de mínimos cuadrados ordinarios simples, los resultados se muestran en la tabla 12.

Con los parámetros anteriores se pronostica la volatilidad para los tres periodos siguientes y se comparan con la volatilidad real. El cálculo de los errores se realiza mediante la desviación media absoluta (MAD), el error medio al cuadrado (MSE) y el porcentaje de error medio absoluto (MAPE). La tabla 13 muestra los resultados.

Tabla 12. Parámetros de la regresión GARCH

		POWA	OWAWA	IPOWA	IOWAWA	IPOWAWA
Escenario 1	β_1	-5.62E-05	-6.66E-05	-2.71E-05	-2.98E-05	-3.00E-05
	β_2	2.23E-01	4.89E-01	3.78E-01	4.35E-01	4.34E-01
	α	1.39E-04	1.08E-04	7.06E-05	8.15E-05	7.90E-05
Escenario 2	β_1	3.37E-05	3.52E-05	5.12E-05	5.37E-05	5.30E-05
	β_2	7.19E-01	7.46E-01	7.42E-01	7.85E-01	7.66E-01
	α	-6.15E-06	-1.55E-06	-1.43E-05	-1.14E-05	-1.38E-05
Escenario 3	β_1	1.39E-04	1.39E-04	-1.95E-05	-1.86E-05	-1.87E-05
	β_2	7.05E-01	7.16E-01	5.71E-01	5.87E-01	5.79E-01
	α	2.42E-05	2.01E-05	4.73E-05	4.02E-05	4.30E-05
Escenario 4	β_1	-3.67E-06	-3.81E-06	-3.33E-06	-3.42E-06	-3.37E-06
	β_2	7.85E-01	7.94E-01	8.08E-01	8.21E-01	8.15E-01
	α	1.32E-05	1.13E-05	8.70E-06	5.89E-06	7.15E-06

Los resultados muestran como el error de pronóstico se eleva en escenarios de alta volatilidad (Escenario 1 y 2), esto es consistente con la literatura que marca la problemática de pronóstico de volatilidad en épocas de gran inestabilidad. Sin embargo, en el caso del escenario 1, se observa como la regresión IPOWAWA-GARCH minimiza el error de la estimación de MCO a casi 50% en época de gran inestabilidad, además, se destaca que la estimación con operadores OWA mejora los resultados en la mayoría de las regresiones excepto en OWAWA-GARCH. Contrario a lo anterior, el escenario 2 responde mejor a la ponderación de pesos, por lo tanto, la regresión OWAWA-GARCH minimiza el error de pronóstico en el periodo analizado, en este periodo todos los resultados que utilizan operadores de agregación mejoran la regresión OLS. Los resultados en el escenario 3 minimizan el error en las regresiones IPOWA-GARCH, IOWAWA-GARCH y IPOWAWA-GARCH. Por su parte, el escenario 3 reacciona mejor a estimadores simples y no inducidos, en este sentido POWA-GARCH y OWAWA-GARCH son los mejores resultados.

Tabla 13. Análisis de resultados

Periodo	Tipo	h_{tc}	h_{tR}	MAD	MSE	MAPE
Escenario 1	POWA-GARCH	0.00015	0.00022	0.00040	0.00000038	0.65977
		0.00019	0.00011			
		0.00010	0.00116			
	OWAWA-GARCH	0.00015		0.00042	0.00000039	0.74584
		0.00022				
		0.00009				
	IPOWA-GARCH	0.00011		0.00041	0.00000039	0.61375
		0.00015				
		0.00008				
	IOWAWA-GARCH	0.00013		0.00041	0.00000038	0.65496
		0.00018				
		0.00010				
	IPOWAWA-GARCH	0.00013		0.00020	0.00000019	0.32562
		0.00018				
		0.00009				
	MCO	0.00015		0.00042	0.00000039	0.72340
		0.00021				
		0.00009				
Escenario 2	POWA-GARCH	0.00001	0.00005	0.00059	0.00000099	0.60911
		0.00008	0.00010			
		0.00006	0.00178			
	OWAWA-GARCH	0.00002		0.00058	0.00000098	0.54365
		0.00009				
		0.00007				
	IPOWA-GARCH	0.00001		0.00059	0.00000099	0.62371
		0.00010				
		0.00006				
	IOWAWA-GARCH	0.00001		0.00059	0.00000098	0.62549
		0.00011				
		0.00007				
	IPOWAWA-GARCH	0.00001		0.00059	0.00000099	0.63059
		0.00011				
		0.00006				
	MCO	0.00001		0.00059	0.00000097	0.69962
		0.00006				
		0.00008				

Tabla 13. cont.

Periodo	Tipo	h_{tc}	h_{tR}	MAD	MSE	MAPE
Escenario 3	POWA-GARCH	0.00016	0.00010	0.00006	0.00000000	0.42575
		0.00010	0.00019			
		0.00019	0.00016			
	OWAWA-GARCH	0.00016		0.00006	0.00000000	0.41931
		0.00009				
		0.00019				
	IPOWA-GARCH	0.00015		0.00005	0.00000000	0.32533
		0.00011				
		0.00015				
	IOWAWA-GARCH	0.00015		0.00005	0.00000000	0.32939
		0.00010				
		0.00015				
	IPOWAWA-GARCH	0.00015		0.00005	0.00000000	0.32763
		0.00010				
		0.00015				
	MCO	0.00016		0.00005	0.00000000	0.36969
		0.00011				
		0.00018				
Escenario 4	POWA-GARCH	0.00009	0.00013	0.00003	0.00000000	0.18651
		0.00012	0.00015			
		0.00013	0.00013			
	OWAWA-GARCH	0.00009		0.00003	0.00000000	0.18979
		0.00012				
		0.00013				
	IPOWA-GARCH	0.00009		0.00003	0.00000000	0.19004
		0.00012				
		0.00013				
	IOWAWA-GARCH	0.00009		0.00003	0.00000000	0.19855
		0.00012				
		0.00013				
	IPOWAWA-GARCH	0.00009		0.00003	0.00000000	0.19507
		0.00012				
		0.00013				
	MCO	0.00009		0.00004	0.00000000	0.26248
		0.00011				
		0.00015				

Abreviaciones en tabla 6.

CAPITULO VI: CONCLUSIONES

CONCLUSIONES GENERALES

El tipo de cambio juega un importante rol en el comercio internacional, la orientación de inversiones y en la situación económica de un país. Los movimientos de los precios de las monedas son causados en muchas ocasiones por situaciones imprecisas y factores ambiguos, lo cual genera volatilidad.

Una alternativa para minimizar la incertidumbre que provoca la volatilidad es la propuesta de modelos y estimadores de parámetros de estos modelos, los cuales tienen como objetivo estimar y pronosticar la volatilidad eficazmente. En la literatura existen un gran número de estudios orientados a solucionar esta problemática. Una de las metodologías recurrentemente utilizadas son los modelos GARCH, los cuales han sido probados y analizados en diferentes instrumentos financieros desde la década de los 80s. Sin embargo, las tendencias actuales apuntan a que los modelos GARCH pueden presentar fallas en ciertos momentos debido a las nuevas tendencias que las series financieras presentan en cuanto a la no-normalidad, no linealidad y comportamientos atípicos.

Ante este panorama, este trabajo propone un estimador de modelos GARCH que incluye mínimos cuadrados ordinarios y operadores OWA en la misma formulación. Esta herramienta ha sido llamada LR-OWA para la regresión lineal simple y MLR-OWA para la regresión lineal múltiple. En su aplicación a modelos GARCH, el estimador se presentan como OWA-ARCH y OWA-GARCH. Note que se han desarrollado numerosas aplicaciones con distintas extensiones del operador OWA, por tanto, han resultado muchas más propuestas.

Después de analizar la serie de tiempo de tipo de cambio peso mexicano-dólar estadounidense y aplicar la nueva metodología en diferentes periodos, los principales hallazgos son los siguientes:

- Como lo muestra el apartado 2.1. el tipo de cambio peso mexicano/dólar estadounidense presenta varios hechos estilizados que vuelven complejo su análisis. Comportamientos atípicos, no normalidad y saltos abruptos provocan que los modelos y estimadores convencionales tiendan a ser deficientes en su pronóstico.
- En este sentido, los modelos de volatilidad ARCH-GARCH tienden a presentar dificultades practicas, esto provoca que otros modelos más complejos como los estocásticos los superen en eficiencia.
- La metodología propuesta estima parámetros en modelos de volatilidad de una forma muy sencilla, sin necesidad de utilizar vías complejas y sin requerir grandes números de datos disponibles.
- Utilizar estimadores con operadores de agregación OWA provoca que los modelos de volatilidad GARCH se adapten mucho mejor a la serie de tiempo estudiada. El hecho de agregar información adicional a la estimación de parámetros resulta en un pronóstico más acorde a la realidad.
- El nivel de éxito de la nueva metodología varia según el operador de agregación OWA utilizado y las características de la muestra. Sin embargo, esta promete minimizar el error de pronóstico en la mayoría de los casos.
- En periodos de alta inestabilidad, donde existen pronunciados comportamientos atípicos y saltos abruptos la regresión OWA-GARCH ha demostrado mejora los resultados en el pronóstico.
- La nueva metodología ha funcionado mejor midiendo la volatilidad mediante desviación estándar en comparación con log-volatilidad y volatilidad realizada.

Los resultados obtenidos confirman la hipótesis de este trabajo, con lo cual se responde a la pregunta central de investigación. Es decir, los modelos GARCH estimados con mínimos cuadrados ordinarios y operadores de agregación OWA mejoran el error de pronóstico en series de tiempo que presentan no linealidad, no normalidad y comportamientos atípicos.

LIMITACIONES

La aplicación de la nueva metodología ha sido efectiva para la volatilidad de tipo de cambio peso mexicano-dólar estadounidense y para los periodos analizados en los diferentes

apartados. Por tanto, no se pueden tomar las afirmaciones descritas en los resultados para describir otro tipo de serie o periodo.

Adicionalmente, se debe tomar en cuenta que los criterios para los vectores de peso se han hecho a través de tomadores de decisión específicos. Estas decisiones se hacen en base a el contexto en el que se trabaja, sin embargo, pueden contener un grado de subjetividad lo cual puede variar en investigaciones subsecuentes.

FUTURAS LÍNEAS DE INVESTIGACIÓN

La nueva metodología propuesta en este trabajo puede ser tomada de base para futuras aplicaciones y extensiones de la misma. En concreto se enlista lo siguiente:

- El estimador OWA-GARCH puede ser aplicado a otras volatilidades en diferentes tipos de cambio. Hacer una comparación entre resultados para diferentes volatilidades pude ser un trabajo interesante.
- Aplicar el estimador MLR-OWA en diversos modelos puede ser favorable para comprobar la funcionalidad de la metodología. Se puede extender el análisis a modelos de diversas áreas económicas, financieras, administrativas y del comportamiento.
- El estimador MLR-OWA se puede extender a toda la gran variedad de operadores OWA que han surgido en los últimos años. Operadores como el operador inducido de promedios móviles OWA (IOWMA) (Merigó y Yager, 2013) y el Bonferroni OWA (BOWA) (Yager, 2009) pueden ser considerados.
- Además de extender el estimador con diferentes operadores OWA, también se puede extender conservando la idea como estimador MCO. En este sentido se pueden desarrollar extensiones de este como mínimos cuadrados por etapas y la extensión a más variables independientes.

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APÉNDICES: ARTÍCULOS PUBLICADOS O ACEPTADOS

Apéndice 1.

Comparison of the GARCH and stochastic models: An application to the mexican peso-us dollar exchange rate. CyAUNAM.

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Comparison of the garch and stochastic models: An application to the mexican peso-us dollar exchange rate

Comparación de modelos GARCH y estocásticos: una aplicación en el tipo de cambio peso mexicano-dólar estadounidense

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Abstract

Forecasting volatility is of great importance an important topic for researchers, entrepreneurs, and policymakers. This work compares different volatility models to ascertain their forecasting efficiency. The models include standard approaches such as Autoregressive Conditional Heteroskedasticity (GARCH), exponential GARCH, and Stochastic Volatility models (SV). For estimation, a comparison between the Frequentist and the Bayesian approaches are made using the maximum likelihood and the Monte Carlo Markov Chains (MCMC) methods. The case analysis considers the Mexican peso/US dollar exchange rate. The results show a favorable behavior between the SV models estimated with the MCMC and the GARCH models in forecasting out of the sample. Additionally, the analysis shows that the current volatility reacts to the data within the last period, despite the former periods.

JEL code: C13; C32; C52; G17

Keywords: GARCH; Stochastic Model; Exchange rate

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Resumen

El pronóstico de la volatilidad es un tema importante para investigadores, empresarios y responsables políticos. Este trabajo compara modelos de volatilidad para determinar su eficiencia en el pronóstico. Los modelos incluyen modelos estándar, como los son, modelos de Heteroscedasticidad condicional autoregresiva (GARCH), exponencial y Volatilidad estocástica (SV). Para la estimación, se realiza una comparación entre los métodos frecuentistas y bayesianos, utilizando máxima verosimilitud y Cadenas de Marcov Montecarlo (MCMC). El análisis es aplicado en el tipo de cambio del peso mexicano-dólar estadounidense. Los resultados muestran que los modelos SV estimados con MCMC se comportan favorablemente frente a los modelos GARCH en el pronóstico de la muestra. Además, el análisis evidencia que la volatilidad actual reacciona a la última información dentro de un período, sin importar los períodos anteriores.

Código JEL: C13; C32; C52; G17
Palabras clave: GARCH; Modelo estocástico; Tipo de cambio

Introduction

Exchange rates play an important role in international trade, the determination of investments, business risk management, as well as in the economic situation within a country (Frankel and Saravelos, 2012; Korol 2014). The variations in currency prices are caused, in many cases, by imprecise and ambiguous factors such as economic, political and psychological conditions (Gabaix and Maggiori, 2015; Della Corte et al. 2016; Pinho and Couto, 2017). The above generates volatility, uncertainty, and risks for the economic agents that interact in financial markets.

Volatility is an important issue in regards to international decision-making, since the expected returns on prices and their high variability have a negative correlation. Therefore, high volatility generates a decrease in yields and significant losses for economic agents (Guo et al. 2014; Bali and Zhou, 2016; Morales et al. 2016). In this regard, some studies are oriented to know both the causes of these fluctuations and the alternatives to minimize uncertainty (Korol, 2014; Gupta and Kashyap, 2016; Lahmiri, 2017).

The difficulty of explaining and forecasting nominal exchange rate movements was systematically reported by Meese and Rogoff (1983); they considered their behavior as a random walk, which means that their growth rates are independent events. Similarly, Fama (1965) developed the efficient market hypothesis, which justifies the impossibility of predicting the returns of financial assets and also supports the idea that the stochastic process underlying the returns is a martingale process.

However, subsequent research has shown how the financial series cannot obey the ethical assumptions of these two proposals. Characteristics such as independence, identical distribution and normality may not appear in the series. Subsequently, the exchange rate series can present some stylized facts like non-linearity, non-normality, volatility clustering, asymmetry and heavy tails (Yang and Chen, 2014; Patton and Sheppard, 2015; Pinho et al. 2016; Byrne et al. 2016) which should be considered when modeling and forecasting volatility.

Most of the research efforts regarding price variability have focused on standard forecast models, where volatility is a key parameter, using conditional heteroskedasticity dependent on time (Korol, 2014; Pinho et al. 2016). This type of volatility models is called General Autoregressive Conditional Heteroskedasticity (GARCH), proposed by Engle (1982) and generalized by Bollerslev (1986) as an alternative to model non-linearity and volatility clusters in a simple way and easily adapting to different scenarios. Autoregressive models propose a better performance in terms of forecasting, and they are easy to combine with estimation methods (West and Cho, 1994; Lahmiri, 2017).

However, there is evidence arguing that GARCH models do not consider stylized facts of the financial series such as trends, heavy tails, and non-seasonality. Thus, stochastic models were proposed by Taylor (1986) whose main advantage is to consider a random component adaptable to abrupt changes. In stochastic models, the volatility estimation process is not directly observable and part of the equation that represents it is not completely known. To do this, an additional likelihood function must be constructed that captures the behavior of the collected data (Jacquier et al. 1994; Sandmann and Koopman, 1998). The likelihood function is the one that has made the difference between stochastic estimates in the last decade. On the one hand, proposals are using maximum likelihood (Ait-Sahalia and Kimmel, 2007; Abanto-Valle et al. 2017). Alzghool (2017) proposes quasi-likelihood and asymptotic quasi-likelihood approaches obtaining favorable results. On the other hand, Bayesian simulations have proven forecasting efficiency in numerous occasions (Raftery et al. 1997; Kastner, 2017). In stochastic volatility models, the Monte Carlo Markov Chains (MCMC) has been generally used in estimation due to its development in algorithms (Jacquier et al. 1994; Kastner and Fruhwirth-Schnatter; 2014; Kastner, 2016).

However, the two types of models implied time-varying volatilities with very different properties. To compare the differences, the literature has mainly focused on their forecasting performance (Rossi, 2013; Clark and Ravazzolo, 2015; Chan and Grant, 2016). Knowing the best model for a financial series is a fundamental issue for making decisions, especially in cases of emerging and free-floating economies where volatility tends to be recurring (Neumeyer and Perri, 2005; Rafi and Ramachandran, 2018). In this study, a comparison of some GARCH and SV models was made. The

main objective is to know which model is best to explain the volatility of the Mexican peso-US dollar exchange rate in terms of minimizing the forecast error. This work is divided into five sections. The second section describes the traditional models used in price volatility. The third section describes the structure of the proposed models and presents the data used to calculate volatility. In the fourth section, the estimation of the models and a comparison of the two most efficient models to predict volatility are made. Finally, conclusions are presented and future studies are suggested.

Volatility models

This section presents a summary of the two traditional volatility models used in this study, in order to evaluate their efficiency.

The GARCH model

The GARCH model (Bollerslev, 1986) is a volatility model where the recent past data provides information on the variance of a period. Therefore, the value of the current forecast is based on past information. GARCH models have been used in different areas of volatility price forecasting, such as the price indices (Kim et al. 2016; Yuan et al. 2017), oil prices (Klein and Walther, 2016; Kristjanpoller and Minutolo, 2016) and exchange rates (Trucios and Hotta, 2016; Gupta and Kashyap, 2016).

The GARCH models for log return series, are given by returns $r_t = log\left(\frac{x_t}{x_{t-1}}\right)$, let a_t is the innovation at time t, as $a_t = r_t - E_{t-1}[r_t]$. Then a_t follows a GARCH (p,q) model if $a_t = \sigma_t \epsilon_t$, where $\{\epsilon_t\}$ is a sequence of independent random variables with equal distribution, average 0 and variance 1, then the volatility model is represented as follows:

$$h_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 h_{t-1}^2, \tag{1}$$

where $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$ and $\sum_{i=1}^{max(p,q)} \quad (\alpha_1 + \beta_1) < 1$. The variance process is always straightly positive and stationary.

The GARCH model family can be obtained from a transformation of the conditional standard deviation (SD) h_t, determined by the transformation $f(.)$ of the innovations a_t, and lagged transformed conditional SDs. This is, the conditional variance h_t^2 in a simple equation follows an AR (1) process. The GARCH (2, 1) model (in which h_t^2 follows an AR (2) process) allows a better variance dynamic, then we have:

$$h_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 h_{t-1}^2 + \beta_2 h_{t-2}^2. \tag{2}$$

As we have already noted, we assume that the parameters $\alpha_0, \alpha_1, \beta_1, \beta_2$ are all positive and $\alpha_1 + \beta_1 + \beta_2 < 1$.

The GARCH model assumes that positive and negative error variations have a symmetric effect on volatility, which means that good and bad news have the same effect on volatility. Patton and Sheppard (2015) show that future volatility is more strongly related to the volatility of past negative returns than to that of positive returns and that the impact of a price jump on volatility depends on the sign of the jump, with negative (positive) jumps leading to higher (lower) future volatility.

An exponential GARCH, EGARCH model (Nelson, 1991) assumes that if the distribution is symmetric, the change in the variance of tomorrow is conditionally not correlated with the excess of yields of today. Therefore, the asymmetry in a GARCH model can be calculated as follows:

$$h_t^2 = \alpha_0 + \beta_1 h_{t-1}^2 + \gamma \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}^2}} + \omega \left[\frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right], \tag{3}$$

where $\alpha_0, \beta_1, \gamma,$ and ω are constant parameters. Since the γ coefficient is typically negative, positive return shocks generate less volatility than negative return shocks. The EGARCH (1,1) suggests an interesting metric to analyze the effect of the news on conditional heteroscedasticity.

In addition, other models have been proposed in asymmetric volatility, such as the QGARCH quadratic introduced by Engle (1990) and Sentana (1995), and the GJR model proposed by Glosten et al. (1993).

Stochastic models

Stochastic volatility models (SV) consider a random variable, in contrast to GARCH models in which the conditional variance is a deterministic function of the parameters and the past data. In financial areas, the SV models are applied in many variables such as inflation (Chan, 2015; Diebold et al. 2017), price indices (Pinho et al. 2016; Pinho and Couto, 2017) and exchange rates (Kastner and Fruhwirth, 2014; Alzghool, 2017).

The basic stochastic model is represented as a linear space state model with logarithmic and chi-square perturbations, its volatility can be represented as an autoregressive AR (1) model. The first model is the standard stochastic volatility model SV (1) and it is represented as follows:

$$y_t = exp\left\{\frac{h_t}{2}\right\}\sigma_\epsilon \varepsilon_t \qquad \varepsilon_t \sim N(0,1),$$
$$h_t^2 = \mu + \phi h_{t-1} + \tau\eta_t \qquad \eta_t \sim N(0,1), \tag{4}$$

where y_t is the response variable, h_t is the unobserved log-volatility of y_t. The use of more than one autoregressive process results in some extensions of the stochastic model. Considering an SV (2) the log-volatility h_t follows a stationary AR (2) process, that is:

$$h_t^2 = \mu + \phi h_{t-1} + \phi h_{t-2} + \tau\eta_t \qquad \eta_t \sim N(0,1). \tag{5}$$

Note that, the estimation process of stochastic volatility is not directly observable. Therefore, an additional likelihood function must be constructed to include the behavior of the collected data. Jaquier, et al (1994) proposes a Bayesian approach, using the Monte Carlo Markov chain (MCMC) technique where the posterior distribution of the parameters is sampled.

The MCMC creates a Markov process whose stationary transition distribution is specified through P (θ | Y), then runs a large enough number of simulations where the distribution of the current process is as close as possible to the stationary transition distribution, thus creating a posterior distribution (Salimans et al. 2015; Ravenzwaaij et al. 2018).

The simulation starts by taking a random draw z_0 from the initial distribution $p(x \mid z)$ and then a random stochastic transition operator z_1 is applied. Then:

$$z_t \sim q(\frac{z_t}{z_{t-1}}, x).$$

By judiciously choosing the transition operator and applying it repeatedly, we have a result that converges to a posterior $p(x \mid z)$ distribution with an optimal result.

A series of algorithms that carry out the basic idea of the MCMC method have been proposed; these generate a large number of repetitions in a short period of time. Among the most widely used algorithms are the Metropolis-Hasting algorithm (Lin et al. 2000; Doucet et al. 2015) and Gibss sampling (Roberts and Rosenthal, 2009; Billio et al. 2016). The MCMC method in stochastic models has been used for price volatility with good performance (Kim et al. 2017; Brix et al. 2018).

