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Pronóstico del precio futuro del maíz mediante la implementación del operador de agregación *BON-POWA*

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Capítulo I. Introducción

1.1. Introducción

Todas las operaciones dentro de la empresa tienen un riesgo, provocado por continuos cambios en la demanda, tipos de cambio, tasas