Data and model analysis

In order to analyze the efficiency of the models described in the previous section, a real case of the exchange rate market is considered. The data are the daily prices of the FIX exchange rate for the US dollar-Mexican peso, during the period 1994-2018. The information was converted to monthly data where the first data corresponds to April 1994 and the last date is October 2018. The prices are taken from the official website of the Mexican Central Bank - Banco de Mexico (BANXICO).

Note that volatility is a variable not directly observed in the market. Therefore, volatility was calculated as log-volatility (Kim et al. 1998; Chan and Grant, 2016; Gatheral et al. 2018). The price returns (R) of the currency are used, is the difference between today's price and yesterday's price logarithm. The formulation is as follows:

$$R = ln(\frac{p_t}{p_{t-1}}) \tag{6}$$

where p_t is the current period price and h_t^2 is the log-volatility analyzed, then log volatility is:

$$hR_t^2 = \left(R_t - \underline{R}\right)^2, \tag{7}$$

where $\underline{R}$ is the mean in price returns. Five volatility models were developed based on the traditional GARCH and SV models, then:

GARCH 1,1: $hR_t^2 = \alpha_0 + \alpha_1 aR_{t-1}^2 + \beta_1 hR_{t-1}^2$

GARCH 1,2: $hR_t^2 = \alpha_0 + \alpha_1 aR_{t-1}^2 + \beta_1 hR_{t-1}^2 + \beta_2 hR_{t-2}^2$

EGARCH: $hR_t^2 = \alpha_0 + \beta_1 log(hR_{t-1}^2) + \gamma \frac{aR_{t-1}}{\sqrt{hR_{t-1}^2}} + \omega \left[\frac{|aR_{t-1}|}{\sqrt{hR_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$

SV 1: $hR_t^2 = \mu + \phi hR_{t-1}$

SV 2: $hR_t^2 = \mu + \phi hR_{t-1} + \phi hR_{t-2}$

Finally, parameter estimation was calculated as follows: GARCH (1,1), GARCH (2,1) and EGARCH were estimated by maximum likelihood with a normal distribution. SV (1) and SV (2) using an exponential distribution and a prior function for hR_{t-1}, $N(0,1)$, and for μ, $N(0.0012,0.1)$.

Results

Models results

First, the GARCH models were developed and then compared, in order to select the best one considering some criteria. The Akaike information criterion (AIC) (Akaike, 1974) is a technique based on a sample fit to estimate the likelihood of a model to predict future values. A good model is the one that has minimum AIC among all the other models. The Schwarz Criterion (SC) (Schwarz, 1978) considers both, the closeness of fit of the points to the model and the number of parameters used by the model. Using this criterion, the best model is the one with the lowest SC.

Table 1

Analysis of results in GARCH models

Model	Model parameters	Parameters significance	Akaike info criterion	Schwarz criterion
GARCH (1.1)	$\alpha_0 = 0.000433$ $\alpha_1 = 0.465281$ $\beta_1 = 0.128459$	0.0000 0.0000 0.0303	-4.314630	-4.264637
GARCH (2,1)	$\alpha_0 = 0.000434$ $\alpha_1 = 0.459434$ $\beta_1 = 0.208128$ $\beta_2 = -0.073937$	0.0000 0.0000 0.0273 0.1361	-4.314609	-4.252118

EGARCH	$\alpha_0 = -3.428093$ $\beta_1 = 0.215611$ $\gamma = 0.483839$ $\omega = 0.553491$	0.0000 0.0005 0.0000 0.0000	-4.403681	-4.341190

The results in Table 1 show that the EGARCH model has the minimum AIC and minimum SC. However, the parameter α_0 is negative, then the assumption of positivity is not met. Therefore, the best model that met the criterion s is the GARCH (1, 1).

To estimate the parameters of the SV models, the Metropolis Hasting algorithm (Metropolis et al. 1953; Hasting, 1970) is used. Monte Carlo standard error (MCSE) is a standard deviation around the posterior mean of the samples. The acceptable size of the MCSE depends on the acceptable uncertainty, then when we compare models, a lower MCSE is better (Flegal et al. 2008).

Table 2

Analysis of results in SV models

Model	Model parameters	Montecarlo standard error parameters	Max Efficiency MCMC
SV (1)	$\mu = 0.0009798$ $\phi = 0.2209891$	0.00097 0.00169	0.1342
SV (2)	$\mu = 0.0007921$ $\phi = 0.183780$ $\rho = 0.172050$	0.00001 0.00215 0.00209	0.1032

Table 2 presents the resulting parameters of stochastic models. The Monte Carlo standard error shows that the parameter μ is better in the second model, but the rest of the parameters are more significant in the first model. The efficiency MCMC demonstrates that SV1 is the best model with 13.42%.

Comparison of models

In this section, the GARCH (1, 1) and the SV (1) models are compared in the forecast for the next seven periods. It is observable that the following period which corresponds to November 2018 is a period of high volatility, while the fourth period which corresponds to February 2019 shows low volatility. We use aR and hR that were calculated previously on dependent variables. To calculate and

analyze the errors in forecasting, we use the Mean Absolute Deviation (MAD), the Root Mean Squared Error (RMSE) and the Mean Absolute Percentage Error (MAPE) methods (Franses, 2016; Khair et al. 2017). The results are in Table 3.

The error indicators show that the SV1 model minimizes the error for forecasting in periods of instability (high or low volatility). It is observable that it minimizes the absolute error in all periods except for the last two. The squared error is small in the stochastic model for most periods, except in periods five and six.

The global results in MAD, RMSE, and MAPE are smaller in the SV1 model than in the GARCH (1.1) model. Therefore, the SV1 model is considered as the best model to predict the variability in the exchange rate of the outside sample.

Conclusions

The exchange rate is a financial variable difficult to predict due to the different inaccuracies that may occur over time. Nevertheless, literature models have proposed a way to know the future volatility. GARCH and SV models have been commonly used for forecasting and estimating volatility. However, no consensus has been reached on which is the best proposal.

This work proposes a comparison between some fundamental models, GARCH and SV. The analysis concluded that the SV model works better than GARCH models; both were used to out-of-sample forecasting volatility of the exchange rate. The results show a decrease in the forecast error in most of the periods analyzed when the stochastic model is used.

In addition, the analysis found that the models of volatility in the series are effective only when the past information is only a period back, because when the models consider two lags their effectiveness decreases. In this sense, both the GARCH and SV models show better adjustments when they only consider a period lag.

Finally, it is suggested that in future research, the functional SV model can be extended in order to minimize the error.

Table 3

Forecast analysis

	Period	Value	Real log-volatility	Absolute error	Squared error	Percentage error	MAD	RMSE	MAPE
GARCH (1,1)	1	0.00051	0.00232715	0.00182	3.313E-06	0.78	0.00135	0.00422	13132.6
	2	0.00158	0.00018830	0.00139	1.939E-06	7.40			
	3	0.00072	0.00340258	0.00268	7.176E-06	0.79			
	4	0.00211	0.00000002	0.00211	4.448E-06	91872			
	5	0.00070	0.00002223	0.00068	4.647E-07	30.6			
	6	0.00053	0.00041893	0.00011	1.319E-08	0.27			
	7	0.00070	0.00003959	0.00066	4.315E-07	16.5			
SV1	1	0.00098	0.00232715	0.00135	1.81E-06	0.57	0.00133	0.00002	10787.4
	2	0.00149	0.00018830	0.00131	1.71E-06	6.91			
	3	0.00102	0.00340258	0.00238	5.67E-06	0.70			
	4	0.00173	0.00000002	0.00173	3.00E-06	7543			
	5	0.00098	0.00002223	0.00096	9.17E-07	43.0			
	6	0.00098	0.00041893	0.00057	3.20E-07	1.31			
	7	0.00107	0.00003959	0.00103	1.07E-06	26.0			

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Apéndice 2.

Induced OWA operators in linear regression. JIFS.

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Induced OWA operators in linear regression

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Abstract. The induced ordered weighted average (IOWA) is an aggregation operator that provides a parameterized family of operators between the minimum and the maximum. This work presents a new application that uses the simple linear regression (LR) and the IOWA operator in the same formulation. We study some of its main properties and particular cases. The main advantage of the linear regression IOWA operator is that it unifies the IOWA operator with the linear regression in the same formulation considering the degree of optimism and pessimism of the decision maker. Thus, we can under- or over-estimate the regression according to complex attitudes that the decision may have in the analysis. The work ends analyzing the applicability of this new approach in a problem regarding exchange rate forecasting. The objective of the new approach is to analyze the information in a more complete way.

Keywords: IOWA operator, linear regression, aggregation operator

1. Introduction

Statistical linear regression (LR) is a tool for the analysis and summarization of information used in developing mathematical models for applications in business [1], economic forecasting [2], natural sciences [3], engineering [4], and so on. The most common statistical method used to estimate the parameters is the least-squares regression (LSR). This method works finding the "best curve" through the data that minimizes the residual sum of squares. LSR has advantages as simplicity and easy to use [5, 6], however atypical data and the complex relationship of the variables can be problematic in the estimation of regression analysis because the consideration of linearity becomes untrue.

The ordered weighted average (OWA) [7] is an aggregation operator used in uncertain environments for analyzing a set of data between the minimum and the maximum [8]. During the last years, several authors have developed a wide range of extensions and generalizations of the OWA operator [9–11]. An interesting extension of the OWA is the induced OWA (IOWA) operator [12], in which the ordering of the arguments is induced by another variable called the order inducing variable. Another extension is the generalized OWA (GOWA) operator [13, 14] that adds an additional parameter controlling the power to which the argument values are raised, using generalized means. In this case, Merigó and Gil-Lafuente [15] introduce an extension of the OWA operator that uses the main characteristics of both the IOWA and the GOWA called induced GOWA (IGOWA) operator. In the last few years, the IGOWA operator has been studied in many approaches [16–18] by using fuzzy information [19, 20], moving average [21], logarithmic means [22], distance measures

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[23], Bonferroni means [24, 25] and prioritized information [26].

The advantages of using OWA and IOWA operators are the possibility of considering a reordering process attached by many factors such as the degree of optimism and psychological aspects [27]. We can represent different scenarios through the distinction between descending and ascending orders. Given that identifying the optimal combination depends on the situation in which we are working, the procedure to select the best alternative depends on the type of knowledge that the decision maker has on the state of the situation [28].

In the case of exchange rate forecasting, there is a discussion to how analyze the economic and financial changes presented by the economies nowadays. On the one hand, we find data transformations such as tendency, filtering and seasonal adjustment caused by the complex relationship between the variables [29, 30]. On the other hand, active risk taking in currency markets is highly concentrated in financial players who present repeated bouts of financial risk intolerance and illiquidity [31, 32]. Therefore, the construction models in the study of forecasting in exchange rates consider the complex relationship between variables and the subjective evaluation of the different scenarios.

In this paper, we present an operator using IGOWA operator in calculation of the parameters in linear regression. That is to say, it uses order inducing variables in the reordering process and generalized means to calculate the variables means. The objective is to obtain a linear model which can be estimated according to a degree of optimism to get different forecast scenarios. Moreover, by using the IOWA operator, the decision maker can under- or over- estimate the information according to a complex attitude that includes the degree of optimism but also psychological and competitive factors. So, the use of induced operators helps us to work with complex variables where not always the highest benefit is the best solution. For example, this may occur depending on the results that competitors get or some personal opinions about the alternatives.

The rest of the paper is organized as follows. Section 2 briefly reviews some basic preliminaries regarding linear regression and the OWA operators. Section 3 presents the new approach regarding the use of OWA and IOWA operators in linear regression. Section 4 further extends this approach by using induced generalized aggregation operators. Section 5 develops an application of the new framework in the analysis of exchange rate forecasting between USD and MXN. Section 6 summarizes the main findings and conclusions of the paper.

2. Preliminaries

This section presents a brief description of the simple linear regression and the operators OWA, IOWA and IGOWA.

2.1. Linear regression

The linear regression purpose is to estimate a dependent variable according to the values or movements of other explanatory variables in a linear form, this relation could be used to describe either the random variables or their realization [33, 34]. Linear regression method can be used for forecasting under the assumption of continuing the relation between the variables in the future. It is defined as follows:

Definition 1. Consider a simple regression problem, given a set of variables (x_k, y_k), given the $k = 1, \ldots, K : x_k \in U^n, y_k \in U$, we have a model $f_\theta : R^n \to R$, parameterized by a parameter vector $\theta \in \Omega \subseteq R^p$, which is composed by α y β. The estimated simple linear regression model is developed as $y_j = \alpha + \beta x_j$. where the value α stand for the value of y_j when $x_j = 0$, and β is the change in y_j when x_j shift in a unit. The objective is to determine the parameters vector θ. In ordinary least squares the optimum parameters vector is found by minimizing as $MinimizeF(\theta) = \sum_{K=1}^{K} w_k (r_k(\theta))^2$. where residuals are $r_k(\theta) = f_\theta(x_k) - y_k$.

To find the values α and β that minimizes the previous expression, differential calculations are used as follows:

$$\beta = \frac{Cov(x, y)}{var(x)} = \frac{\sum_{k=1}^{k} (x_k - \bar{x})(y_k - \bar{y})}{\sum_{k=1}^{k} (x_k - \bar{x})^2} \quad (1)$$

$$\alpha = \bar{y} - \beta\bar{x} \quad (2)$$

Where $\bar{x}$ and $\bar{y}$ are the averages in the sets x_k, y_k respectively.

2.2. OWA operator

The OWA operator was introduced by Yager [7], and provides a method to add several arguments that

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lie between the maximum and minimum operators. It is defined as follows:

Definition 2. An OWA operator with dimensions n is a model $OWA : R^n \to R$ such that it has associated weights vector $W = [w_1, w_2, .., w_n]^T$ thus $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, So we have:

$$OWA\,(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \qquad (3)$$

where b_j is the jth largest in $a_1, a_2, .., a_n$.

The OWA operator is a mean operator that satisfies the conditions:

1) Monotonicity: $F(a_1, \ldots, a_n) \geq F(\hat{a}_2, \ldots, \hat{a}_n)$ if $a_i \geq \hat{a}_i$ for i.
2) Commutativity: The initial ordering of the arguments does not matter.
3) Idempotency: if $a_j = a$ for all j, then $F(a_i, \ldots, a_n) = a$.

2.3. IOWA operator

The induced ordered weighted average operator (IOWA) was introduced by Yager and Filev [12], and uses argument pairs, called IOWA pairs. In this operator one component is used to induce an ordering over the second component and then aggregated. This provides an aggregation in scenarios with multiple variables and a mixture of these.

Definition 3. An IOWA operator of dimension n is a mapping $IOWA : R^n \to R$ such that it has an associated weights vector $W = [w_1, w_2, .., w_n]^T$ where $w_i =\in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, then:

$$IOWA\,(\langle u_1, a_1\rangle \langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \sum_{j=1}^{n} w_j b_j, \qquad (4)$$

where u_i is the variable that induced order and a_i is the argument of the variable. In this case b_j is the value a_i in the IOWA that have the jth largest u_i.

The IOWA also satisfies the conditions of monotonicity, commutativity and idempotency.

2.4. IGOWA operator

The induced generalized OWA operator (IGOWA) was introduced by Merigó and Gil-Lafuente [15], and uses the generalized means in the OWA operator proposed by Yager [14]. At the same time, it is considered an inducing variable to reorder the arguments. The IGOWA is defined as follows:

Definition 4. An IGOWA operator of dimension n is a mapping $IGOWA : R^n \to R$ such that it has a vector of associated weights $W = [w_1, w_2, .., w_n]^T$ were it is accomplished $w_i =\in [0, 1]$ y $\sum_{i=1}^{n} w_i = 1$, So we have:

$$IGOWA\,(\langle u_1, a_1\rangle \langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda}\right)^{1/\lambda}, \qquad (5)$$

where u_i is the order induced variable and a_i is the variable argument, then b_j is the value a_i in the IOWA pair that have the jth largest u_i. It satisfies the conditions of monotonicity, commutativity and idempotency.

3. Induced OWA operators in linear regression

The OWA linear regression is an extension of the OWA operator whose means are calculated by steps of ordering the arguments [35]. We can see that depending on the order we use, we can obtain different means adapted to the level of optimism or pessimism and distinguishing between the descending (DOWA) operator and the ascending (AOWA) order. It can be defined as follows:

Definition 5. An OWA linear regression of dimension n is a model $OWA : R^n \to R$ such that it has an associated weighting vector $W = [w_1, w_2, .., w_n]^T$ where $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, so we have:

$$y_{owa} = \alpha_{OWA} + \beta_{OWA} x_j. \qquad (6)$$

Then we calculate the parameter β as follows:

$$\beta_{OWA} = \frac{Cov_{owa}\,(x, y)}{var_{OWA}\,(x)} = \frac{\sum_{k=1}^{k} w_j\,(x_j - \mu)\,(y_j - \upsilon)}{\sum_{k=1}^{k} w_j\,(x_j - \mu)^2}, \qquad (7)$$

where x_j y y_j is the jth largest arguments in the variables x and y severally, μ and ν are means

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(OWA operator x,y), and the arguments $(a_i - \mu)^2$ and $(x_j - \mu)\ (y_j - \upsilon)$ have some associated weights (WA) w_j with $\sum_{i=1}^{n} w_j = 1$ y $w_j \in [0, 1]$. We can see that the estimator β_{OWA} can be calculated in different ways depending on whether the operator OWA is being considered in some of its components or not. Therefore, we can get seven cases in particular as Table 1 shows. For further reading on the variance OWA (var-OWA) and the covariance OWA (covar-OWA), see for example Merigó et al. [36] and Yager [37, 38].

In this sense the parameter α is:

$$\alpha_{OWA} = \upsilon - \beta_{OWA}\mu, \tag{8}$$

where μ and ν are the means x and y (the OWA operator). Different combinations can be obtained according to the type of OWA average we use in each component of the formula, obtaining three different additional forms as show in Table 2.

We can see that we can obtain more different combinations by uniting α (Table 2) and β (Table 1). Particularly, we could build $7 \times 4 = 28$ cases.

Special cases of OWA operator are considered depending on the ordering weights: the maximum $[w = (1, 0, \ldots, 0)]$, the minimum $[w = (0, \ldots, 0, 1)]$, and the arithmetic mean $\left[w = \left(\left(\frac{1}{K}\right), \ldots, \left(\frac{1}{K}\right)\right)\right]$. In this way, we can obtain many other interesting cases, like additive neat OWA (ANOWA) operator [39], determined by weighting function $f(x)$ with weights $w_f = (w_1, w_2, \ldots, w_n)$ defined as $w_i = f(x_i) \Big/ \sum_{j=1}^{n} f(x_i)$.

This new approach shares the proprieties of OWA operators:

- Monotonic with respect to arguments values, this is, let a second ordered argument vector $w = [b_1, b_2, \ldots, b_n]$ such that for each j, if $a_j \geq b_j$ then:

$$F(y_{owa}(a_1, a_2, \ldots, a_n)) \geq F(y_{owa}(b_1, b_2, \ldots, b_3))$$

- Symmetry (commutativity) implied by equal importance of criteria, this is, if A and A´ are the ordered argument vectors of $(a_1, a_2, \ldots, a_n)$ and $(a'_1, a'_2, \ldots, a'_n)$ respectively then $A = A'$. Hence:

$$F(y_{owa}(a_1, a_2, \ldots, a_n)) = F(y_{owa}(a'_1, a'_2, \ldots, a'_n))$$

- Idempotent in the sense that if $a_j = a$, for all $j = 1, \ldots, n$, then:

$$F(y_{owa}(a_1, a_2, \ldots, a_n) = a$$

In the case of IOWA linear regression β and α are calculated with the IOWA means. Here one component is used to induce an ordering over the arguments. It can include an extensive range of factors in the reordering process rather than consider the values of the arguments. Therefore, we can use more complex variables related to other. β and α IOWA develops as follows:

Definition 6. Is a IOWA linear regression of dimension n if we have a model $IOWA : Rn \times Rn \rightarrow R$ such that it has associated weights vector $W = [w_1, w_2, .., w_n]^T$ thus $w_i = \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, So we have:

$$y_{Iowa} = \alpha_{IOWA} + \beta_{IOWA} x_j. \tag{9}$$

The parameters can be defined as follows:

$$\beta_{IOWA} = \frac{Cov_{IOWA}(x, y)}{var_{IOWA}(x)} = \frac{\sum_{k=1}^{k} w_j \left(x_j - \mu\right)\left(y_j - \upsilon\right)}{\sum_{k=1}^{k} w_j \left(x_j - \mu\right)^2} \tag{10}$$

$$\alpha_{IOWA} = \upsilon - \beta_{IOWA}\mu, \tag{11}$$

where x_j y y_j are the induced values with the IOWA pair (u_i, x_i), μ and ν are the means (OWA operator x, y),where (u_i, x_i) is the jth largest u_i, u_i is the order-inducing variable, and the arguments $(a_i - \mu)^2$ and $(x_j - \mu)\ (y_j - \upsilon)$ has some associated weights (WA) w_j with $\sum_{i=1}^{n} w_j = 1$ and $w_j \in [0, 1]$.

We can see that β_{IOWA} and α_{IOWA} allow the same number of combinations that are considered previously according to the portion of the formula used by the IOWA operator. Reviewing, $\bar{x}$ and $\bar{y}$ represent the arithmetic means used in calculation of beta and alpha traditionally. But it is possible to consider either OWA means or classical means. Table 3 presents different cases that could be considered in the aggregation of a linear regression IOWA process.

Note that the fourth case when all the variables use the classical means is omitted.

The special cases of maximum and minimum are as follows with IOWA operators: maximum when $w_p = 1$ and if p is the ordered position of the largest argument and minimum if $w_p = 1$ and p is the

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Table 1
Analysis of β parameter

Case	Formula	Description
1	$\beta_{OWA} = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	In the covariance, normal means are used in the differences of the variable x while in the differences of the variable y OWA means are used. In the calculation of the variance it is calculated with OWA means.
2	$\beta_{OWA} = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	In the covariance, normal means are used in the differences of the variable y while in the differences of the variable x OWA means are used. In the calculation of the variance it is calculated with OWA means.
3	$\beta_{OWA} = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	In the covariance, OWA means are used in the differences of the variable y and x. in the calculation of the variance it is calculated with normal means.
4	$\beta_{OWA} = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	In the covariance, normal means are used in the differences of the variable y while in the differences of the variable x OWA means are used. In the calculation of the variance it is calculated with OWA means.
5	$\beta_{OWA} = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	In the covariance, normal means are used in the differences of the variable y while in the differences of the variable x OWA means are used. In the calculation of the variance it is calculated with normal means.
6	$\beta_{OWA} = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	In the covariance, normal means are used in the differences of the variable x while in the differences of the variable y OWA means are used. In the calculation of the variance it is calculated with normal means.
7	$\beta_{OWA} = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	In the calculation of variance and covariance, OWA means are used in all its sections.

Table 2
Analysis of α parameter

Case	Formula	Description
1	$\alpha_{OWA} = \bar{y} - \beta_{OWA}\mu$	It is calculated with y normal mean and with x OWA mean.
2	$\alpha_{OWA} = \upsilon - \beta_{OWA}\bar{x}$	It is calculated with y OWA mean and with x normal mean.
3	$\alpha_{OWA} = \upsilon - \beta_{OWA}\mu$	It is calculated with y OWA mean and with x OWA mean.
4	$_{OWA} = \bar{y} -_{OWA} \bar{x}$	Both y and x use normal mean.

ordered position of the smallest argument. In this way, we have LR-IOWA special cases:

- The maximum β when $\bar{x} = \mu$ and $\mu = maximum\ IOWA$, then $\bar{y} = \nu$ and $\nu = minimum\ IOWA$
- The maximum α is when $\bar{y} = \nu$, $\nu = maximum\ IOWA$ and we use minimum β_{IOWA}
- The minimum α is when $\bar{y} = \nu$, $\nu = minimum\ IOWA$ and β_{IOWA} is the maximum.

The IOWA operator in linear regression preserves the OWA properties. First it is monotonic if we assume that we have a second vector IOWA pair $\langle u_i, b_i\rangle$, such that for each j, we reorder according to the values of u_i. Let A and B be the respective ordered argument vectors. Since the u_i are the same for both collections, if $a_j \geq b_j$ then:

$$F\left(y_{IOWA}\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle\right) \geq F\left(y_{IOWA}\langle u_i, b_i\rangle, \ldots \langle u_n, b_n\rangle\right)$$

Proof. Let.

$$f\left(y_{IOWA}\langle u_1, a_1\rangle, \ldots, \langle u_n a_n\rangle\right) = \alpha_{IOWA} + \beta_{IOWA} x_i$$

and

$$f\left(y_{IOWA}\langle u_1, e_1\rangle, \ldots, \langle u_n e_n\rangle\right) = \alpha_{IOWA} + \beta_{IOWA} x_i$$

Commutativity: Let $f(y_i)$ be an application of IOWA operator and where $f(\langle u_1, a_1\rangle, \ldots, \langle u_n, a_n\rangle)$ is a permutation of the arguments $f(\langle u_i, e_i\rangle, \ldots, \langle u_n, e_n\rangle)$. Then.

$$F\left(y_{IOWA}\langle u_i, a_i\rangle, \ldots, \langle u_n, a_n\rangle\right) = F\left(y_{IOWA}\langle u_i, b_i\rangle, \ldots \langle u_n, b_n\rangle\right)$$

Proof. Let.

$$f\left(y_{IOWA}\langle u_1, a_1\rangle, \ldots, \langle u_n a_n\rangle\right) = \alpha_{IOWA} + \beta_{IOWA} x_i$$

and

$$f\left(y_{IOWA}\langle u_1, e_1\rangle, \ldots, \langle u_n e_n\rangle\right) = \alpha_{IOWA} + \beta_{IOWA} x_i$$

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Table 3
Analysis of combinations with β and α

		β	α	
1	1.1	$\beta_{IOWA}1 = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}1 = \bar{y} - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance in the difference of the variable x and in the mean of y the calculation of alpha. in all others sections are used OWA means.
	1.2	$\beta_{IOWA}2 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}1 = \bar{y} - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance in the difference of the variable y and in the mean of y the calculation of alpha. in all others sections are used OWA means.
	1.3	$\beta_{IOWA}3 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	$\alpha_{IOWA}1 = \bar{y} - \beta_{IOWA}\mu$	Normal means are used in the calculation of the variance and in the mean of y the calculation of alpha. in all others sections are used OWA means.
	1.4	$\beta_{IOWA}4 = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}1 = \bar{y} - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance and in the mean of y the calculation of alpha. in all others sections are used OWA means.
	1.5	$\beta_{IOWA}5 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	$\alpha_{IOWA}1 = \bar{y} - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance in the difference of the variable y and in the variance, also in the mean of y calculation of alpha. in all others sections are used OWA means.
	1.6	$\beta_{IOWA}6 = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	$\alpha_{IOWA}1 = \bar{y} - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance in the difference of the variable x and in the variance, also in the mean of y the calculation of alpha. in all others sections are used OWA means.
	1.7	$\beta_{IOWA}7 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}1 = \bar{y} - \beta_{IOWA}\mu$	Normal means are used in the calculation of alpha. in all others sections are used OWA means.
2	2.1	$\beta_{IOWA}1 = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}2 = \upsilon - \beta_{IOWA}\bar{x}$	Normal means are used in the calculation of the covariance in the difference of the variable x and in the mean of x the calculation of alpha. in all others sections are used OWA means.
	2.2	$\beta_{IOWA}2 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}2 = \upsilon - \beta_{IOWA}\bar{x}$	Normal means are used in the calculation of the covariance in the difference of the variable x and in the mean of x the calculation of alpha. in all others sections are used OWA means.
	2.3	$\beta_{IOWA}3 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	$\alpha_{IOWA}2 = \upsilon - \beta_{IOWA}\bar{x}$	Normal means are used in the calculation of the variance and in the mean of x the calculation of alpha. in all others sections are used OWA means.
	2.4	$\beta_{IOWA}4 = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}2 = \upsilon - \beta_{IOWA}\bar{x}$	Normal means are used in the calculation of the covariance and in the mean of x the calculation of alpha. in all others sections are used OWA means.
	2.5	$\beta_{IOWA}5 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	$\alpha_{IOWA}2 = \upsilon - \beta_{IOWA}\bar{x}$	Normal means are used in the calculation of the covariance in the difference of the variable y and in the variance, also in the mean of x the calculation of alpha. in all others sections are used OWA means.
	2.6	$\beta_{IOWA}6 = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	$\alpha_{IOWA}2 = \upsilon - \beta_{IOWA}\bar{x}$	Normal means are used in the calculation of the covariance in the difference of the variable x and in the variance, also in the mean of x the calculation of alpha. in all others sections are used OWA means.

(*Continued*)

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Table 3
(*Continued*)

		β	α	
	2.7	$\beta_{IOWA}7 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}2 = \upsilon - \beta_{IOWA}\bar{x}$	Normal means are used in the calculation in the x of alpha. in all others sections are used OWA means.
3	3.1	$\beta_{IOWA}1 = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}3 = \upsilon - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance in the difference of the variable x. in all others sections are used OWA means.
	3.2	$\beta_{IOWA}2 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}3 = \upsilon - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance in the difference of the variable y. in all others sections are used OWA means.
	3.3	$\beta_{IOWA}3 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	$\alpha_{IOWA}3 = \upsilon - \beta_{IOWA}\mu$	Normal means are used in the calculation of the variance. in all others sections are used OWA means.
	3.4	$\beta_{IOWA}4 = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}3 = \upsilon - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance. in all others sections are used OWA means.
	3.5	$\beta_{IOWA}5 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\bar{y})}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	$\alpha_{IOWA}3 = \upsilon - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance in the difference of the variable y and in the variance. in all others sections are used OWA means.
	3.6	$\beta_{IOWA}6 = \frac{\sum_{k=1}^{k} w_j(x_j-\bar{x})(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\bar{x})^2}$	$\alpha_{IOWA}3 = \upsilon - \beta_{IOWA}\mu$	Normal means are used in the calculation of the covariance in the difference of the variable x and in the variance. in all others sections are used OWA means.
	3.7	$\beta_{IOWA}7 = \frac{\sum_{k=1}^{k} w_j(x_j-\mu)(y_j-\upsilon)}{\sum_{k=1}^{k} w_j(x_j-\mu)^2}$	$\alpha_{IOWA}3 = \upsilon - \beta_{IOWA}\mu$	All sections for calculated alpha and beta are used OWA means.

Boundary condition: Assume that the ordered argument vector B induced by the order inducing variable can be ordered as $min_i\left[a_j\right]$ or $max_j\left[a_j\right]$, then:

$$y_{IOWA}\left(min_i\left[a_j\right]\right) \leq F\left(y_{IOWA}u_i, a_i\right) \leq y_{IOWA}\left(max_j\left[a_j\right]\right)$$

Idempotency: Let $f(y_i)$ be an application of IOWA operator. If $a_i = a$ for all a_i then:

$$f\left(y_{IOWA}\left\langle u_1, a_1\right\rangle, \ldots, \left\langle u_n a_n\right\rangle\right) = a$$

The weights vector can be calculated with different measurements developed in the OWA literature such as the degree of orness, the entropy of dispersion [7], the divergence of W [40] and the balance operator [41]. The degree of orness can be defined in the following way:

$$\alpha(W) = \sum_{j=1}^{n} w_j^* \left(\frac{n-j}{n-1}\right), \tag{12}$$

where w_j^* is the $w_j$weight with the jth largest u_i in a IOWA pair.

For the entropy of dispersion, we get:

$$H(W) = -\sum_{j=1}^{n} w_j \ln\left(w_j\right). \tag{13}$$

For the balance operator, we have:

$$BAL(W) = \sum_{j-1}^{n} \left(\frac{n+1-2j}{n-1}\right) w_j. \tag{14}$$

For the divergence of W, we get:

$$DIV(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} - \alpha(W)\right)^2. \tag{15}$$

We can use this measurement to combine in calculation of alpha and beta linear regression and obtain numerous additional combinations.

4. Induced generalized aggregation operators in linear regression

Merigó and Gil-Lafuente [15] proposed a generalization of the IOWA operator by using generalized

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means and order inducing variables called the induced generalized ordered weighted averaging (IGOWA) operator, the objective is getting a more general formulation of the reordering process that it is able to consider more complex situations. This operator is defined as follows:

Definition 7. Is an IGOWA linear regression of dimension n if a model $IGOWA : Rn \times Rn \rightarrow R$ such that it has associated weights vector $W = [w_1, w_2, .., w_n]^T$ thus $w_i = \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, So we have:

$$y_{IGOWA} = \alpha_{IGOWA} + \beta_{IGOWA} x_j. \quad (16)$$

The parameters can be defined as follows:

$$\beta_{IGOWA} = \frac{Cov_{IGOWA}(x, y)}{var_{IGOWA}(x)} = \frac{\sum_{k=1}^{k} w_j (x_j - \mu)(y_j - \upsilon)}{\sum_{k=1}^{k} w_j (x_j - \mu)^2} \quad (17)$$

$$\alpha_{IGOWA} = \upsilon - \beta_{IGOWA} \mu, \quad (18)$$

where $\lambda \in (-\infty. + \infty)$-0, b_j is a parameter of the IOWA pair (u_i, x_i) with the jth largest u_i, u_i is the order-inducing variable, μ y ν are the means (OWA operator x,y), and the arguments $(a_i - \mu)^2$ and $(x_j - \mu)$ $(y_j - \upsilon)$ has some associated weights (WA) w_j with $\sum_{i=1}^{n} w_j = 1$ and $w_j \in [0, 1]$.

Some special cases in IGOWA operators are presented when λ takes different values; when $\lambda = 1$, the IGOWA operators becomes the IOWA operator, then $IGOWA(\langle u_1, a_1 \rangle, \langle u_2 a_2 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j$.

- When $\lambda = 0$, the IGOWA operator is the IOWG operator, this is, $IGOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \ldots, \langle u_n, u_a \rangle) = \prod_{j=1}^{n} b_j^{w_j}$.
- If $\lambda = -1$ the aggregation is the IOWHA operator, we have $IGOWA(\langle u_1, u_2 \rangle, \langle u_2 a_2 \rangle, \ldots, \langle u_n, a_2 \rangle) = \frac{1}{\sum_{j=1}^{n} \frac{w_j}{b_j}}$.
- When $\lambda = 2$, we form the IOWQA operator, then, $IGOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \ldots, \langle u_n, a_n \rangle) = \left(\sum_{j=1}^{n} w_j b_j^2\right)^{1/2}$.

An interesting case is the generalization of the IOWA operator by using quasi-arithmetic means to obtain the quasi-IOWA, in this we replace b^{λ} with a general continuous function $g(b)$, then, $QIGOWA(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) = g^{-1}\left(\sum_{j=1}^{n} w_j g(b_j)\right)$. We form Quasi-LR-IOWA. These cases are more explained in Table 4.

Table 4
Analysis of λ in LR-IGOWA

λ	LR-IGOWA	formula
$\lambda = 1$	LR-IOWA	$y_{Iowa} = \alpha_{IOWA} + \beta_{IOWA} x_j$
$\lambda = 2$	LR- IOWQA	$y_{Iowa} = \alpha_{\mathrm{IOWQA}} + \beta_{\mathrm{IOWQA}} x_j$
$\lambda = 0$	LR- IOWG	$y_{Iowa} = \alpha_{\mathrm{IOWG}} + \beta_{\mathrm{IOWG}} x_j$
$\lambda = -1$	LR- IOWHA	$y_{Iowa} = \alpha_{\mathrm{IOWHA}} + \beta_{\mathrm{IOWHA}} x_j$

We can get twenty-one different cases as follow and combine each with the five special cases of the IGOWA operator. This application shares the properties of OWA operators; Monotonicity, commutativity and idempotency.

5. Forecasting the USD/MXN exchange rate using linear regression with OWA operators

5.1. The exchange rate forecasting

Exchange rate is among the most important price in open economies, due to their relationship with companies, investors and policy makers [42, 43]. The movement of exchange rate is affected by many factors (economic, political and psychological), which are impressed and ambiguous [44, 45].

The difficulty of explaining and forecasting nominal exchange rates was systematically reported by Meese and Rogoff [46], where the behavior has been used as a random walk. However, the evolution of the series of exchange rates has encouraged nonlinear models as a better option for the forecast [47, 48].

The forecast models of the exchange rate are subject to the instability of the parameters that derive from the constant changes in the economies [29, 49]. These changes reflect the set of decisions of the agents in the financial markets represented by speculators and investors [50].

The fuzzy regression analysis should be applied to many real-life problems in which the strict assumptions of classical (statistical) regression analysis cannot be met. Moreover, it can be applied in forecasting with an uncertain environment [51, 52].

We propose a tool where we take the virtues of the linear regression forecast and add the family of ordered weight average (OWA) operators. The objective is to estimate linear regression parameters which can be adapted to optimism or future pessimism scenarios.

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In this research, we use the simple linear regression to forecast exchange rate USD/MXN. The information used to generate the forecast is from the 2017 fix exchange rate. This is:

$$y_{fer} = \alpha + \beta x_{fer-1}, \tag{19}$$

where y_{fer} is future fix exchange rate and x_{fer-1} is spot exchange rate with one lag. This model is applied for each of the operators previously seen as follows:

1. Linear regression with OWA operator

$$y_{fer} = \alpha_{OWA} + \beta_{OWA} x_{fer-1}, \tag{20}$$

where α and β are calculated with OWA averages

2. Linear regression with IOWA operator

$$y_{fer} = \alpha_{IOWA} + \beta_{IOWA} x_{fer-1}, \tag{21}$$

where α and β are calculated with IOWA averages

3. Linear regression with IGOWA operator

$$y_{fer} = \alpha_{IGOWA} + \beta_{IGOWA} x_{fer-1}, \tag{22}$$

where α and β are calculated with IGOWA averages

5.2. Calculation of the LR-IOWA parameters

For the calculation of OWA parameters in the linear regression we perform the following steps:

Step 1: Define the number of elements in the matrix B and its components. This is done for variable x (B_x) and variable y (B_y). In this estimation, we use a sequence $n = 10$ because the decision maker believes that this are the period that influences the determination of the predicted variable. We select two different vectors, one belongs to the independent variable x, which is the exchange rate with a delay period, and another vector of the dependent variable as the current exchange rate.

Step 2: Continue with the assignment of weights constructing the matrix W for the variables x and y, which can be the same or different according to the criteria considered. It should be noted that the larger weights are assigned to the closest months to the similar months of the previous year. In the case of the induced OWA we change the highest prices with the highest weights to denote a maximum, we use the same weight vector for both variables. You can see the vector values in Table 5.

We calculate the OWA means (OWA, IOWA and IGOWA) resulting in the following:

Table 5
Vector value for OWA calculation

y_{fer}		x_{fer-1}	
Price	W1	Price-1	W2
19.18	0.01	18.91	0.01
20.29	0.08	20.29	0.08
19.30	0.05	19.30	0.05
21.39	0.25	20.52	0.25
18.92	0.2	21.38	0.2
18.82	0.15	18.81	0.15
18.79	0.03	18.78	0.03
18.76	0.03	18.75	0.03
18.13	0.05	18.13	0.05
17.84	0.05	17.83	0.05
17.83	0.05	17.82	0.05
17.81	0.05	17.80	0.05

- OWA

$\mu_{OWA} = 19.24$

$\upsilon_{OWA} = 18.90$

- IOWA

$\mu_{IOWA} = 19.75$

$\upsilon_{IOWA} = 18.85$

- IGOWA a) $\lambda = -1$

$\mu_{a)IGOWA} = 19.68$

$\upsilon_{a)IGOWA} = 19.19$

- IGOWA b) $\lambda = 2$

$\mu_{b)IGOWA} = 19.78$

$\upsilon_{b)IGOWA} = 19.26$

- IGOWA c) $\lambda = 0$

$\mu_{c)IGOWA} = 19.71$

$\upsilon_{c)IGOWA} = 19.21$

Step 3: We calculate the beta parameter of the linear regression using one of the equations mentioned in the Table 1 where in one or more components the traditional means are replaced by OWA means. In this case we use equation 3.7 (see Table 3) of them, where OWA means are used in all sections. We have:

$$\beta_{OWA} = 0.83$$

$$\beta_{a)IGOWA} = 0.94$$

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Table 6
Comparative results of linear regressions

	Simple linear regression	Regression OWA	Regression IOWA	Regression IGOWA $\lambda = -1$	Regression IGOWA $\lambda = 2$	Regression IGOWA $\lambda = 0$
FER1	19.03	18.8569	18.13	19.57	19.69	19.68
FER2	19.92	18.5868	19.08	20.13	20.21	20.62
FER3	19.83	18.3619	19.04	20.57	20.71	21.06

$$\beta_{IOWA} = 0.91$$

$$\beta_{c)IGOWA} = 0.95$$

$$\beta_{b)IGOWA} = 0.98$$

Step 4: We continue with the beta parameter calculation using one of the equations mentioned in the Table 2. The case selected when in one or more components the traditional means are replaced by OWA means. We use the form 3.7 in the Table 3, where OWA are used in all sections. The results are:

$$\alpha_{OWA} = 2.88$$

$$\alpha_{a)IGOWA} = 1.52$$

$$\alpha_{b)IGOWA} = 0.76$$

$$\alpha_{c)IGOWA} = 1.27$$

Step 5: we construct the linear regression model with the corresponding betas and alphas, then:

$$y_{OWA} = 2.88 + 0.83x_{fer-1}$$

$$y_{IOWA} = 1.57 + 0.91x_{fer-1}$$

$$y_{a)IGOWA} = 1.52 + 0.94x_{fer-1}$$

$$y_{b)IOWA} = 0.76 + 0.98x_{fer-1}$$

$$y_{b)IOWA} = 1.28 + 0.95x_{fer-1}$$

We substitute the values of x in the equations to obtain y, the Table 6 shows the forecasts corresponding to the three subsequent periods.

We can see in the table that the predicted values for each one of the regressions oscillate between 18 and 19 pesos, a figure in which the price of the dollar in Mexico was in those periods. it can be noted that the predicted prices with generalized induced OWA corresponds to a much more pessimistic scenario as these moves away from the others especially whenλ is 0.

6. Conclusions

Linear regression is a tool used to estimate and predict data in a simple and parsimonious way. However, in reality the financial series are characterized by atypical movements and constant changes caused by uncertainty. For this causes the simple methods for forecasting tend to commit mistakes. OWA aggregation operators are an alternative that propose the weighting of various attributes in such a way that we can overestimate or underestimate them according to the attitudinal character of the decision maker. The induced OWA operator works with more complex situations in which one variable can be affected by another. This helps us to consider complicated scenarios represented with complex attitudinal characters as the financial scenarios.

In this paper, we have presented linear regression using OWA means called the LR-IOWA and the LR-IGOWA operators. This tool calculates the parameters of the linear regression using the least squares method, adding OWA components in its calculation with which it is possible to add more complex situations to the estimation of the linear regression. The main characteristic of LR-OWA is that its parameters can be estimated according to the optimism or pessimism scenarios that are considered by the decision maker.

In order to test the new approach, the work develops a simple illustrative example regarding exchange rate forecasting between USD/MXN. The analysis uses linear regression methods that incorporates in the aggregation data from the OWA and the IOWA operator. The objective is to analyze how the use of these operators can under- or over- estimate the classical results according to the attitudinal character of the decision maker. Due to uncertainty, this approach is very interesting because often the information may have distortions or simply the decision maker wants to consider an optimistic or pessimistic scenario of the results.

To continue developing this idea, in future research we propose its application in other extensions of OWA operators [42, 53] such as the induced or- dered

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weighted averaging weighted average (IOWAWA), the induced probabilistic ordered weighted average (IPOWA) [54], the induced ordered weighted moving average (IOWMA) [55] and OWA-expertons [56]. In addition, we can use different variants of linear regression, such as multiple linear regression and stepwise regression.

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Apéndice 3.

A bibliometric overview of volatility. JIFS.

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A bibliometric overview of volatility

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Abstract. Price volatility is a matter of importance for making decisions in the finance world. The growing studies regarding volatility have focused on minimizing the risks through modeling, estimating and forecasting. This paper presents a bibliometric overview of the most important authors, institutions and countries that work on the topic. Additionally, a historical analysis of how the agents have interrelated is presented. For the purposes of the analysis and the design of tables and graphics, tools from the Web of Science Core Collection and the VOSviewer software were used. The results show the importance of volatility in the study of business economics and decision making.

Keywords: Volatility, bibliometrics, Web of Science, VOSviewer

1. Introduction

In the last decades, a growing literature has emerged regarding modeling and forecasting dynamics in the volatility of financial markets. The global financial crises have generated a significant instability in the prices of financial assets [1, 2]. Measuring volatility forecasting and making decisions to hedge are important tasks for the policymakers, researchers and financial practitioners [3–5].

Volatility can be caused by many interconnected variables. Political-economic problems such as a financial crisis, major changes in the markets, generation of speculative bubbles, or switches in the monetary and debt management policies can cause instability [6, 7]. Additionally, domestic volatility can be caused by volatilities in different markets around the world [5, 8].

However, since volatility is not directly observable, it is usually estimated through parametric models [9, 10]. Practitioners and researchers need to build a model with the capability to reproduce stylized facts based on observed time series that exhibit desirable statistical properties. In this sense, literature offers a set of models that adapt to one or more stylized facts. The existing parametric models for estimating the volatility include the popular ARCH–GARCH family [11, 12], the stochastic volatility models [13, 14] to a lesser extent, fuzzy models [15, 16], and artificial neural network models [17, 18].

Many reviews have been presented trying to compare different ways of volatility modeling. Poon and Granger [19] compare the volatility forecasting of existing models. Asai et al. [20] review specification, estimation, and evaluation of multivariate stochastic volatility models. Ait-Sahalia et al. [21] study the implied volatility in the model for forecasting. McAleer and Medeiros [22] review the volatility measure in high-frequency data. However, there is not a consensus on the best measures and volatility forecasting, due to the different instruments, horizons and scenarios.

This article proposes a bibliometric review of price volatility models in business economics and decision

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making. The main advantage of the paper is to offer a deep analysis of volatility studies and their interrelation. As part of this research, all the documents regarding volatility published between 1975 and 2018 were analyzed using the Web of Science Core Collection.

The results reveal a considerable increase of volatility studies in the last decades, resulting in more than twenty thousand papers in the area of business economics and decision making. The analysis reveals four facts: 1) The most important paper is the "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation" proposed by Engle. 2) The most representative authors are McAleer M, Gupta R. and Bollerslev T. 3) The main institutions are the New York University, Columbia University and Duke University; and 4) The countries with the most publications regarding volatility are the USA, United Kingdom and China. Additionally, the most influential financial institutions on the topic are analyzed.

However, some fundamental investigations could be omitted if they are not found in the database or if another nomenclature has been considered to describe the phenomenon.

This work is organized as follows: 2) A description of the bibliometric methods used in the analysis; 3) An analysis of the publications structure, showing tables with the rankings of the best papers, authors, countries and institutions that study volatility; 4) A graphical analysis of the bibliographic data, and 5) A summary and conclusions.

2. Bibliometric methods

Bibliometric methods offer a convenient and non-reactive tool for studying collaboration in research. Pritchard [23] defines bibliometric analysis as the application of mathematical and communication methods to books and another media communication. A bibliometric study can produce many indicators such as productivity of authors, countries and universities. Additionally, this work includes information about co-authorship with journals, countries and universities. In the literature, many bibliometric studies have been analyzed in the areas of finance [24, 25], management [26, 27] and economics [28, 29].

To measure productivity, bibliometrics usually use the number of publications and their influence measured by the number of citations [30]. A very popular indicator to measure productivity is the h -index which combines publications and citations in the same measurement [31]. Another alternative is to consider the ratio citation/papers or the impact factor by obtaining the mean of citation per paper. For the purpose of this work, we analyze both, the h -index and the impact factor.

The analysis considers other bibliometric indicators such as citation structure, which includes the number of papers that have surpassed a citation threshold, cites per paper, cites per year, citing articles, and their evolution through time. For the country analysis, a ratio of paper/million inhabitants is used. To analyze universities' productivity, the Academic Ranking of World Universities (ARWU) and the Quacquarelli and Symonds (QS) University Ranking are being considered.

To schematize the bibliometric analysis, we used the VOSviewer software [32]. The VOSviewer presents the information graphically, in terms of bibliographic coupling, citation, co-citation, co-authorship and co-occurrence of author keywords.

The study used the Web of Science Collection database that is currently owned by Clarivate Analytics with the indexes SCI-EXPANDED, SSCI, A&HCI and ESCI. The search topic keywords were "conditional heteroskedasticit* OR return volatility* OR conditional heteroscedasticit* OR stochastic volatility* OR volatility*", which for purposes of this work will be considered as "volatility". The search took place in May 2019. Note that the analysis only considered articles, reviews, letters and notes.

3. Results

3.1. Publication and citation structure

The number of publications regarding volatility from 1975 to 2018 were 22,004. Figure 1 shows the annual number of documents that have been

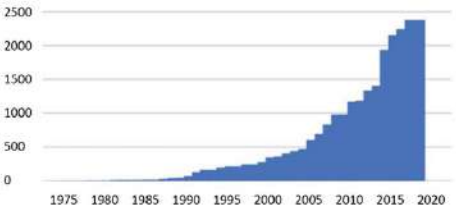

Fig. 1. Volatility papers published annually.

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Table 1
The 40 most cited papers regarding volatility between 1975–2018

R	TC	Title	Author/s	Year	C/Y
1	7332	Autoregressive conditional heteroscedasticity with estimates of the variance of United-Kingdom inflation	Engle, RF	1982	192.9
2	6954	Generalized autoregressive conditional heteroskedasticity	Bollerslev, T	1986	204.5
3	3167	Conditional heteroskedasticity in asset returns – a new approach	Nelson, DB	1991	109.2
4	2737	A closed-form solution for options with stochastic volatility with applications to bond and currency options	Heston, SL	1993	101.3
5	2548	On the relation between the expected value and the volatility of the nominal excess return on stocks	Glosten, LR; Jagannathan, R; Runkle, DE	1993	94.3
6	1962	Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models	Engle, R	2002	109
7	1542	ARCH modeling in finance: – a review of the theory and empirical-evidence	Bollerlev, T; Chou, RY; Kroner, KF	1992	55.0
8	1475	Investor psychology and security market under- and overreactions	Daniel, K; Hirshleifer, D; Subrahmanyam, A	1998	67.0
9	1316	No contagion, only interdependence: Measuring stock market comovements	Forbes, KJ; Rigobon, R	2002	73.1
10	1316	Stock markets, banks, and economic growth	Levine, R; Zervos, S	1998	59.8
11	1260	Expected stock returns and volatility	French, KR; Schwert, GW; Stambaugh, RF	1987	38.1
12	1249	By force of habit: A consumption-based explanation of aggregate stock market behavior	Campbell, JY; Cochrane, JH	1999	59.4
13	1230	Measuring and testing the impact of news on volatility	Engle, RF; Ng, VK	1993	45.5
14	1213	The quality of accruals and earnings: The role of accrual estimation errors	Dechow, PM; Dichev, ID	2002	67.3
15	1190	Stock market Prices do not follow random walks: Evidence from a simple specification test	Lo, Andrew W.; MacKinlay, A. Craig	1988	37.1
16	1174	The dividend price ratio and expectations of future dividends and discount factors	Campbell, John Y.; Shiller, Robert J.	1988	36.6
17	1072	Investor sentiment and the cross-section of stock returns	Baker, Malcolm; Wurgler, Jeffrey	2006	76.5
18	1069	The pricing of options on assets with stochastic volatilities	Hull, J; White, A	1987	32.3
19	1049	Modeling and forecasting realized volatility	Andersen, TG; Bollerslev, T; Diebold, FX; Labys, P	2003	61.7
20	1022	Market liquidity and funding liquidity	Brunnermeier, Markus K.; Pedersen, Lasse Heje	2009	92.9
21	996	Why does stock-market volatility change over time	Schwert, GW	1989	32.1
22	961	Answering the skeptics: Yes, standard volatility models do provide accurate forecasts	Andersen, TG; Bollerslev, T	1998	43.6
23	923	The cross-section of volatility and expected returns	Ang, A; Hodrick, RJ; Xing, YH; Zhang, XY	2006	65.9
24	907	Corporate disclosure policy and analyst behavior	Lang, MH; Lundholm, RJ	1996	37.7
25	893	Empirical performance of alternative option pricing models	Bakshi, G; Cao, C; Chen, ZW	1997	38.8
26	870	Market-based assets and shareholder value: A framework for analysis	Srivastava, RK; Shervani, TA; Fahey, L	1998	39.5
27	869	Extreme correlation of international equity markets	Longin, F; Solnik, B	2001	45.7
28	810	Risks for the long run: A potential resolution of asset pricing puzzles	Bansal, R; Yaron, A	2004	50.6
29	803	Fractionally integrated generalized autoregressive conditional heteroskedasticity	Baillie, RT; Bollerslev, T; Mikkelsen, HO	1996	33.4
30	798	The impact of uncertainty shocks	Bloom, Nicholas	2009	72.5
31	792	The territorial trap: the geographical assumptions of international relations theory	Agnew, John	1994	30.4
32	759	Giving content to investor sentiment: The role of media in the stock market	Tetlock, Paul C.	2007	58.3
33	756	Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk	Campbell, JY; Lettau, M; Malkiel, BG; Xu, YX	2001	39.7
34	756	Corporate-debt value, bond covenants, and optimal capital structure	Leland, HE	1994	29.1
35	750	Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options	Bates, DS	1996	31.2

(*Continued*)

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Table 1
(*Continued*)

R	TC	Title	Author/s	Year	C/Y
36	748	The distribution of realized stock return volatility	Andersen, TG; Bollerslev, T; Diebold, FX; Ebens, H	2001	39.3
37	744	The stochastic behavior of commodity prices: Implications for valuation and hedging	Schwartz, ES	1997	32.3
38	744	Accounting earnings and cash flows as measures of firm performance – the role of accounting accruals	Dechow, PM	1994	28.6
39	726	Modelling asymmetric exchange rate dependence	Patton, AJ	2006	51.8
40	713	Stochastic volatility: Likelihood inference and comparison with ARCH models	Kim, S; Shephard, N; Chib, S	1998	32.4

Abbreviations: R = Rank; TC = Total citations; C/Y = Citations per year.

Table 2
The most cited publications regarding volatility

R	Year	Cited reference (only first author)	Type	citation	TLS
1	1986	bollerslev t, j econometrics, v31, p307	A	2586	2571
2	1982	engle rf, econometrica, v50, p987	A	2399	2386
3	1991	nelson db, econometrica, v59, p347	A	1674	1669
4	1993	glosten lr, j financ, v48, p1779	A	1471	1464
5	1973	black f, j polit econ, v81, p637	A	1432	1396
6	1993	heston sl, rev financ stud, v6, p327	A	1255	1235
7	1987	newey wk, econometrica, v55, p703	A	1103	1091
8	2002	engle r, j bus econ stat, v20, p339	A	913	906
9	1993	fama ef, j financ econ, v33, p3	A	876	871
10	1987	french kr, j financ econ, v19, p3	A	796	796
11	1993	engle rf, j financ, v48, p1749	A	722	722
12	2003	andersen tg, econometrica, v71, p579	A	700	700
13	1995	engle rf, economet theor, v11, p122	A	695	690
14	1992	bollerslev t, j econometrics, v52, p5	A	684	682
15	1992	bollerslev t., 1992, economet rev, v11, p143	A	668	666
16	1985	cox jc, econometrica, v53, p385	A	643	635
17	1989	schwert gw, j financ, v44, p1115	A	623	621
18	1989	hamilton jd, econometrica, v57, p357	A	615	606
19	1998	andersen tg, int econ rev, v39, p885,	A	611	611
20	1990	bollerslev t, rev econ stat, v72, p498	A	588	587
21	1985	kyle as, econometrica, v53, p1315	A	587	574
22	1994	hamilton j. d., time series anal	B	546	530
23	1997	bakshi g, j financ, v52, p2003	A	540	537
24	1973	merton rc, bell j econ, v4, p141	A	540	533
25	1993	ding z., j empir financ, v1, p83	A	537	537
26	1976	black f., p 1976 m am stat ass, p177	B	532	531
27	1987	engle rf, econometrica, v55, p251	A	531	517
28	1979	dickey da, j am stat assoc, v74, p427	A	523	518
29	1995	diebold fx, j bus econ stat, v13, p253	A	514	510
30	2006	ang a, j financ, v61, p259	A	508	508
31	2000	duffie d, econometrica, v68, p1343	A	508	507
32	1976	merton rc, j financ econ, v3, p125,	A	505	498
33	1973	fama ef, j polit econ, v81, p607	A	501	498
34	2001	andersen tg, j am stat assoc, v96, p42	A	497	497
35	1988	phillips pcb, biometrika, v75, p335	A	489	488
36	2001	andersen tg, j financ econ, v61, p43	A	480	480
37	2002	forbes kj, j financ, v57, p2223	A	478	474
38	1982	christie aa, j financ econ, v10, p407	A	476	476
39	1988	bollerslev t, j polit econ, v96, p116	A	467	467
40	1987	hull j, j financ, v42, p281	A	467	464

Abbreviations: A = Article B = Book; TLS = Total Link Strength.

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published and it is observable that the number has considerably grown, especially in the last two decades.

In the first 17 years, publications were less than 100 per year. Then, an increase was observed in the decade of the nineties. The growth was accentuated in the first decade of the millennium. Since 2011 publications exceeded 1,000 per year. In 2016 they went up to 2,000. Nowadays, the growth is considerably significant.

Table 1 presents the forty most cited papers from 1975 to 2018. Note that the first places on the table correspond to works that study volatility modeling. The most cited paper was written by Robert Engle, winner of the Nobel Prize in economics in 2003 for his methods of analyzing economic time series with time-varying volatility, ARCH. The second most cited paper belongs to Tim Bollerslev who generalized the ARCH methods of Engle and proposed the GARCH model. In the third position, we find the work of Nelson that provides a new approach based on ARCH-GARCH models. On the other hand, the authors with the most cited papers are Bollerslev with 5 papers, Campbell with 4, Engle with 3 and Andersen with 3.

In order to analyze the most cited documents and their relation with the topic of volatility we used the VOSviewer software and obtained the co-citations presented in Table 2. The most cited document was written by Bollerslev in [12], followed by the contribution of Engle. Note that Bollerslev and Engle lead this list with five publications in the table.

With the objective to analyze the citations structure, this work established thresholds and presented the information from 1991 to 2018 as seen on Table 3. It is observable that the majority of the most cited papers were published between 1990 and 2009. Only 1.5% of the publications have more than two hundred citations and 34% more than ten citations. While 80% of the papers about volatility have been cited at least once.

3.2. Leading authors, institutions and countries

In this section, authors, institutions and universities that have publications regarding volatility are

Table 3
Papers citation structure per year (1991–2018)

Year	TP	TC	≥200	≥100	≥50	≥20	≥10	≥5	≥1
1991	73	7089	6	14	23	36	42	54	67
1992	132	8894	10	21	36	63	86	100	122
1993	161	16243	17	33	60	89	108	125	149
1994	150	11236	15	26	46	76	94	103	139
1995	194	9007	11	24	43	83	109	140	178
1996	217	15231	23	40	68	104	137	159	201
1997	201	9568	10	27	49	82	115	148	192
1998	243	17043	18	37	63	106	149	179	224
1999	221	11344	13	29	58	108	149	174	203
2000	278	17533	21	48	83	150	187	221	259
2001	350	1934	24	52	101	175	246	285	335
2002	365	23481	22	55	99	192	245	305	349
2003	403	21503	27	56	104	192	259	330	288
2004	439	19125	17	47	102	213	294	372	423
2005	472	18411	15	44	92	209	300	374	456
2006	606	23269	14	48	119	264	381	467	576
2007	688	21049	9	52	121	266	393	511	646
2008	832	22146	12	43	121	300	471	621	783
2009	974	25734	17	39	130	324	522	692	922
2010	973	19111	4	30	93	277	465	682	903
2011	1171	21795	7	33	100	317	527	755	1074
2012	1187	18785	6	20	82	266	489	760	1091
2013	1332	16147	0	15	55	231	494	792	1212
2014	1407	14844	3	8	39	203	458	795	1261
2015	1932	11305	0	2	22	104	300	696	1588
2016	2151	9442	1	3	10	66	228	592	1618
2017	2245	5424	0	0	2	24	110	339	1416
2018	2376	1858	0	0	0	3	18	79	835

Abbreviations: TP and TC = Total papers and citations; ≥200, ≥100, ≥50, ≥20, ≥10, ≥5, ≥1 = number of papers with equal or more than 200, 100, 50, 20, 10, 5 and 1 citations.

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Table 4
The most productive authors regarding volatility

R	Author name	University	Country	TP	TC	H	C/P	≥200	≥100	≥50
1	McAleer, Michael	Univ Western Australia	Australia	78	2135	23	27.37	1	4	12
2	Gupta, Rangan	Univ Pretoria	South Africa	78	503	12	6.44	0	0	0
3	Bollerslev, Tim	Duke Univ	USA	57	17305	39	303.59	13	23	35
4	Hammoudeh, Shawkat	Drexel Univ	USA	54	1041		19.27	0	2	5
5	Balcilar, Mehmet	Eastern Mediterranean Univ	Turkey	41	324	9	7.90	0	0	0
6	Bali, Turan G.	Georgetown Univ	USA	40	1591	22	39.77	1	3	9
7	Ghysels, Eric	Univ N Carolina	USA	38	2431	21	63.97	3	9	14
8	Bahmani-Oskooee, Mohsen	Univ Wisconsin	USA	37	195	7	5.27	0	0	0
9	Engle, Robert F.	NYU	USA	37	13350	29	360.81	10	16	25
10	Carr, Peter	NYU	USA	36	2178	22	60.5	3	8	12
11	Fabozzi, Frank J	EDHEC Business Sch	France	35	247	8	7.05	0	0	0
12	Duc Khuong Nguyen	IPAG Business Sch	France	35	1002	17	28.62	0	1	8
13	Andersen, Torben G.	Northwestern Univ	USA	34	6608	24	194.35	10	16	20
14	Narayan, Paresh Kumar	Deakin Univ, Deakin Business Sch	Australia	33	784	15	23.75	0	2	6
15	Tauchen, George	Duke Univ	USA	33	2568	21	77.81	4	6	13
16	Caporale, Guglielmo Maria	Brunel Univ London	England	31	327	10	10.54	0	0	1
17	Choudhry, Taufiq	Univ Southampton	England	31	266	9	8.58	0	0	0
18	Wohar, Mark	Univ Nebraska	USA	31	269	7	8.67	0	0	1
19	Bekaert, Geert	Columbia Univ	USA	30	4467	24	148.9	6	14	19
20	Ryu, Doojin	Sungkyunkwan Univ	South Korea	30	434	13	14.46	0	0	0
21	Serletis, Apostolos	Univ Calgary	Canada	30	532	12	17.73	0	1	2
22	Chevallier, Julien	IPAG Business Sch	France	29	351	9	12.10	0	1	1
23	Madan, Dilip B.	Univ Maryland	USA	29	829	10	28.58	1	2	5
24	Christoffersen, Peter	Univ Toronto	Canada	28	1224	16	43.71	0	3	11
25	Jacobs, Kris	Univ Houston	USA	28	1299		46.39	0	3	11
26	Kutan, Ali M.	Southern Illinois Univ	USA	28	255	9	9.10	0	0	1
27	McMillan, David G.	Univ Stirling	Scotland	28	100	5	3.57	0	0	0
28	Wong, Hoi Ying	Chinese Univ Hong Kong	China	28	233	10	8.32	0	0	0
29	Wu, Chongfeng	Shanghai Jiao Tong Univ	China	28	420	11	15	0	1	2
30	Brooks, Chris	Univ Reading	England	27	769	15	28.48	0	0	6
31	Herwartz, Helmut	Univ Kiel	Germany	27	189	7	7	0	0	1
32	Lee, Bong Soo	Henry W Bloch Sch Management	USA	27	473	11	17.51	0	1	3
33	Shephard, Neil	Harvard Univ	USA	27	3530	21	130.74	6	12	15
34	Tse, Yiuman	Univ Missouri	USA	27	482	13	17.85	0	0	3
35	Lien, Donald	Univ Texas San Antonio	USA	26	369	10	14.19	0	0	1
36	Aizenman, Joshua	Univ Southern Calif	USA	25	594	13	23.76	0	1	4
37	Caporin, Massimiliano	Univ Padua	Italy	25	189	6	7.56	0	0	1
38	Laurent, Sebastien	Aix Marseille Univ	France	25	1474	15	58.96	1	4	7
39	Pierdzioch, Christian	Helmut Schmidt Univ	Germany	25	146	6	5.84	0	0	0
40	Todorov, Viktor	Northwestern Univ	USA	25	651	14	26.04	0	1	3

Abbreviations available in Tables 1 and 3 except: C/P=Cites per paper; H = h-index.

analyzed. Table 4 shows the most productive authors. Michael McAleer from University of Western Australia is the most productive author with seventy-eight publications and he has been cited in two thousand one hundred thirty-five papers. In second place is Rangan Gupta with equal publications but fewer citations.

The list shows some authors with high h-index as Bollerslev, Engle, Andersen, Bekaert and Bali. It is important to note that almost half of the authors listed on the table are from the USA.

Table 5 analyzes the performance of the most productive authors through time; the information is presented in four periods of eleven years each. The productivity during the first period was low with an average of three publications. Engle and Schiller were the most cited authors during this period. Between 1986–1996 the two most productive authors were Engle and Nelson. In the third period, Bollerslev ascended to the first position with twenty-three publications. In the last eleven years, Gupta has been the most productive author with seventy-seven papers on volatility.

In order to list the most productive and influential universities, we considered the last institution where the most productive authors have published their works.

In Table 6, USA universities appear with more frequency in the first three periods. However, from 2008 to 2018 European Universities became more relevant.

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Table 5
The most productive authors and their performance by period

	1975–1985			1986–1996		
R	Author	TP	TC	Author	TP	TC
1	Kawai, M	3	42	Engle, RF	13	2894
2	Stock, D	3	6	Nelson, DB	12	3805
3	Anderson, RW	2	190	Bollerslev, T	11	10448
4	Hegde, SP	2	3	Baillie, RT	9	2533
5	Livingston, M	2	5	Brorsen, BW	9	182
6	Park, HY	2	16	Booth, GG	7	459
7	Schrems, EL	2	5	Harvey, CR	7	924
8	Schwartz, RA	2	15	Whaley, RE	7	746
9	Sears, RS	2	16	Chan, K	6	400
10	Shiller, RJ	2	413	Chan, KC	6	1094
11	Snyder, NH	2	44	Karolyi, GA	6	1395
12	Trennepohl, GL	2	13	Lastrapes, WD	6	495
13	Whitcomb, DK	2	15	Lee, JH	6	575
14	Engle, RF	1	7333	Seguin, PJ	6	603
15	Garman, MB	1	415	Tauchen, G	6	1174
16	Klass, MJ	1	415	Tucker, AL	6	92
17	Flavin, MA	1	107	Gallant, AR	5	1132
18	Pyburn, Philip J.	1	103	NG, VK	5	1429
19	Schmalensee, R	1	94	Kroner, KF	5	2124
20	Trippi, RR	1	94	Schwert, GW	5	3061

	1997–2007			2008–2018		
R	Author	TP	TC	Author	TP	TC
1	Bollerslev, T	23	5676	Gupta, Rangan	77	496
2	Bali, TG	18	550	McAleer, Michael	63	1130
3	Brooks, C	18	581	Hammoudeh, Shawkat	54	1042
4	Andersen, TG	17	5678	Balcilar, Mehmet	39	306
5	Ghysels, E	17	1710	Bahmani-Oskooee, Mohsen	36	173
6	McAleer, M	15	1005	Duc,Khuong Nguyen	35	1003
7	Subrahmanyam, A	14	3688	Fabozzi, Frank J.	33	244
8	Bekaert, G	13	3224	Narayan,Paresh Kumar	32	681
9	Diebold, FX	13	4517	Ryu, Doojin	30	434
10	Carr, P	12	1431	Chevallier, Julien	29	352
11	Devereux, MB	12	653	Wohar, Mark	29	219
12	Hong, YM	12	683	Wu, Chongfeng	27	417
13	Kanas, A	12	145	Wang, Yaw-Huei	26	376
14	Aizenman, J	11	333	Wong, Hoi Ying	26	190
15	Henry, OT	11	332	Caporale, Guglielmo Maria	24	244
16	Shephard, N	11	1933	Caporin, Massimiliano	24	183
17	Timmermann, A	11	778	Carr, Peter	24	747
18	Brandt, MW	10	979	Christoffersen, Peter	24	963
19	Laurent, S	10	1153	Jacobs, Kris	24	1038
20	Bakshi, G	10	1996	Todorov, Viktor	24	624

Abbreviations are available in the previous tables.

Since volatility is an important financial topic, this study also analyzes the most productive institutions that are not universities.

Table 7 shows the National Bureau of Economic Research leading the list with 574 publications. Note that the institutions publishing about volatility are the policy makers of influential countries.

Table 8 shows the 40 countries with most publications regarding the topic of volatility. The USA is the most productive country followed by the United Kingdom and China. If we consider the number of papers and cites per million inhabitants, the most productive countries are Australia, Switzerland, Singapore, Denmark, New Zealand, Cyprus and Luxembourg with more than 50 publications per millioninhabitants. Switzerland, Singapore and Denmark lead in the number of citations with more than 1000 cites per million inhabitants.

4. Graphical analysis of the information with VOSviewer

This section presents the graphical maps that resulted from the bibliometric networks analysis

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Table 6
The most productive universities by period

	1975–1985			1986–1996		
R	University	TP	TC	University	TP	TC
1	University of California Berkeley	4	505	University of Chicago	33	8245
2	Johns Hopkins University	3	42	University of Pennsylvania	33	5485
3	Massachusetts Institute of Technology (MIT)	3	196	Duke University	27	4047
4	New York University	3	85	New York University	26	1700
5	University of Georgia	3	41	University of California San Diego	26	12927
6	University of Michigan	3	47	University of Michigan	23	4420
7	University of Oklahoma - Norman	3	6	Northwestern University	22	4249
8	University of Virginia	3	151	University of Wisconsin Madison	21	1169
9	Arizona State University	2	13	Columbia University	20	3936
10	Baruch College (CUNY)	2	19	Ohio State University	20	2201
11	George Washington University	2	3	Stanford University	20	809
12	Harvard University	2	25	University of California Berkeley	20	1840
13	Rutgers State University New Brunswick	2	15	Louisiana State University	19	830
14	Stanford University	2	53	Harvard University	18	1533
15	University of California San Diego	2	7427	University of Rochester	18	4597
16	University of Illinois Urbana-Champaign	2	16	Massachusetts Institute of Technology (MIT)	17	2680
17	University of Pennsylvania	2	413	Princeton University	17	4253
18	University of Wisconsin Parkside	2	5	University of North Carolina	17	2484
19	Washington University (WUSTL)	2	114	Vanderbilt University	16	1933
20	London School Economics & Political Science	1	7333	Arizona State University	14	967

	1997–2007			2008–2018		
R	University	TP	TC	University	TP	TC
1	University of London	114	5578	New York University	158	6445
2	Duke University	76	13873	Erasmus University Rotterdam	150	2049
3	Columbia University	65	8424	University of Amsterdam	150	2707
4	University of Pennsylvania	61	10472	Monash University	145	1397
5	University of California Los Angeles	58	7372	Columbia University	144	4134
6	University of California Berkeley	54	3606	Aarhus University	142	2490
7	Harvard University	52	8069	University of Oxford	141	3446
8	University of Chicago	52	8620	City University London	137	1794
9	Cornell University	50	3234	London School Economics & Political Science	130	2722
10	Massachusetts Institute of Technology (MIT)	49	7629	University of New South Wales Sydney	124	1240
11	University of Montreal	49	1950	University of Chicago	120	3637
12	Stanford University	48	7930	University of Sydney	118	888
13	Hong Kong University of Science & Technology	45	2683	Duke University	116	3820
14	University of Michigan	45	6262	University of Technology Sydney	116	882
15	University of North Carolina Chapel Hill	45	4142	IPAG Business School	114	1105
16	London School Economics & Political Science	44	3011	National University of Singapore	109	1345
17	University of California San Diego	43	6105	University of Montreal	106	940
18	University of Manchester	42	1700	University of Toronto	104	1559
19	Northwestern University	41	9208	University of Manchester	102	894
20	University of Maryland College Park	41	3442	Xiamen University	99	795

Abbreviation are available in Tables 1 and 3.

Table 7
The most productive institutions

R	Institution	Country	TP	TC	H	C/P	≥250	≥100	≥50
1	National Bureau of Economic Research	USA	574	54282	123	94.56	60	147	236
2	Federal Reserve System	USA	550	22814	73	41.48	15	40	100
3	International Monetary Fund	USA	231	5316	39	23.01	2	11	31
4	Centre National de la Recherche Scientifique (CNRS)	France	225	2746	29	12.20	0	2	11
5	Centre for Economic Policy Research	England	160	5798	39	36.23	4	15	32
6	European Central Bank	UE	124	4266	30	34.40	3	7	20
7	Federal Reserve System Board of Governors	USA	115	2633	27	22.89	0	6	15
8	The World Bank	USA	96	2123	21	22.11	0	6	13

Abbreviations are available in Tables 1 and 3.

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Table 8
The most productive and influential countries regarding volatility

R	Country	TP	TC	H	C/P	≥250	≥100	≥50	P/Po	c/Po
1	USA	8194	315969	236	38.56	214	693	1372	25.076	966.96
2	United Kingdom	3132	65627	153	20.95	26	97	253	47.046	985.78
3	China	1849	19703	58	10.65	4	24	81	1.307	13.92
4	Germany	1442	20795	60	14.42	5	31	88	17.523	252.69
5	Australia	1423	17419	58	12.24	4	22	76	57.443	703.17
6	Canada	1249	23388	70	18.72	8	42	118	33.799	632.90
7	France	1242	18702	60	15.05	6	29	80	19.039	286.69
8	Taiwan	907	7481	39	8.24	0	5	28	38.280	315.73
9	Italy	898	10597	47	11.80	2	10	46	15.146	178.73
10	Spain	798	8984	43	11.25	2	8	37	17.199	193.63
11	Netherlands	673	14399	63	21.39	1	24	81	39.393	842.81
12	South Korea	640	7124	40	11.13	3	10	34	12.509	139.24
13	Switzerland	498	9107	48	18.28	2	19	46	58.286	1065.89
14	Turkey	458	4901	31	10.70	2	10	19	5.591	59.83
15	Japan	448	4289	29	9.57	1	7	18	3.522	33.72
16	Greece	387	3796	31	9.80	0	2	13	34.733	340.69
17	India	369	1478	17	4.00	0	1	3	0.273	1.09
18	Singapore	328	7075	39	21.57	4	17	34	56.631	1221.53
19	Denmark	327	15367	46	46.99	6	20	44	56.827	2670.50
20	Belgium	300	5661	37	18.87	1	9	25	26.090	492.32
21	New Zealand	270	2536	26	9.39	0	2	10	56.847	533.94
22	Sweden	252	4447	32	17.64	3	8	18	25.244	445.47
23	South Africa	227	928	16	4.08	0	0	0	3.955	16.17
24	Brazil	223	1946	21	8.72	0	4	7	1.058	9.23
25	Norway	212	2188	24	10.32	0	1	9	39.601	408.72
26	Czech Republic	193	1213	17	6.28	0	0	2	18.122	113.90
27	Austria	191	2361	24	12.36	1	3	11	21.824	269.77
28	Portugal	182	1985	20	10.90	0	4	7	17.685	192.88
29	Tunisia	177	1819	22	10.27	0	1	2	15.181	156.01
30	Finland	175	1836	25	10.49	0	0	5	31.574	331.26
31	Ireland	164	2341	24	14.27	1	2	10	34.140	487.33
32	Malaysia	164	996	16	6.07	0	1	3	5.118	31.08
33	Poland	151	600	13	3.97	0	0	0	3.963	15.75
34	Israel	132	1936	24	14.66	0	3	9	15.616	229.04
35	Pakistan	110	279	9	2.53	0	0	0	0.548	1.39
36	Chile	103	852	16	8.27	0	1	2	5.660	46.82
37	Romania	100	226	7	2.26	0	0	0	5.107	11.54
38	Russia	99	646	13	6.52	0	0	5	0.688	4.49
39	Saudi Arabia	94	623	14	6.62	0	0	2	2.801	18.57
40	U Arab Emirates	86	884	15	10.27	0	0	5	9.013	92.65

Abbreviations are available in previous tables, except $\frac{P}{PO}$, $\frac{C}{PO}$ papers and cites per million inhabitants.

using the software VOSviewer [32]. The maps include indexes such as co-citation, bibliographic coupling, citation analysis and co-authorship in order to analyze the influence and relationship among journals, publications of authors, institutions, countries and keywords.

In Fig. 2, the study analyzes the co-citation of journals regarding volatility. It presents the general analysis of the period 1975–2018 with a minimum threshold of five hundred cites and the one hundred most representative connections. Note that clusters are identified with different colors. One cluster regards the journals of mathematics, foreground publications in Econometrica, Journal of Econometrics and Physic A.

Another group of influential co-citations regards Economic Theory. The most cited journals in this field are the American Economic Review, the Journal of International Money and Finance, and the Journal of Monetary Economics. The leading journals in co-citation in the financial cluster are the Journal on Finance and The Review of Financial Studies. A fourth cluster includes the journals regarding management and business topics. While examining co-citation between clusters, it is observable that economic and econometric journals have a great tendency to cite financial journals.

Another interesting analysis is the bibliographic coupling among authors who publish about volatility. The resulting graph shown in Fig. 3 used data

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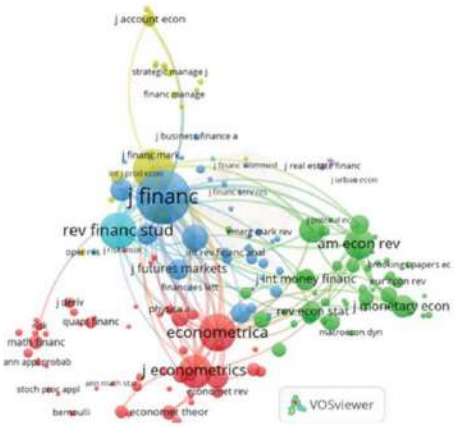

Fig. 2. Co-citation of journals regarding volatility.

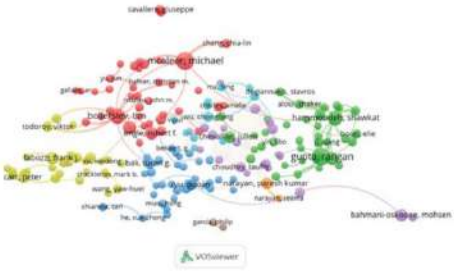

Fig. 3. Bibliographic coupling among authors publishing about volatility.

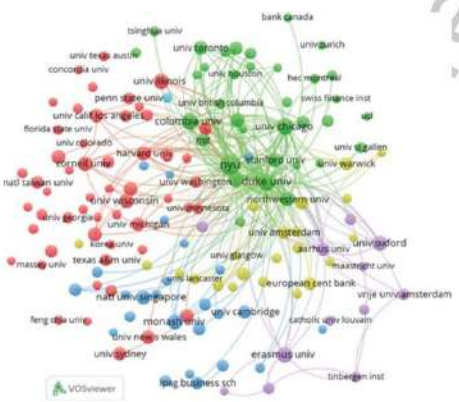

Fig. 4. Citation analysis by institutions.

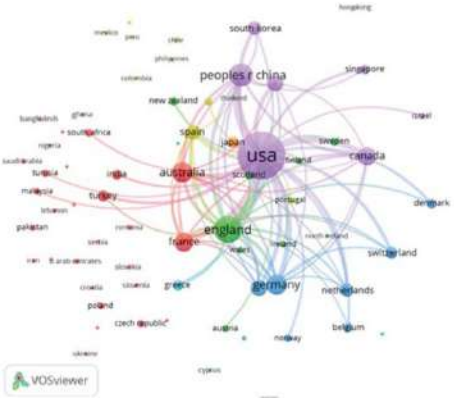

Fig. 5. Bibliographic coupling by countries.

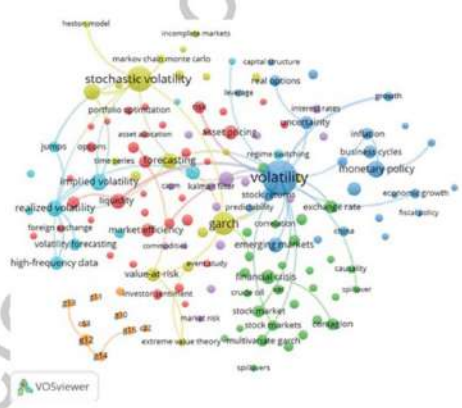

Fig. 6. Co-occurrence of author keywords in published documents regarding volatility.

with a minimum threshold of twelve papers and sixty cites. The featured authors in the graph are Michael Mcaleer, Tim Bollerslev and Rangan Gupta, also mentioned as the most productive authors in Table 4. Authors collaborate among themselves by countries or geographical areas. Note that in very few cases collaboration happens outside or beyond their research groups or areas.

Figure 4 shows an analysis of the citations by institutions, with a minimum threshold of fifty papers, five hundred cites and one hundred connections. The results are mostly grouped by geographical areas. The two largest groups are universities from the United States and they have great interaction among them

Table 9
Most common author keywords occurrences regarding volatility

	Global			1986–1996			1997–2007			2008–2018		
R	Keyword	OC	CO	Keyword	OC	CO	Keyword	OC	CO	Keyword	OC	CO
1	volatility	1303	903	volatility	26	16	volatility	237	162	volatility	1040	688
2	stochastic volatility	681	400	garch	20	14	stochastic volatility	135	92	stochastic volatility	529	262
3	garch	551	418	stochastic volatility	17	17	garch	112	82	garch	420	306
4	implied volatility	290	177	arch	14	13	exchange rates	68	54	realized volatility	258	191
5	monetary policy	287	151	exchange rates	10	7	option pricing	56	41	implied volatility	256	150
6	forecasting	286	203	option pricing	9	9	forecasting	49	45	monetary policy	236	112
7	option pricing	284	167	forecasting	8	8	monetary policy	49	33	forecasting	229	149
8	realized volatility	283	214	asset pricing	7	5	exchange rate volatility	38	19	option pricing	219	112
9	emerging markets	235	158	term structure	7	7	long memory	37	28	financial crisis	211	135
10	exchange rates	230	165	generalized method of moments	6	6	business cycles	34	12	emerging markets	202	136
11	liquidity	222	153	arch models	5	5	emerging markets	33	20	liquidity	202	133
12	financial crisis	215	148	business cycles	5	1	market microstructure	32	18	asset pricing	173	104
13	asset pricing	209	140	heteroskedasticity	5	5	implied volatility	30	21	long memory	165	123
14	long memory	203	153	options	5	4	risk	30	15	uncertainty	162	105
15	market efficiency	190	99	price volatility	5	3	asset pricing	29	23	value-at-risk	162	115
16	value-at-risk	190	144	cointegration	4	4	high-frequency data	29	23	market efficiency	161	75
17	uncertainty	189	130	conditional heteroskedasticity	4	3	trading volume	28	22	high-frequency data	156	115
18	exchange rate volatility	188	75	exchange rate volatility	4	4	value-at-risk	28	22	risk management	154	97
19	high-frequency data	185	141	finance	4	4	investment	27	17	exchange rate	153	101
20	exchange rate	179	123	futures markets	4	3	garch models	26	14	exchange rates	151	103
21	risk management	177	117	implied volatility	4	3	inflation	26	23	stock returns	150	99
22	stock returns	174	126	market microstructure	4	3	market efficiency	26	17	exchange rate volatility	146	59
23	business cycles	172	80	nonlinear filtering	4	4	price volatility	26	10	stock market	140	101
24	risk	163	99	stock returns	4	4	real options	26	10	volatility forecasting	140	98
25	market microstructure	156	100	unit root	4	4	term structure	26	17	contagion	138	98
26	volatility forecasting	156	110	chaos	3	2	uncertainty	26	20	g12	136	103
27	stock market	154	111	conditional volatility	3	1	arch	25	23	business cycles	133	54
28	contagion	150	109	garch models	3	3	exchange rate	25	20	risk	133	76
29	inflation	147	107	general equilibrium	3	3	kalman filter	25	19	inflation	121	85
30	real options	138	60	interest rates	3	3	markov chain monte carlo	25	23	market microstructure	120	78
31	g12	137	106	international	3	3	realized volatility	25	21	jumps	119	105
32	multivariate garch	134	95	market efficiency	3	1	hedging	24	15	multivariate garch	115	82
33	jumps	132	120	mean reversion	3	3	conditional heteroskedasticity	23	7	stock markets	115	88
34	garch models	126	76	quasi-maximum likelihood	3	3	learning	23	15	investor sentiment	114	68
35	hedging	126	87	smoothing	3	2	interest rates	22	15	structural breaks	114	85
36	stock markets	125	96	stock index futures	3	3	risk management	22	15	real options	112	46
37	value at risk	123	95	term structure of interest rates	3	1	garch model	21	11	china	110	55
38	price volatility	119	50	time-varying volatility	3	2	value at risk	21	16	volatility spillovers	109	70
39	asymmetry	118	89	vector autoregression	3	1	liquidity	20	13	vix	107	67
40	cointegration	118	63	volume	3	3	regime switching	20	14	economic growth	105	52

Abbreviations: OC = Author keyword occurrences; CO = Author keyword co-occurrences links.

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as observed. European universities are part of the other three small groups, which have little interrelation with each other. Most of the universities in the United States are connected to the rest of the representative universities.

Figure 5 shows the bibliographic coupling of countries with a threshold of twenty documents and one hundred connections. It is observable that the United States has a close relationship with Scotland, Japan and Finland. Collaborative works regarding volatility can be found among the most representative countries: USA, China, England, Australia, Canada, Germany and France.

In order to analyze the words used by the authors, Fig. 6 shows the leading keywords in volatility papers. The Co-occurrence of author keywords analysis took into account a minimum threshold of fifty papers and one hundred connections. The most influential words are volatility, stochastic volatility, GARCH, realized volatility, forecasting and monetary policy. The words linkable to this study are estimation, forecast and modeling volatility. Words tend to be grouped into different clusters depending on other subtopics such as business and economics, financial markets, modeling and measures. The interrelationship between the representative words mainly occurs between the areas of modeling and volatility measurement.

Finally, in order to analyze the most common words through time, Table 9 shows the forty most used words along the general period and during three different periods: 1986–1996, 1997–2007 and 2008–2018. Volatility has been the most used keyword in all periods; GARCH was the second most used in the first period, while stochastic volatility is ranked in second place for the global period.

5. Summary and conclusions

Volatility is an important topic for financial management and decision making. Since volatility has an impact on prices, it becomes an influential indicator for investors, researchers and policymakers. Open markets and free trade have created continuous instability in the prices of financial assets. Exchange rates, indices and derivatives prices have been the most affected. In this sense, the literature has developed many proposals to counteract such problems.

Modeling, estimating and forecasting are the most important issues regarding volatility. This paper proposes a bibliometric analysis of volatility, modeling and forecasting through time. The work comprises forty-four years between 1975–2018.

The paper also proposes an analysis of authors, institutions and countries who work in volatility, using comparative tables and graphical visualizations. Note that the information was retrieved from the database of the Web of Science Core Collection, and the maps were built with VOSviewer software. The analysis includes bibliographic coupling, co-authorship and co-citations, wherewith clusters and leading trends can be analyzed.

The most cited paper was written by Engle with 7,332 cites; his paper proposes an innovation in modeling and forecasting volatility. The USA is the most productive country through time with an annual number of publications well above the rest of the countries. Therefore, it is not strange that American universities lead the list of the most productive institutions regarding volatility. Other countries with high productivity are the United Kingdom and Germany while China has considerably increased its productivity in recent years.

This paper highlights the importance of studying volatility and how authors and institutions tend to work in groups by countries and geographical areas. It also shows how volatility has been studied from mathematical, financial and economic approaches over time.

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Apéndice 4.

Exchange rate and volatility: a bibliometric review. IJFE.

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RESEARCH ARTICLE

WILEY

Exchange rate and volatility: A bibliometric review

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Abstract

The exchange rate is one of the most important prices in open economies. Exchange rate volatility (ERV) has been studied in terms of its measurement, forecast and impact and relationship with other variables. This article proposes a bibliometric analysis of ERV compared with two databases Web of Science and Scopus. The number of data obtained reflects the importance of the topic in scientific research. In addition, we identify authors, institutions and countries of great influence studying currency volatility. The evolution of the study through time shows the increase in attention on the topic. VOS viewer software has been used to create graphic maps and visualize the connections existing in the study.

KEYWORDS

bibliometric, exchange rate, scopus, volatility, web of science

1 | INTRODUCTION

The currency market is the largest and most liquid financial market in the world (Balcilar, Gupta, Kyei, & Wohar, 2016; FOREX, 2020). The exchange rate is one of the most important prices in open economies due to its relationship with companies, investors and policymakers, this through the determination of investments, risk management and economic decisions of governments (Avdjiev, Bruno, Koch, & Song Shin, 2019; Korol, 2014; Leon-Castro, Avilez-Ochoa, & Gil-Lafuente, 2016). The price of currencies has an impact on the relative value of domestic and foreign goods, which influences numerous macroeconomic variables.

Transcendent events over time have accentuated the ERV in the world, such as the breaking of the Bretton-Woods agreement (Asteriou, Masatci, & Pilbeam, 2016; Ouyang, Rajan, & Li, 2016) and the global financial crises (Kholer, 2019; Pinho, Franco, & Silva, 2016; Stiglitz, 2010). Table 1 shows the volatility in the currencies of some countries and the European Union with the U.S. dollar. Japan and Mexico have the highest currencies volatility presenting values up to 170 and 36%, respectively. While the European Union and Canada have low volatility, around 1%.

Price movements in financial assets are originated in an event-based approach, such as news or announcement of political-economic origin. The events influence in economic agent decisions to maintain national or foreign assets (Andersen, Bollerslev, & Dievold, 2007; Ayadi, Omrane, Wang, & Welch, 2019). Factors such as inflation and interest rate differentials between countries generate volatility in the exchange rate (Jeelani, Tomar, Das, & Das, 2019). Also, the variation in the exchange rate in one country may be caused by situations in other economies (spillovers) as monetary or fiscal policy (Fisher & Chairman, 2015; Hung, 2019). Therefore, ERV is experienced by each country at different levels according to the political-economic characteristics of each one. The country creates conditions both to generate internal volatility and to create scenarios vulnerable to external impacts.

In general, ERV has varied effects on the economy. First, the level of exchange rates might have a direct effect on international trade earnings and trade volumes. Some theories suggest that there is a negative relationship between ERV and international trade (Arize, Osang, & Slottje, 2000; Asteriou et al., 2016; Bahmani-

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TABLE 1 Analysis of ERV per country

	2014-1	2014-2	2015-1	2015-2	2016-1	2016-2	2017-1	2017-2	2018-1	2018-2	2019-1	2019-2
China	1.90%	1.23%	1.17%	2.83%	2.28%	2.87%	1.67%	3.26%	4.18%	3.07%	3.23%	3.05%
UE	0.68%	0.93%	1.59%	1.21%	1.10%	1.02%	1.02%	0.95%	1.02%	0.67%	0.50%	0.57%
Japan	56.01%	116.08%	92.23%	101.22%	156.30%	170.60%	117.67%	92.01%	90.84%	70.18%	53.04%	52.25%
México	8.07%	11.36%	15.01%	20.40%	29.60%	36.11%	26.69%	24.73%	26.72%	28.70%	17.19%	18.70%
Brazil	2.43%	3.78%	6.54%	7.92%	6.58%	4.99%	3.54%	3.66%	4.81%	7.84%	4.11%	6.37%
Canada	0.77%	0.69%	1.61%	1.19%	1.62%	1.04%	1.02%	1.15%	1.05%	0.92%	0.69%	0.61%

Note: Based on Federal Reserve data, calculated by the standard deviation.

Oskooee & Gelan, 2018). Exchange rate fluctuations also influence economic policy, in the cases of countries that are adopting an inflation-targeting regime, the level and volatility in the exchange rate can affect those objectives (Ghosh, Ostry, & Chamon, 2016; Minella, Springer de Freitas, Goldfajn, & Kfoury Muinhos, 2003; Osabuohien, Obiekwe, Urhie, & Osabohien, 2018). Additionally, there is evidence that the investor portfolio flows change with the degree of uncertainty in the foreign exchange market. Normally, risk-averse traders who are expecting wild swings in the exchange rate might prefer to halt trading in the short term until the restoration of stability (Caporale, Ali, Spagnolo, & Spagnolo, 2017; Fidora, Freatzscher, & Thimann, 2007). Economic growth is another indicator that is affected by changes in the exchange rate, the analysis tended to find evidence in favour of a negative impact of exchange rate instability on growth (Alagidede & Ibrahim, 2016; Schnabl, 2008).

However, ERV has been studied in different ways according to particular objectives. Because volatility is not directly observable, its measurement has been analysed (Andersen, Bollerslev, Dievold, & Labys, 2001; Daly, 2008). Intending to consider future behaviours, anticipating situations and improving decision making, a large part of the ERV research focuses on the forecast (Andersen, Bollerslev, Diebold, & Labys, 2003; Wang, Pan, & Wu, 2017; West & Cho, 1995). Also, it is important to know the causative factors and their behaviour over time to test relationships with other variables and create models (Curran & Velic, 2019; Diebold & Nerlove, 1989; Flood & Rose, 1995) (Aghion, Bacchetta, Ranciere, & Rogoff, 2009; Campa & Goldberg, 2005; Chang & Bhutto, 2019). Studies by Gali and Monacelli (2005) and Thornton and Vasilakis (2019) have provided empirical evidence for the relationship between currency volatilities and monetary policy. Therefore, the ERV becomes interesting to analyse from a quantitative point of view, as this field has been developing over time.

The bibliometric method is increasingly used as a tool for research performance evaluation, its advances

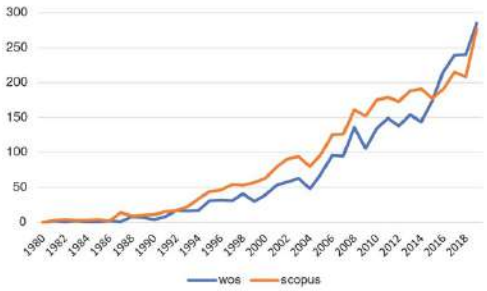


FIGURE 1 The annual number of ERV papers published [Colour figure can be viewed at wileyonlinelibrary.com]

and trends (Merigó, Cancino, Coronado, & Urbano, 2016; Wallin, 2005). One of the objectives in the bibliometric analysis is to arrive at a consistent and standardized set of indicators based on bibliographic databases (Hood, 2003). As more and more scientific discoveries occur and quoted by other researchers, bibliometric indicators are becoming important. Bibliometric indicators are especially important for researchers and organizations. For researchers, bibliometric indicators are important because they allow objective measurements of the diffusion and the impact of the articles published in a topic (Durieux & Gevenois, 2010). For organizations, the bibliometric method helps to measurements of the quality of particular research, individual researchers or a researcher group (Garner, Hirsch, Albuquerque, & Fargen, 2017; Narin & Hamilton, 1996).

Bibliometric studies are fairly common in a wide range of research fields such as financial studies (Merigó, Yang, & Xu, 2015), volatility spillovers (Chen & Yang, 2019), international competitiveness (Olczyk, 2016) and currencies (Merediz-Sola & Bariviera, 2019).

Therefore, the aim of the article is to provide a broad quantitative and qualitative view of ERV research, based

TABLE 2 documents most cited in ERV

Web of Science

R	TC	Title	Author/s	Year	C/Y
1	1,160	Modelling and forecasting realized volatility	Andersen, TG; Bollerslev, T; Diebold, FX; Labys, P	2003	64.4
2	1,041	Answering the skeptics: Yes, standard volatility models do provide accurate forecasts	Andersen, TG; Bollerslev, T	1998	45.2
3	854	Fractionally integrated generalized autoregressive conditional heteroskedasticity	Baillie, RT; Bollerslev, T; Mikkelsen, HO	1996	34.1
4	828	The distribution of realized exchange rate volatility	Andersen, TG; Bollerslev, T; Diebold, FX; Labys, P	2001	41.4
5	792	Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options	Bates, DS	1996	31.6
6	784	Modelling asymmetric exchange rate dependence	Patton, AJ	2006	52.2
7	646	Forecasting volatility in financial markets: A review	Poon, SH; Granger, CWJ	2003	35.8
8	526	A forecast comparison of volatility models: Does anything beat a GARCH (1, 1)?	Hansen, PR; Lunde, A	2005	32.8
9	504	Monetary policy and exchange rate volatility in a small open economy	Gali, J; Monacelli, T	2005	31.5
10	442	Micro effects of macro announcements: Real-time price discovery in foreign exchange	Andersen, TG; Bollerslev, T; Diebold, FX; Vega, C	2003	24.5
11	412	A new approach to measuring financial contagion	Bae, KH; Karolyi, GA; Stulz, RM	2003	22.8
12	405	Range-based estimation of stochastic volatility models	Alizadeh, S; Brandt, MW; Diebold, FX	2002	21.3
13	365	One money, one market: the effect of common currencies on trade	Rose, AK	2000	17.3
14	358	Why is it so difficult to beat the random walk forecast of exchange rates?	Kilian, L; Taylor, MP	2003	19.8
15	356	International business cycles: World, region and country-specific factors	Kose, MA; Otrok, C; Whiteman, CH	2003	19.7
16	354	Real-time price discovery in global stock, bond and foreign exchange markets	Andersen, Torben G.; Bollerslev, Tim; Diebold, Francis X.; Vega, Clara	2007	25.2
17	316	Exchange rate pass-through into import prices	Campa, JM; Goldberg, LS	2005	19.7
18	316	Business cycles in emerging economies: the role of interest rates	Neumeyer, PA; Perri, F	2005	19.7
19	307	Modelling volatility persistence of speculative returns: A new approach	Ding, ZX; Granger, CWJ	1996	12.2
20	303	Bivariate GARCH estimation of the optimal commodity hedge	BAILLIE, RT; MYERS, RJ	1991	10.1
21	268	Classifying exchange rate regimes: Deeds versus words	Levy-Yeyati, E; Sturzenegger, F	2005	16.7
22	266	Financial development and economic growth: The role of stock markets	Arestis, P; Demetriades, PO; Luintel, KB	2001	13.3
23	259	Nonlinear aspects of goods-market arbitrage and adjustment: Heckscher's commodity points revisited	Obstfeld, M; Taylor, AM	1997	10.7
24	256	The dynamics of exchange rate volatility—A multivariate latent factor ARCH model	DIEBOLD, FX; NERLOVE, M	1989	8
25	242	Volatility spillover effects from Japan and the United States to the Pacific-Basin	Ng, A	2000	11.5

Scopus

R	TC	Title	Author/s	Year	C/Y
1	1,343	Modelling and forecasting realized volatility		2003	79.0

(Continues)

TABLE 2 (Continued)

Scopus					
R	TC	Title	Author/s	Year	C/Y
			Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P.		
2	1,008	Fractionally integrated generalized autoregressive conditional heteroskedasticity	Baillie, R.T., Bollerslev, T., Mikkelsen, H.O.	1996	42.0
3	932	Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options	Bates, D.S.	1996	38.8
4	922	The distribution of realized exchange rate volatility	Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P.	2001	48.5
5	688	Bayesian analysis of stochastic volatility models	Jacouier, E., Polson, N.G., Rossl, P.E.	1994	26.4
6	629	Asymmetric dynamics in the correlations of global equity and bond returns	Cappiello, L., Engle, R.F., Sheppard, K.	2006	44.9
7	604	A forecast comparison of volatility models: Does anything beat a GARCH (1, 1)?	Hansen, P.R., Lunde, A.	2005	40.2
8	604	Monetary policy and exchange rate volatility in a small open economy	Galí, J., Monacelli, T.	2005	40.2
9	594	Roughing it up: Including jump components in the measurement, modelling and forecasting of return volatility	Andersen, T.G., Bollerslev, T., Diebold, F.X.	2007	45.6
10	581	Multivariate stochastic variance models	Harvey, A.	1994	22.3
11	525	Econometrics of testing for jumps in financial economics using bipower variation	Barndorff-Nielsen, O.E., Shephard, N.	2006	37.5
12	511	Institutional causes, macroeconomic symptoms: Volatility, crises and growth	Acemoglu, D., Johnson, S., Robinson, J., Thaicharoen, Y.	2003	30.0
13	480	A new approach to measuring financial contagion	Bae, K.-H., Karolyi, G.A., Stulz, R.M.	2003	28.2
14	466	Range-based estimation of stochastic volatility models	Alizadeh, S., Brandt, M.W., Diebold, F.X.	2002	25.8
15	393	Real-time price discovery in global stock, bond and foreign exchange markets	Andersen, T.G., Bollerslev, T., Diebold, F.X., Vega, C.	2007	30.2
16	381	Exchange rate pass-through into import prices	Campa, J.M., Goldberg, L.S.	2005	25.4
17	368	Was it real? The exchange rate-interest differential relation over the modern floating-rate period	MEESE, R., ROGOFP, K.	1988	11.5
18	363	Modelling volatility persistence of speculative returns: A new approach	Ding, Z., Granger, C.W.J.	1996	15.1
19	352	Classifying exchange rate regimes: Deeds versus words	Levy-Yeyati, E., Sturzenegger, F.	2005	23.4
20	336	Pricing foreign currency options with stochastic volatility	Melino, A., Turnbull, S.M.	1990	11.2
21	333	The dynamics of exchange rate volatility: A multivariate latent factor ARCH model	Diebold, F.X., Nerlove, M.	1989	10.7
22	330	Heterogeneous information arrivals and return volatility dynamics: Uncovering the Long-run in high-frequency returns	Andersen, T.G., Bollerslev, T.	1997	14.3
23	327	Testing the covariance stationarity of heavy-tailed time series: An overview of the theory with applications to several financial datasets	Loretan, M., Phillips, P.C.B.	1994	12.5
24	320	Volatility spillover effects from Japan and the US to the Pacific-Basin	Ng, A.	2000	16.0
25	316	Modelling stochastic volatility: A review and comparative study	Taylor, S.J.	1994	12.1

Abbreviation: C/Y cites per papers.

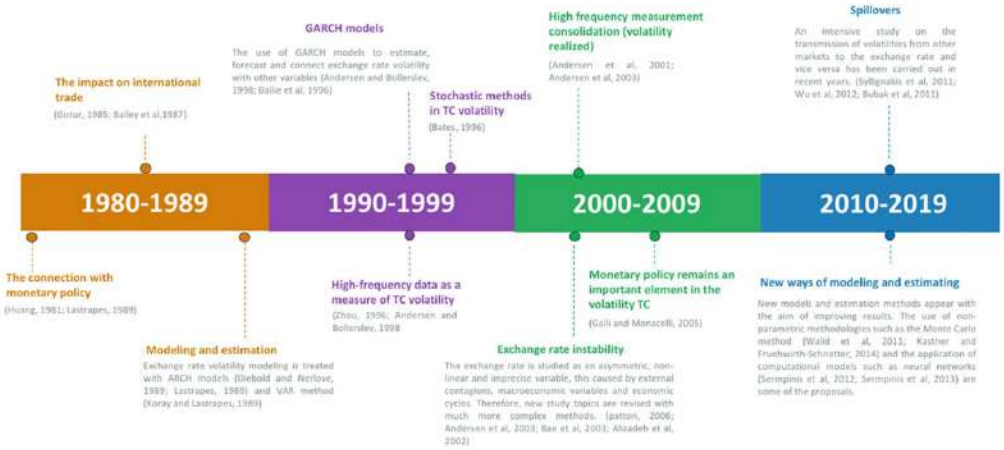


FIGURE 2 Evolution of the ERV study [Colour figure can be viewed at wileyonlinelibrary.com]

on bibliometric approaches. Additionally, an analysis of bibliometric performance and a graphic mapping of the topic is presented in order to show the most productive and influential studies and the connections between the different participants. The objective is to identify the authors, publications, universities and countries with more productivity in the topic and make a thorough analysis of these. The work compares the results between the Web of Science (WoS) and Scopus databases in the period 1980–2018.

The results of the bibliometric performance analysis and graphic mapping show the importance of the topic in the number of investigations obtained and the influence of the main actors. Modelling and forecasting realized volatility by Andersen are the most cited paper in ERV research. Bahmani-Oskooee of the University of Wisconsin-Milwaukee leaders the table, in the most productive author. In this sense, the University of Wisconsin-Milwaukee and the USA are the most influential university and country, respectively. The National Bureau of Economic Research is the institution with most publications on the topic. The study clusters of ERV are formed according to geographic areas where North America, Europe and Asia are in the foreground.

This article is organized as follows; Section 2 presents the bibliometric methods used in this work. Section 3 shows the analysis results of WoS and Scopus in authors, universities, institutions, countries and journals. Section 4 develops a graphical analysis of the bibliographic data in WoS and Scopus. Section 5 concludes the main findings.

2 | BIBLIOMETRIC METHODS

Bibliometric analysis can be used for two main tasks: the evaluation of research performance in a topic (Broadus, 1987; Garfield, 1979) and the study of science as a system of communication and knowledge generation (van Ran, 2005). Peer review is typically a qualitative assessment of research performance but bibliometric indicators discussed here represent the quantitative side.

This study considers several of the bibliometric indicators, such as the numbers of papers and citations as indicators of productivity and efficiency. An indicator that unifies productivity and efficiency is the h-index (Hirsch, 2005), it is a simple representative measure of individual scientific achievement (Bornmann & Daniel, 2005). H-index has been considered in authors, universities, institutions, countries and journals.

The work also presents other bibliometric indicators as cites per paper, thresholds according to high levels to cite and the evolution through time. For countries, we use the number of inhabitants and calculates the number of papers and citations per millions of inhabitants. The analysis of universities is added to the Academic Ranking of World Universities (ARWU) and the Quacquarelli and Symonds (QS) university ranking.

The Vos Viewer software (Van Eck & Waltman, 2010) has been used to represent graphic maps. The Vos Viewer can represent graphically the information in terms of bibliographic coupling, citation, co-citation and co-occurrence of author keywords.

The article analyses a comparison between the WoS database and Scopus for the same period and keywords.

TABLE 3 documents most cited by ERV publications

Web of Science

R	Year	cited reference	Type	Citations	TLS
1	1986	bollerslev t, j econometrics, v31, p307	A	383	373
2	1982	engle rf, econometrica, v50, p987	A	327	323
3	1991	nelson db, econometrica, v59, p347	A	198	195
4	1983	meese ra, j int econ, v14, p3	A	160	149
5	1987	engle rf, econometrica, v55, p251	A	147	144
6	1988	degrauwe p, int monet fund s pap, v35, p63	A	133	132
7	1993	glosten lr, j financ, v48, p1779	A	131	130
8	2001	pesaran mh, j appl econom, v16, p289	A	131	115
9	1979	dickey da, j am stat assoc, v74, p427	A	130	130
10	1978	hooper p, j int econ, v8, p483	A	128	127
11	1999	mckenzie m. d., j econ surv, v13, p71	A	120	118
12	2001	andersen, t.g., bollerslev, t., diebold, f.x., labys, p., j am stat assoc, v96, p42	A	118	114
13	1976	dornbusch r, j polit econ, v84, p1161	A	117	106
14	2000	arize ac, j bus econ stat, v18, p10	A	110	110
15	1993	chowdhury ar, rev econ stat, v75, p700	A	109	108
16	2002	engle r, j bus econ stat, v20, p339	A	109	108
17	1992	bollerslev t, j econometrics, v52	A	108	106
18	1988	phillips pcb, biometrika, v75, p335	A	106	106
19	2002	calvo ga, q j econ, v117, p379	A	105	91
20	1988	johansen s, j econ dyn control, v12, p231	A	99	98

Scopus

R	Year	Cited reference	Type	Citations	TLS
1	1986	bollerslev, t., journal of econometrics, 31, pp. 307–327	A	288	196
2	1991	nelson, d.b., econometrica, 59, pp. 347–370	A	75	59
3	1982	engle, r.f., econometrica, 50, pp. 987–1007	A	58	43
4	1976	dornbusch, r., journal of political economy, 84, pp. 1161–1176	A	54	32
5	1989	hamilton, j.d., econometrica, 57, pp. 357–384	A	48	27
6	1988	de grauwe, p., imf staff papers, 35, pp. 63–84	A	40	32
7	1998	andersen, t.g., bollerslev, t., international economic review, 39, pp. 885–905	A	39	33
8	1998	dominguez, k.m., journal of international money and finance, 17, pp. 161–190	A	38	20
9	1995	obstfeld, m., rogoff, k., journal of political economy, 103, pp. 624–660	A	37	13
10	1995	engle, r.f., kroner, k.f., econometric theory, 11, pp. 122–150	A	36	22
11	1982	engle, r.f., econometrica, 50 (4), pp. 987–1007	A	35	24
12	1999	mckenzie, m.d., journal of economic surveys, 13 (1), pp. 71–106	A	35	27
13	1991	nelson, d.b., econometrica, 59 (2), pp. 347–370	A	33	20
14	2001	andersen, t.g., bollerslev, t., diebold, f.x., labys, p., j am stat assoc, 96, pp. 42–55	A	32	25
15	1982	engle, r.f., econometrica, 50, pp. 987–1007	A	32	25
16	1991	franke, g., journal of international money and finance, 10, pp. 292–307	A	32	31
17	1978	hooper, p., kohlhagen, s.w., journal of international economics, 8, pp. 483–511	A	32	28
18	2001	pesaran, m.h., shin, y., smith, r.j., journal of applied econometrics, 16, pp. 289–326	A	32	21
19	1988	phillips, p.c.b., perron, p., biometrika, 75, pp. 335–346	A	32	22
20	1996	rogoff, k., journal of economic literature, 34, pp. 647–668	A	32	19

Abbreviations: A, article; TLS, total link strength.

TABLE 4 Citing articles of ERV: authors, organizations, country and journal

Web of Science								
R	**Author**	**TP**	**Organization**	**TP**	**Country**	**TP**	**Journal**	**TP**
1	Gupta R	110	National Bureau of Economic Research	589	USA	6,856	J Int money and Finance	769
2	Bahmani-Oskooee M	94	Federal Reserve System—USA	511	China	3,023	Physica A Stat Mech App	598
3	McAleer M	61	University of London	507	England	2,988	Economic Modeling	594
4	Bollerslev T	58	International Monetary Fund	356	Germany	1,922	Applied Economics	584
5	Hammoudeh S	57	Centre National de la Recherche Scientifique (CNRS)	264	Australia	1,471	J Banking Finance	468
6	Wohar ME	56	Centre for Economic Policy Research—UK	240	France	1,335	J Economics	384
7	Su CW	55	University of Oxford	212	Canada	1,193	Energy Economics	364
8	Tiwari AK	55	London School Economics & Political Science	210	Italy	1,050	J Futures Markets	356
9	Zhang TY	55	European Central Bank	209	Taiwan	1,029	J Int Economics	337
10	Gil-Alana LA	51	Duke University	202	Spain	1,004	Int rev Economics Finance	323
11	Chang TY	49	Chinese Academy of Sciences	194	South Korea	769	Emerging Markets Fin Tade	306
12	Pierdzioch C	49	Monash University	194	Turkey	746	Quantitative Finance	277
13	Balcilar M	46	Aarhus University	190	Japan	665	App Economics Letters	262
14	Narayan PK	46	Erasmus University Rotterdam	178	Netherlands	635	J Economic Dynamics Cont	250
15	Stanley HE	44	Columbia University	175	Switzerland	596	Int J Finance Economics	218
16	Sarno L	45	City University London	173	India	577	Economics Letters	212
17	Bouri E	41	University of Warwick	169	Greece	464	Int Rev Financial Analysis	206
18	Ma F	41	Harvard University	162	Malaysia	401	North American J Eco Fin	204
19	Shahbaz M	41	New York University	160	Brazil	398	J int Fin Markets Ins Money	203
20	chen CWS	40	University of Sydney	159	Belgium	391		202
Scopus								
R	**Author**	**TP**	**organization**	**TP**	**Country**	**TP**	**Journal**	**TP**
1	Bollerslev, T.	65	National Bureau of Economic Research	257	USA	4,033	J Int Money Finance	387
2	McMillan, D.G.	43	International Monetary Fund	207	China	1,297	J Econometrics	348
3	Shephard, N.	40	Duke University	179	Germany	919	Physica A Stat Mech App	310
4	Andersen, T.G.	38	Aarhus Universitet	158	Australia	879	J Banking Finance	279
5	Ma, F.	35	Centre for Economic Policy Research, London	135	France	858	Economic Modelling	239
6	Caporin, M.	34	University of Oxford	124	Canada	692	J Empirical Finance	217

(Continues)

TABLE 4 (Continued)

Scopus								
R	**Author**	**TP**	**organization**	**TP**	**Country**	**TP**	**Journal**	**TP**
7	Asai, M.	31	University of Cambridge	123	Italy	629	Applied Economics	205
8	Laurent, S.	31	London School of Economics and Political Science	120	Taiwan	512	J Futures Markets	202
9	McAleer, M.	31	The University of Chicago	117	Spain	478	Quantitative Finance	189
10	Caporale, G.M.	30	New York University	110	Japan	409	App Financial Economics	180
11	Degiannakis, S.	30	Erasmus University Rotterdam	109	South Korea	395	J int Fin Markets Ins Money	165
12	Diebold, F.X.	30	Federal Reserve System	107	Netherlands	373	J Business And Economic Statistics	157
13	Gupta, R.	30	Australian National University	106	Switzerland	362	Int Rev Economics And Finance	147
14	Dungey, M.	29	Columbia University in the City of New York	105	India	298	J International Economics	145
15	Todorov, V.	29	Monash University	101	Greece	290	Energy Economics	138
16	Cheong, C.W.	28	The University of Sydney	96	Hong Kong	280	Int Rev Financial Analysis	136
17	Tauchen, G.	28	University of New South Wales UNSW Australia	93	Turkey	279	J Forecasting	125
18	Wei, Y.	28	CNRS Centre National de la Recherche Scientifique	91	Denmark	250	J Financial Econometrics	123
19	Hammoudeh, S.	26	City University of London, Cass Business School	90	Brazil	248	North American J Eco Fin	118
20	Taylor, M.P.	26	European Central Bank	87	Austria	89	Int J Forecasting	114

Abbreviation: TP, total publications.

WoS is used as a Core Collection database that is currently owned by Clarivate Analytics with the index SCI-EXPANDED, SSCI, A&HCI, BKCI-S, BKCI-SSH and ESCI. The topic is exchange rate volatility (ERV) searched with the keywords 'exchange rate' and volatility*. The study has taken place in January 2020 for the period 1980–2019. Note that the analysis only considers articles, reviews and letters.

3 | RESULTS

3.1 | Publication and citation structure

The study of ERV starts in 1980 obtaining 2,889 results in WoS and 3,449 results in Scopus. Figure 1 shows the annual number of documents that work about the topic in the period 1980–2019. Note that the results have grown considerably since the middle of the 1990 decade. In the first 16 years, publications on ERV were less than 50 per year in both WoS and Scopus. The number of publications per year is greater in Scopus for the majority of years, until 2016 where WoS begins to be more productive in the topic. One can see that there was a rebound in publications in the 2008 crisis, where the topic became relevant due to its impact on the decision making of financial agents.

To know which paper has the greatest impact on the topic, Table 2 presents the 25 most cited papers on ERV by WoS and Scopus. The most cited paper corresponds to 'Modeling and forecasting realized volatility' proposed by Andersen, Bollerslev, Diebold and Labys. This article analyses GACRH modelling and high-frequency data, which appears in both databases with more than a thousand citations. The second place in WoS corresponds to Andersen and Bollerslev, which does not appear in Scopus. The third most cited article in WoS is for Baillie, Bollerslev and Mikkelsen, which appears in second place in Scopus. Note that among the first publications, we find a large majority study about modelling and forecasting of ERV, while a smaller part studies on monetary policy.

In order to analyse the ERV study over time, the period examined has been divided into four decades and the most important documents in each have been

TABLE 5 Most productive authors in ERV

Web of Science										
R	**Author**	**Institution**	**Country**	**TP**	**TC**	**H**	**C/P**	**≥200**	**≥100**	**≥50**
1	Bahmani-Oskooee, M	Univ Wisconsin-Milwaukee	USA	41	250	8	6.1	0	0	0
2	Devereux, MB	Univ British Columbia	Canada	15	695	9	46.3	1	1	5
3	Bollerslev, Tim	Duke Univ	USA	14	5,259	14	375.6	6	10	12
4	Aizenman, Joshua	Univ Southern Calif	USA	13	392	11	30.2	0	0	2
5	Cheung, Yin-Wong	City Univ Hong Kong	China	12	471	9	39.3	0	1	3
6	Hegerty, Scott W	NE Illinois Univ	USA	12	99	5	8.3	0	0	0
7	Kocenda, Evzen	Charles Univ Prague	Czech Republic	12	202	6	16.8	0	0	1
8	Pierdzioch, C	Univ Saarland	Germany	12	74	3	6.2	0	0	0
9	Caporale, Guglielmo Maria	Brunel Univ London	England	11	134	7	12.2	0	0	0
10	Rajan, Ramkishen S	NUS	Singapore	11	64	6	5.8	0	0	0
11	Beine, Michel	Univ Luxembourg	Luxembourg	10	285	8	28.5	0	0	2
12	Gupta, Rangan	Univ Pretoria	South Africa	10	70	5	7.0	0	0	0
13	Zivkov, Dejan	Univ Novi Sad	Serbia	10	30	4	3.0	0	0	0
14	Baillie, Richard T	Michigan State Univ	USA	9	1,516	8	168.4	2	3	5
15	Diebold, Francis X	Univ Penn	USA	9	3,670	9	407.8	6	6	9
16	Kumar, Satish	ICFAI Fdn Higher Educ	India	9	12	2	1.3	0	0	0
17	Neely, Christopher J	Fed Reserve Bank	USA	9	141	5	15.7	0	0	1
18	Shin, Dong Wan	Ewha Womans Univ	South Korea	9	50	6	5.6	0	0	0
19	Rose, Andrew K	Univ Calif Berkeley	USA	8	785	6	98.1	1	2	4
20	Melvin, Michael	Barclays Global Investors	USA	8	310	6	38.8	0	1	3
21	Caglayan, Mustafa	Heriot Watt Univ	Scotland	8	154	7	19.3	0	0	1
22	Lecourt, Christelle	Aix Marseille Univ	France	8	164	5	20.5	0	0	1
23	Narayan, Paresh Kumar	Deakin Univ	Australia	8	132	4	16.5	0	0	1
24	Herwartz, Helmut	Georg August Univ Gottingen	Germany	8	83	5	10.4	0	0	1
25	Wohar, Mark E	Univ Nebraska	USA	8	81	5	10.1	0	0	0
Scopus										
R	**Author**	**Institution**	**Country**	**TP**	**TC**	**H**	**C/P**	**≥200**	**≥100**	**≥50**
1	Bahmani-Oskooee, M.	The University of Wisconsin-Milwaukee	USA	50	317	12	6.3	0	1	2
2	Hegerty, S.W.	The University of Wisconsin-Milwaukee	USA	18	224	7	12.4	0	1	2
3	Arize, A.C.	College of Business and Technology	USA	14	339	11	24.2			
4	Bollerslev, T.	Duke University	USA	14	4,065	14	290.4	7	11	13
5	Devereux, M.B.	Vancouver School of Economics	Canada	14	714	11	51.0	1	4	5
6	Kim, S.-J.	The University of Sydney	Australia	14	193	8	13.8	0	0	0
7	Speight, A.E.H	Swansea University	UK	14	139	7	9.9	0	0	0
8	Caporale, G.M.	South Bank University	London	13	117	7	9.0	0	0	0
9	Kanas, A.	University of Crete	Greece	13	188	6	14.5	0	1	1
10	Laurent, S.	Universite Catholique de Louvain	Belgium	12	395	7	32.9	0	1	2
11	McMillan, D.G.	University of Aberdeen	UK	12	129	6	10.8	0	0	0

(Continues)

TABLE 5 (Continued)

Scopus										
R	Author	Institution	Country	TP	TC	H	C/P	≥200	≥100	≥50
12	Aizenman, J.	University of Southern California	USA	11	332	8	30.2	0	2	2
13	Beine, M.	University of Luxembourg	Germany	11	311	9	28.3	0	0	3
14	Cheung, Y.-W	University of California	USA	11	458	8	41.6	1	1	3
15	Baillie, R.T.	Michigan State University	USA	10	1,540	10	154.0	2	4	5
16	Gupta, R.	University of Pretoria	South Africa	10	60	4	6.0	0	0	0
17	Belke, A.	University of Vienna	Austria	9	51	5	5.7	0	0	0
18	Lecourt, C.	University of Lille II	France	9	170	6	18.9	0	0	2
19	Morales-Zumaquero, A.	University of Málaga	Spain	9	46	4	5.1	0	0	0
20	Neely, C.J.	Federal Reserve Bank of St. Louis	USA	9	187	7	20.8	0	0	1
21	Sariannidis, N.	Technological Educational Institute of West Macedonia	Greece	9	29	3	3.2	0	0	0
22	Rose, Andrew K.	University of California, Berkeley	USA	8	465	6	58.1	1	1	4
23	Melvin, M.	Arizona State University	USA	8	23	6	2.9	0	1	4
24	Aftab, M.	University of Malaya	Malasya	8	68	4	8.5	0	0	0
25	Rajan, R.S.	University of Surrey	UK	8	68	4	8.5	0	0	0

Abbreviations:≥200, ≥100, ≥50, number of papers with at least 200 and 100; C/P, citation per paper; H, h-index; TC, total citations; TP, total publications.

selected. The topics studied are summarized in Figure 2. One can observe in the first decade the predominant study of the fundamental variables and their impact with international trade, at the end of this period the focus on modelling and estimating ERV begins. With the study of ARCH models, the decade of the 90s was characterized by a deepening in mathematical analysis, the study in GARCH models, stochastic methodologies and the use of intra-day data predominate. At the beginning of the millennium, financial instabilities around the world focused the study on the statistical behaviour of the exchange rate series and finding the causes of these instabilities. In recent years, the analysis of the transmission of volatility from one region or market to others has been the focus of ERV research. Additionally, the use of hybrid methodologies for modelling and estimation have been proposed as alternatives to new statistical behaviours.

Those documents that receive most citations by papers and study ERV are shown in Table 3. The data presented is the first 20 co-citations of references extracted from the VOS Viewer software with the two databases. The most cited document is the one proposed by Bollerslev, winner of Novel prize in 2003, where generalized models of conditional heteroskedasticity (GARCH) are suggested. In WoS, papers published by Engle and Nelson for volatility estimation and forecast are followed. While in Scopus the places are inverse; follow by Nelson and Engle. Among the most recent papers cited by the topic are Andersen, Bollerslev, Diebold and Labys in 2001 and Calvo in 2002. As the previous table, the papers most searched by researchers on the topic are those related to modelling and forecasting.

Next, let us into the top 20 authors, organizations, countries and journals that cite the papers in the topic. Table 4 presents the results, the first author from the list is Gupta in WoS and Bollerselv in Scopus. Frequently, Gupta studies analyse the forecast and transmission of volatility, while Bollerslev studies forecasting and measures of volatility. The National Bureau of Economic Research is the organization that has most papers that ERV in both databases. The USA and the Journal of International Money and Finance are the first to lead the list in terms of country and journal.

3.2 | Leading authors, institutions and countries

In this section, the analysis is about authors, institutions and universities with more publications in ERV. Table 5 shows the most productive authors. In both databases, Bahmani-Oskooee from the University of Wisconsin-Milwaukee is the most productive author with 41 publications and 250 citations in WoS, while in Scopus he has

TABLE 6 Temporal evolution of the most productive authors in ERV

Web of Science												
1980–1989				1990–1999			2000–2009			2010–2019		
R	Author	TP	TC	Author	TP	TC	Author	TP	TC	Author	TP	TC
1	Lastrapes WD	2	153	Bollerslev T	5	2,155	Devereux, Michael B.	12	685	Bahmani-Oskooee, Mohsen	35	147
2	Diebold FX	1	256	Baillie RT	5	1,295	Bollerslev, Tim	9	3,210	Gupta, Rangan	10	70
3	Nerlove M	1	256	Campa JM	4	277	Beine, Michel	8	188	Hegerty, Scott W.	10	48
4	Koray F	1	90	Pittis N	4	13	Cheung, Yin-Wong	8	412	Zivkov, Dejan	10	31
5	Gotur P	1	80	Rose AK	3	262	Diebold, FrancisX	8	3,418	Kumar, Satish	9	13
6	Bailey MJ	1	58	Bakaert G	3	86	Andersen, Torben G	6	2,923	Shin, Dong Wan	9	50
7	Tavlas GS	1	58	Arize AC	3	85	Selcuk, Faruk	6	203	Aftab, Muhammad	8	60
8	Ulan M	1	58	Savvides A	3	38	Bahmani-Oskooee, Mohsen	6	103	Aizenman, Joshua	8	222
9	Huang	1	33	Sengupta	3	26	Pierdzioch, Christian	6	59	Njegic, Jovan	8	25
10	Willett TD	1	25	Caporale	3	12	Rajan, Ramkishen S.	6	53	Hammoudeh, Shawkat	7	196
Scopus												
1980–1989				1990–1999			2000–2009			2010–2019		
R	Author	TP	TC	Author	TP	TC	Author	TP	TC	Author	TP	TC
1	Diebold, F.X.	2	342	Arize, A.C.	11	188	Devereux, M.B.	13	704	Bahmani-Oskooee, M.	41	156
2	Flood, R.P.	2	35	Bollerslev, T.	7	1,781	Kanas, A.	12	188	Hegerty, S.W	15	70
3	Kawai, M.	2	9	Baillie, R.T.	5	1,369	Beine, M.	10	231	Gupta, R.	10	60
4	Meese, R	1	368	Peel, D.A.	4	144	McMillan, D.G.	9	97	Sariannidis, N.	9	29
5	Rogofp, K	1	368	Rose, A.K.	4	298	Speight, A.E.H.	9	78	Aftab, M.	8	68
6	Eun, C.S.	1	180	Sultan, J.	4	22	Belke, A.	8	49	Caporale, G.M.	8	52
7	Resnick, B. G	1	180	Taylor, S.J.	3	490	Bollerslev, T.	8	2,727	Harvey, H.	8	43
8	Chesney, M.	1	143	Campa, J.M.	3	313	Kim, S.J.	8	155	Mirdala, R.	8	14
9	Scott, L.	1	143	McKenzie, M.D.	3	255	Laurent, S.	8	345	Aizenman, J.	7	199
10	Ito, T.	1	120	Melvin, M.	3	152	Cheung, Y.W	7	393	Chortareas, G.	7	58

Note: The abbreviation is available in previous tables.

50 papers and 370 citations. Bahmani-Oskooee mainly studies volatility transmission and ERV connections with international trade. The second place on the WoS list is Devereux, which is the fifth in Scopus. Note that the majority of the authors come from the USA.

In order to see the most productive authors through time, Table 6 shows the leading authors in 10 years periods. Bollerslev appears in two decades with a high number of citing. In the last decade 2010–2019, the authors with more publications in ERV are Bahmani-Oskooee, Gupta and Hegerty in WoS. One can see that these authors are the same in Scopus but different order. The previously mentioned is consistent with Figure 2. Because in the '90s the studies focused on modelling and forecasting, while in recent years the study on volatility

TABLE 7 The most productive and influential university in the ERV

Web of science

R	University	Country	TP	TC	H	C/P	≥200	≥100	≥50	ARWU	QS
1	University of Wisconsin-Milwaukee	USA	41	252	8	6.15	0	0	0	400–450	-
2	Charles University Prague	Czech Republic.	33	408	11	12.4	0	0	1	200–250	317
3	National University of Singapore	Singapore	28	486	9	17.4	0	2	2	67	11
4	Duke University	USA	27	4,938	20	183	5	8	13	28	26
5	City University of Hong Kong	China	25	464	13	18.6	0	0	3	200–250	55
6	Monash University	Australia	22	306	8	13.9	0	0	3	73	59
7	Erasmus University Rotterdam	Netherlands	21	377	11	18	0	0	2	68	179
8	New York University	USA	21	1,199	14	57.1	1	3	8	30	43
9	University of New South Wales Sydney	Australia	21	183	7	8.71	0	0	0	94	45
10	University of Wisconsin Madison	USA	21	858	14	40.9	0	2	5	27	53
11	University of Oxford	England	20	620	9	31	0	1	4	7	5
12	Brunel University	England	19	539	10	28.4	1	1	2	–	332
13	University of California Berkeley	USA	19	1,370	13	72.1	2	4	7	5	27
14	University of Pennsylvania	USA	19	4,696	15	247	7	7	12	17	19
15	University of Sydney	Australia	19	219	9	11.5	0	0	1	80	42
16	Australian National University	Australia	18	85	6	4.72	0	0	0	76	24
17	University of Malaya	Malaysia	18	102	5	5.67	0	0	0	300–350	-
18	University of British Columbia	Canada	18	895	11	49.7	1	2	6	35	47
19	Ihsan Dogramaci Bilkent University	Turkey	17	259	6	15.2	0	1	2	–	456
20	London School Economics & Political Science	England	17	1,567	9	92.2	2	3	5	100–150	38
21	University of California Santa Cruz	USA	17	741	14	43.6	0	1	5	100–150	–
22	University of Warwick	England	16	1,109	8	69.3	2	3	5	101	54
23	Harvard University	USA	16	934	12	58.4	0	4	6	1	3
24	Chinese University of Hong Kong	China	16	461	9	28.8	1	1	2	100–150	49
25	City University London	England	16	269	10	16.8	0	0	1	–	351

Scopus

R	University	Country	TP	TC	H	C/P	≥200	≥100	≥50	ARWU	QS
1	University of Wisconsin-Milwaukee	USA	51	319	12	6.3	0	1	2	401	
2	National University of Singapore	Singapore	30	415	11	13.8	0	1	1	67	11
3	University of New South Wales	Australia	27	312	11	11.6	0	0	0	94	45
4	Duke University	USA	24	3,342	16	139.3	4	8	14	28	26
5	Charles University	Czech Republic	24	332	11	13.8	0	0	1	201	317
6	Universite Catholique de Louvain	Belgium	22	567	11	25.8	0	1	4	151	165
7	Erasmus University Rotterdam	Netherlands	22	522	12	23.7	0	2	3	68	179
8	City University of Hong Kong	Hong Kong	21	366	11	17.4	0	0	2	201	55
9	Monash University	Australia	21	344	9	16.4	0	0	3	73	59
10	Chinese University of Hong Kong	Hong Kong	19	516	10	27.2	1	1	2	101	49
11	Athens Univ. of Econ. and Business	Greece	19	231	9	12.2	0	0	1		801–1,000
12	University of Sydney	Australia	19	229	9	12.1	0	0	0	80	42
13	University of Oxford	UK	19	1,503	7	79.1	3	4	4	7	5
14	Swansea University	UK	18	166	8	9.2	0	0	0	401	435
15	University of British Columbia	Canada	18	971	13	53.9	2	5	6	35	47

TABLE 7 (Continued)

Scopus											
R	University	Country	TP	TC	H	C/P	≥200	≥100	≥50	ARWU	QS
16	University of California, Berkeley	USA	18	1,488	12	82.7	3	4	8	5	27
17	London School Economics & Political Science	UK	17	939	10	55.2	1	1	3	151	38
18	University of Pennsylvania	USA	17	4,373	14	257.2	8	9	10	17	19
19	Australian National University	Australia	17	335	8	19.7	1	1	1	76	24
20	London Business School	UK	17	276	10	16.2	0	0	1		
21	Brunel University	UK	17	236	8	13.9	0	0	0		332
22	University of California, Santa Cruz	USA	17	701	12	41.2	1	3	5	101	336
23	University of Chicago	USA	16	1,172	11	73.3	2	2	5	10	9
24	Université libre de Bruxelles ULB	Belgium	16	236	9	14.8	0	0	2		
25	University Putra Malaysia	Malaysia	16	76	5	4.8	0	0	0		202

Note: Abbreviations are available in previous tables.

TABLE 8 The most productive institution in ERV

Web of science									
R	Institution	Country	TP	TC	H	C/P	≥200	≥100	≥50
1	National Bureau of Economic Research	USA	87	9,124	37	104.9	10	20	29
2	International Monetary Fund	USA	70	1701	20	24.3	1	3	10
3	Federal Reserve System	USA	64	2,598	23	40.6	3	6	15
4	Centre for Economic Policy Research	England	32	1711	17	53.5	2	4	9
5	Centre National de la Recherche Scientifique (CNRS)	France	26	244	10	9.4	0	0	0
6	The World Bank	USA	23	371	7	16.1	0	1	2
7	Ifo Institut	Germany	17	310	7	18.2	0	0	3
8	European Central Bank	Germany	13	820	11	63.1	1	2	5
9	Czech Natl Bank	Czech Republic	12	121	5	10.1	0	0	0
10	Central Bank of the Republic of Turkey	Turkey	23	181	7	7.9	0	0	1
Scopus									
R	Institution	Country	TP	TC	H	C/P	≥200	≥100	≥50
1	National Bureau of Economic Research	USA	65	4,530	30	69.7	6	18	24
2	International Monetary Fund	USA	64	1,223	15	19.1	2	3	6
3	CEPR	London	37	1,401	20	37.9	0	7	10
4	World Bank	USA	21	260	7	12.4	0	0	1
5	Federal Reserve System	USA	20	1,216	14	60.8	2	5	7
6	Czech National Bank	Czech Republic	15	150	8	10.0	0	0	0
7	European Central Bank	Germany	13	1,260	10	96.9	2	4	5
8	Centre National de la Recherche Scientifique (CNRS)	France	12	96	6	8.0	0	0	0
9	Central Bank of the Republic of Turkey	Turkey	11	55	3	5.0	0	0	0
10	Banco Central do Brasil	Brrazil	10	91	6	9.1	0	0	0

Note: Abbreviations are available in previous tables.

TABLE 9 The most productive and influential countries in ERV

Web of Science										
R	Country	TP	TC	H	C/P	≥200	≥100	≥50	P/PO	C/CO
1	USA	885	28,999	81	32.8	24	60	128	2.67	87.61
2	United Kingdom	366	8,880	44	24.3	6	14	40	5.39	130.81
3	China	246	2,714	25	11.0	1	5	12	0.17	1.89
4	Australia	183	1,837	20	10.0	0	1	9	7.18	72.04
5	Germany	177	2,965	27	16.8	1	6	14	2.11	35.39
6	France	144	1,770	22	12.3	0	2	8	2.21	27.12
7	Canada	132	2,809	25	21.3	2	6	15	3.50	74.43
8	Turkey	109	762	13	7.0	0	1	3	1.29	9.03
9	South Korea	104	624	14	6.0	0	0	0	2.03	12.17
10	Taiwan	99	888	13	9.0	0	2	3	4.16	37.28
11	Japan	88	979	17	11.1	0	0	3	0.70	7.74
12	India	84	500	12	6.0	0	0	2	0.06	0.36
13	Italy	76	1,205	18	15.9	0	2	6	1.26	19.93
14	Spain	75	718	12	9.6	0	2	2	1.60	15.36
15	Czech Republic	61	525	13	8.6	0	0	1	5.70	49.02
16	Malasya	59	213	8	3.6	0	0	0	1.82	6.58
17	Greece	58	706	13	12.2	0	1	3	5.56	67.73
18	Belgium	55	980	17	17.8	0	1	5	4.75	84.56
19	Netherlands	51	936	16	18.4	0	1	7	2.98	54.63
20	Switzerland	51	1,053	14	20.6	0	2	7	5.89	121.67
21	Singapore	47	659	11	14.0	0	2	3	8.03	112.64
22	South Africa	47	169	8	3.6	0	0	0	0.79	2.85
23	Brazil	39	373	9	9.6	0	1	2	0.18	1.75
24	Tunisa	37	392	9	10.6	0	1	2	3.13	33.17
25	Pakistan	36	109	5	3.0	0	0	0	0.16	0.49
Scopus										
R	Country	TP	TC	H	C/P	≥200	≥100	≥50	P/PO	C/CO
1	USA	944	20,183	81	21.4	28	68	135	2.85	60.98
2	Australia	207	2,294	24	11.1	1	2	10	8.12	89.96
3	Germany	199	3,429	30	17.2	1	7	18	2.38	40.93
4	China	188	1,393	51	7.4	0	3	6	0.13	0.97
5	France	160	2,141	25	13.4	1	3	8	2.45	32.80
6	Canada	126	2,625	24	20.8	4	9	11	3.34	69.55
7	India	124	623	15	5.0	0	0	2	0.09	0.45
8	Taiwan	110	819	14	7.4	0	1	2	4.62	34.39
9	Greece	98	901	16	9.2	0	1	3	9.40	86.44
10	Turkey	96	637	13	6.6	0	1	2	1.14	7.55
11	Japan	93	975	19	10.5	0	1	3	0.74	7.71
12	South Korea	92	1,112	16	12.1	1	1	2	1.79	21.69
13	Italy	84	2,157	23	25.7	3	5	9	1.39	35.68
14	Malaysia	78	306	10	3.9	0	0	0	2.41	9.45
15	Netherlands	77	1,874	25	24.3	0	6	3	4.49	109.37
16	Brazil	75	425	11	5.7	0	0	1	0.35	2.00
17	Spain	74	2,216	16	29.9	3	4	6	1.58	47.40
18	Belgium	71	1,440	22	20.3	0	3	10	6.13	124.25
19	South Africa	68	265	9	3.9	0	0	0	1.15	4.47
20	Czech Republic	65	436	12	6.7	0	0	1	6.07	40.71
21	Hong Kong	61	1,495	18	24.5	2	3	6	8.14	199.41
22	Singapore	50	660	13	13.2	0	1	3	8.55	112.81
23	Switzerland	49	1,506	17	30.7	3	6	9	5.66	174.01
24	Pakistan	47	225	7	4.8	0	0	1	0.21	1.02
25	Nigeria	43	121	6	2.8	0	0	0	0.21	0.59

Note: Abbreviations available in previous tables.

TABLE 10 Most productive journals in ERV

Web of science								
R	**Journal**	**TP**	**TC**	**H**	**C/P**	**≥200**	**≥100**	**≥50**
1	Journal of International Money and Finance	183	4,219	37	23.1	1	5	24
2	Applied Economics	103	671	13	6.5	0	0	2
3	Journal of International Economics	79	3,693	32	46.7	3	10	24
4	Economic Modeling	74	550	13	7.4	0	0	0
5	Physica A Statistical Mechanics and its applications	56	473	14	8.4	0	0	0
6	Applied Economics Letters	50	205	8	4.1	0	0	0
7	International Journal of Finance Economics	44	428	10	9.7	0	1	1
8	International Review of Economics & Finance	42	548	10	13.0	0	1	2
9	Economics Letters	41	508	13	12.4	0	0	2
10	Open Economies Review	38	195	7	5.1	0	0	0
11	Journal of Banking & Finance	37	798	15	21.6	0	0	4
12	Journal of Forecasting	35	566	13	16.2	0	0	3
13	Emerging Markets Finance and Trade	31	129	7	4.2	0	0	0
14	International Review of Financial Analysis	31	248	9	8.0	0	0	0
15	journal of International Financial Markets Institutions Money	31	160	8	5.2	0	0	0
16	Journal of the Japanese and International Economies	30	729	13	24.3	1	1	3
17	Energy Economics	28	783	16	28.0	0	2	5
18	Journal of Econometrics	28	3,032	22	108.3	3	10	16
19	North American Journal of Economics and Finance	27	157	7	5.8	0	0	0
20	Journal of Monetary Economics	26	1,534	15	59.0	1	5	9
21	Journal of Macroeconomics	25	197	8	7.9	0	0	0
22	World Economy	24	225	8	9.4	0	0	1
23	Japan and the World Economy	23	175	8	7.6	0	0	0
24	European Economic Review	22	593	11	27.0	1	1	2
25	Journal of Economic Dynamics & Control	22	359	8	16.3	0	0	1
Scopus								
R	**Journal**	**TP**	**TC**	**H**	**C/P**	**≥200**	**≥100**	**≥50**
1	Journal of International Money and Finance	166	3,427	37	20.6	3	10	50
2	Applied Economics	93	646	14	6.9	0	0	3
3	Journal of International Economics	63	2,867	31	45.5	2	10	25
4	Applied Financial Economics	60	440	11	7.3	0	0	0
5	Economic Modelling	59	491	14	8.3	0	0	0
6	Journal of International Financial Markets Institutions and Money	58	819	18	14.1	0	0	2
7	Applied Economics Letters	53	265	9	5.0	0	0	1
8	International Journal of Finance and Economics	46	622	13	13.5	0	1	2
9	Physica A Statistical Mechanics and its Applications	44	497	13	11.3	0	1	2
10	International Review of Economics and Finance	43	493	13	11.5	0	1	2
11	Economics Letters	40	570	14	14.3	0	1	4
12	Journal of Banking and Finance	38	1,554	20	40.9	2	3	10
13	Open Economies Review	37	207	8	5.6	0	0	0
14	International Review of Financial Analysis	31	356	9	11.5	0	0	2

(Continues)

TABLE 10 (Continued)

Scopus								
R	**Journal**	**TP**	**TC**	**H**	**C/P**	**≥200**	**≥100**	**≥50**
15	Journal of Monetary Economics	30	1,888	16	62.9	3	9	11
16	Research in International Business and Finance	29	377	10	13.0	0	0	2
17	Journal of Econometrics	27	2,657	15	98.4	4	9	12
18	Journal of the Japanese and International Economies	26	796	14	30.6	1	2	3
19	North American Journal of Economics and Finance	26	181	9	7.0	0	0	0
20	Economics Bulletin	25	54	4	2.2	0	0	0
21	International Economic Journal	25	62	5	2.5	0	0	0
22	Journal of Forecasting	25	512	11	20.5	0	2	4
23	Journal of Asian Economics	24	187	9	7.8	0	0	0
24	Journal of Empirical Finance	23	1,111	14	48.3	1	3	7
25	Journal of Applied Econometrics	22	1,770	15	80.5	3	5	9

Note: Abbreviations available in previous tables.

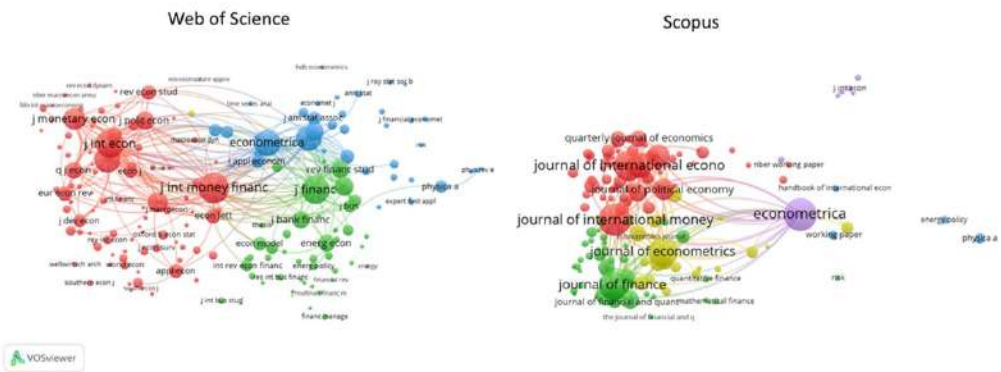


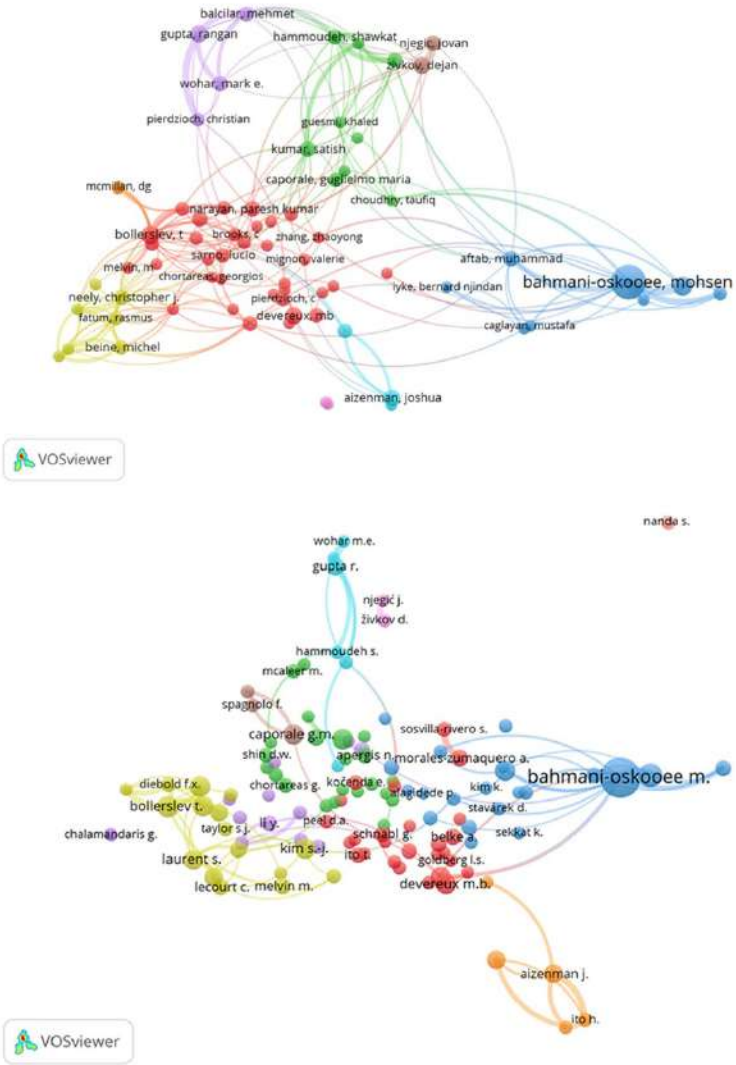
FIGURE 3 Co-citation of journal cited in ERV [Colour figure can be viewed at wileyonlinelibrary.com]

transmissions in financial markets has been more popular.

Another interesting issue is to consider the most productive and influential university. Note that the university select is the last institution with which the author has published. Table 7 shows the results, where the University of Wisconsin-Milwaukee is the most productive university in ERV by both databases. This is consistent with the most productive author since he belongs to that university. Another university with a presence is the National University of Singapore, which has among its most popular publications works to the GARCH modelling. Duke University is another of the most influential universities in the topic, whose main subjects of study are modelling and forecasting since it is the University of Bollerslev. The USA, England and Australia are the countries with more universities in the table.

ERV is an important topic in financial analysis, then we also study the institutions most productive that no are university. One can see in Table 8 that the National Bureau of Economic Research lead the list with 87 publications in WoS and 65 papers in Scopus. This institution has collaborated in recent years with influential authors on the topic, such as Andersen, Bollerslev and Aizenman. The second place is for the International Monetary Fund (IMF) with 70 papers published in WoS and 64 publications in Scopus. The IMF has a wide range of collaborating authors around the world, whose contribution to the

FIGURE 4 Bibliographic coupling of authors publishes in ERV [Colour figure can be viewed at wileyonlinelibrary.com]



institution is to a lesser extent. Noted that the first places are for institutions policy markets in influential countries.

Next, let us analyse the countries with most publishing in ERV. Table 9 shows that the USA is the most productive country in both databases, where the main categories of study are economics, business finance and social science mathematics methods. The list is followed by the United Kingdom in WoS and Australia in Scopus. If we considering papers per million inhabitants the most productive countries are Australia, Singapore and Greece. While in cites per million inhabitants the United Kingdom, Switzerland and Singapore top the list.

Finally, let us summarize the journals that have the most publications in ERV. Table 10 presents the results. One can see that the first three places are the same in both databases WoS and Scopus. Journal of International Money and Finance leads the list with 183 publications in WoS and 166 in Scopus. It is followed by Applied Economics and the Journal of International

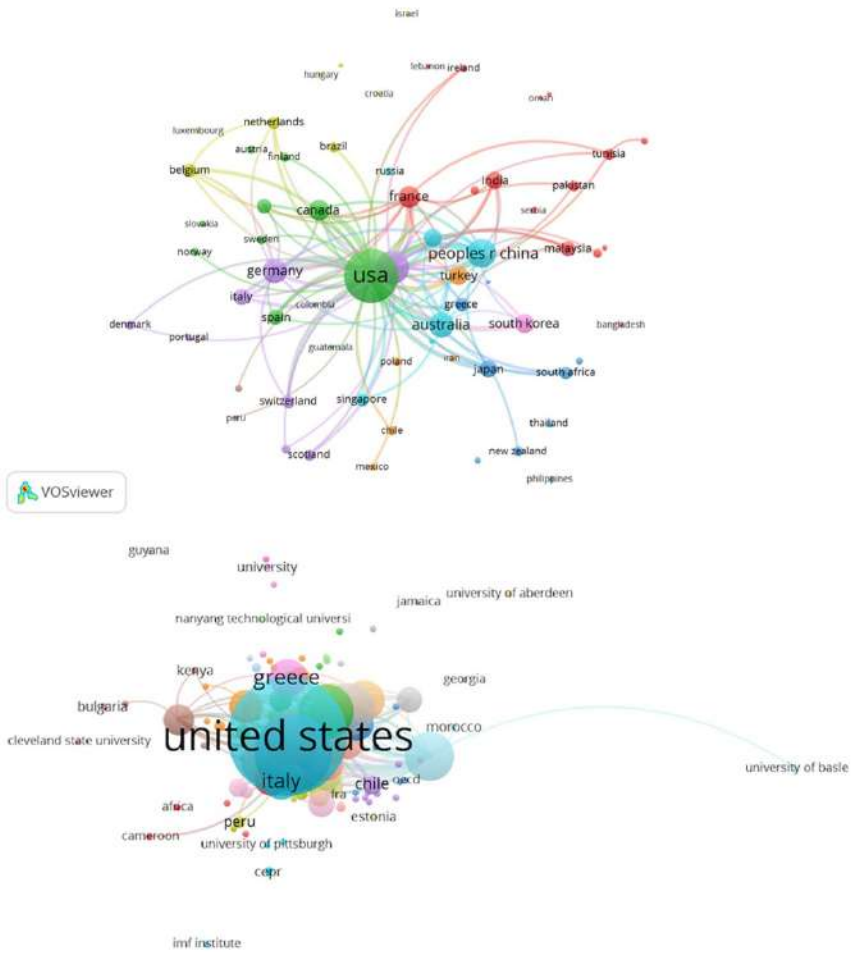


FIGURE 5 Bibliographic coupling of countries that publishing in ERV [Colour figure can be viewed at wileyonlinelibrary.com]

Economics. Other journals with high impact H-index included in the list are the Journal of Econometrics, Journal of Banking and Finance and Journal of Monetary Economics.

4 | GRAPHICAL ANALYSIS OF INFORMATION SCIENCES WITH VOS VIEWER

In this section, some VOS Viewer (Van Eck & Waltman, 2010) graphical maps are presented. The analysis is made in the index as co-citation, bibliographic coupling and co-occurrence with the objective to know the influence and relationship of journals, publications of authors, institutions, countries and keywords.

First, let us analyse the co-citation of journals cited in ERV. Co-citation occurs when two documents published in different journals receive a citation from the same third source (Small, 1973). Figure 3 shows the comparison co-citation maps of WoS and Scopus. One can see that the Journal of International Money and Finance is influential on the topic as we saw in the previous section. Co-citations are grouped according to the object of study in each journal, financial economic groups and mathematical are identified.

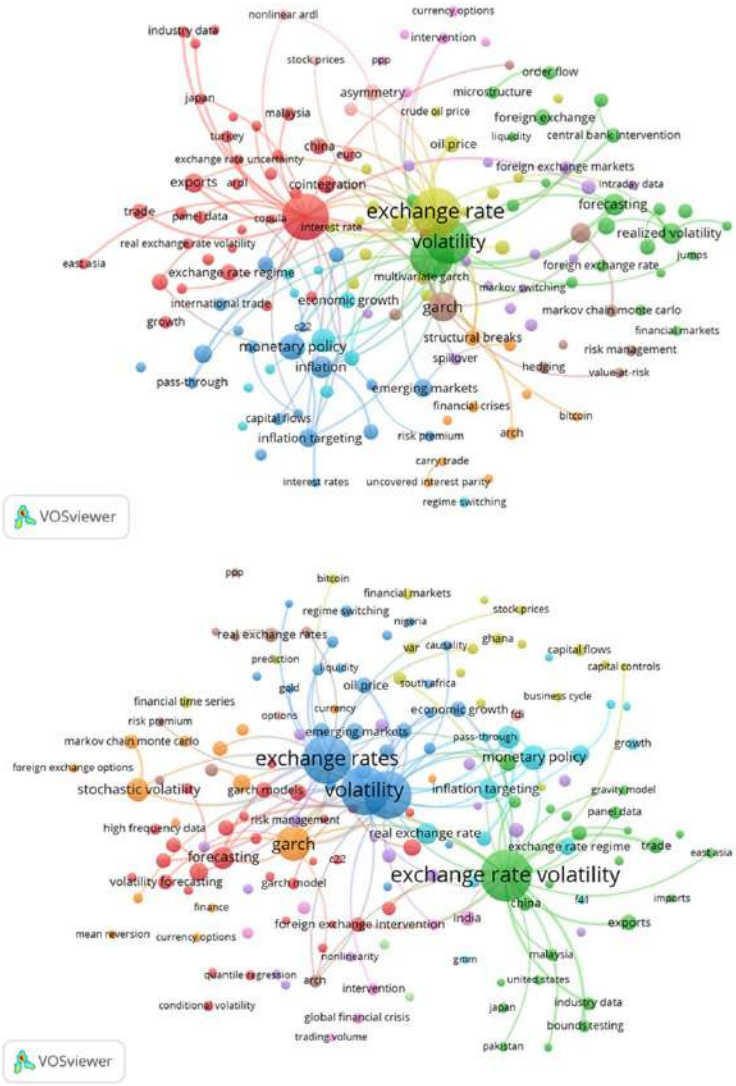


FIGURE 6 Co-occurrence of author keywords of documents published in ERV [Colour figure can be viewed at wileyonlinelibrary.com]

Another interesting issue is to study the bibliographic coupling of author publishing in ERV. Bibliographic coupling of authors occurs when the authors of two documents cite the same third document (Kessler, 1963). Figure 4 verifies that Bahmani-Oskooee is the most productive author according to Table 5. Note that the author that connects are those that have similar research profiles. The first graphic belongs to WoS and the second to Scopus.

In order to analyse the results per country, Figure 5 shows the bibliographic coupling. One can verify that the United States is the country with the most publications in ERV. Countries form clusters around their work relationship, the map shows how this organization tends to be by geographic areas.

The words used by the authors are presented by the co-occurrence of author keywords described in Figure 6. In addition to 'exchange rate volatility', other words associated

TABLE 11 Most common author keywords occurrences in ERV

Web of Science												
Global Keyword				**1990–1999**			**2000–2009**			**2010–2019**		
R	**Keyword**	**Oc**	**Co**	**Keyword**	**Oc**	**Co**	**Keyword**	**Oc**	**Co**	**Keyword**	**Oc**	**Co**
1	Exchange rate	244	182	Exchange rates	20	13	Exchange rates	59	38	Exchange rate	195	130
2	Exchange rate volatility	216	159	Volatility	9	5	Exchange rate volatility	50	30	Exchange rate volatility	159	99
3	Volatility	197	166	Garch	8	5	Exchange rate	47	34	Volatility	145	106
4	Exchange rates	187	142	Exchange rate volatility	7	3	Volatility	43	32	Exchange rates	108	69
5	Garch	86	68	Arch	4	3	Garch	26	18	Garch	52	36
6	Monetary policy	64	46	Forecasting	4	3	Forecasting	17	15	Monetary policy	47	33
7	Real exchange rate	56	42	Currency substitution	3	3	Monetary policy	16	7	Real exchange rate	42	30
8	Forecasting	50	42	Stochastic volatility	3	3	Realized volatility	15	13	Inflation	39	32
9	Inflation	47	41	Central bank intervention	2	1	Exchange rate regimes	14	6	Exports	29	25
10	Realized volatility	44	34	Chaos	2	1	Real exchange rate	14	9	Forecasting	29	22
11	Stochastic volatility	42	25	Correlation dimension	2	1	Stochastic volatility	14	8	Inflation targeting	29	21
12	Exports	36	31	Exchange rate	2	1	Foreign direct investment	11	6	Realized volatility	29	19
13	Cointegration	32	26	Exchange rate regime	2	2	Central bank intervention	10	9	Emerging markets	26	19
14	Emerging markets	32	25	Exchange rate returns	2	1	Foreign exchange	10	6	Financial crisis	26	19
15	Exchange rate regimes	32	18	Exchange rate variability	2	1	Market microstructure	10	8	Oil price	26	20
16	Inflation targeting	32	25	Generalized method of moments	2	2	Arch	8	4	Asymmetry	25	23
17	Financial crisis	29	23	Heteroscedasticity	2	1	Econophysics	8	3	China	25	21
18	Long memory	29	21	Heteroskedasticity	2	2	Exchange rate regime	8	3	Cointegration	25	18
19	China	28	25	Leptokurtosis	2	2	Foreign exchange intervention	8	5	Stochastic volatility	25	13
20	Exchange rate regime	28	16	Market efficiency	2	2	Inflation	8	6	Economic growth	24	17
Scopus												
Global Keyword				**1990–1999**			**2000–2009**			**2010–2019**		
R	**Keyword**	**Oc**	**Co**	**Keyword**	**Oc**	**Co**	**Keyword**	**Oc**	**Co**	**Keyword**	**Oc**	**Co**
1	Volatility	2	2	Exchange rates	35	35	Exchange rates	85	46	Exchange rate volatility	235	68
2	Currency options	1	1	Volatility	16	16	Volatility	83	57	Exchange rate	211	117
3		1	1		14	14		70	30	Volatility	187	115

TABLE 11 (Continued)

Scopus												
Global Keyword				1990–1999			2000–2009			2010–2019		
R	Keyword	Oc	Co	Keyword	Oc	Co	Keyword	Oc	Co	Keyword	Oc	Co
	Exchange rates			Exchange rate volatility			Exchange rate volatility					
4	Forecasting	1	1	f31	11	10	Exchange rate	64	35	Exchange rates	170	70
5	Foreign exchange	1	1	Garch	7	7	Garch	55	37	Garch	79	51
6	Futures prices	1	1	Stochastic volatility	7	7	Stochastic volatility	25	9	Monetary policy	54	29
7	Immunization	1	1	c22	6	6	Exchange rate regimes	18	7	Inflation	52	31
8	Maturity gaps	1	1	Forecasting	6	6	Monetary policy	18	8	Inflation targeting	41	27
9	Options	1	1	Arch	5	5	Forecasting	17	14	Real exchange rate	39	20
10	Statistical models	1	1	Exchange rate	5	5	Central bank intervention	16	11	Forecasting	35	24
11	Swap rate	1	1	Conditional volatility	4	4	Realized volatility	16	10	Stochastic volatility	33	11
12	–			g14	4	4	Real exchange rate	15	5	Emerging markets	32	14
13	–			International trade	4	4	Foreign exchange	13	10	Volatility spillover	29	17
14	–			Arch models	3	3	Long memory	13	9	Oil price	28	19
15	–			c53	3	3	Trade	13	9	Stock market	28	19
16	–			Cointegration	3	3	Foreign exchange intervention	12	11	Economic growth	26	21
17	–			g15	3	3	f31	11	4	Financial crisis	26	12
18	–			Heteroscedasticity	3	3	High-frequency data	11	10	India	26	17
19	–			Heteroskedasticity	3	3	Implied volatility	11	7	Realized volatility	26	8
20	–			Leptokurtosis	3	3	Inflation	11	7	Exports	25	21

with great influence are GARCH, monetary policy, inflation, forecast, stochastic among others. In both databases, the clusters formed are consistent with the objective of forecasting study, measurement, impact on other variables and causes.

In order to analyse the keywords of authors in more depth, Table 11 shows the 20 keywords most used in the topic in both databases WoS and Scopus. Additionally, the table analyses the most common words through time by four periods. The exchange rate has been the most keyword used in all periods. The evolution of the main keywords by periods is consistent with the evolution of the study in ERV. In the 1990s, ARCH-GARCH and stochastic forecasting studies were mostly attended. For the next decade the forecast remains important, but terms such as monetary policy, realized volatility and market microstructure appear on the list. In recent years, the terms related to monetary policy have become relevant, additionally, new relevant topics appear such as emerging markets, asymmetry, cointegration and volatility spillovers.

5 | CONCLUSION

Exchange rate volatility is an important financial topic studied by companies, investors and decision makers. Because volatility is a phenomenon with mainly effects on investment decisions, it has been investigated in terms of its measurement, estimation and forecast. In addition,

ERV has been studied based on its relationship with other financial and economic variables and the use for the development of knowledge.

The work proposes an analysis of authors, institutions, countries and journals that work in ERV using comparative tables and graphical visualizations. The data come from the WoS database and Scopus database. Additionally, graphical maps are made with the Vos Viewer software. The graphical analysis uses bibliographic coupling, co-citations and co-occurrence, where we can to analyse clusters and leading trends.

One can see the importance of the topic for the number of results and the impact of the first positions in each table. The ERV study in the four decades presented has evolved from an analysis of the economic fundamentals, modelling and estimation and the causes generated in a globalized world with a variety of agents interacting. Among the influential agents in the topic are Bahmani-Oskooee in author, the University of Wisconsin-Milwaukee in university, the National Bureau of Economic Research as the most productive institution, the EU as the most influential country.

Additionally, the work analyses the clusters and connections formed by authors, journals and countries. It also shows how ERV has been studied from mathematical, financial and economic approaches over time. The main words used by the authors on the topic have been listed globally and over time.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available in web of science and Scopus official website. These data were derived from the following resources available in the public domains:

- http://apps.webofknowledge.com.dti.sibucsc.cl/WOS_GeneralSearch_input.do?product=WOS&search_mode=GeneralSearch&SID=7BsA4f7IziWE2vG3ldM&preferencesSaved=
- https://www-scopus-com.dti.sibucsc.cl/search/form.uri?display=basic

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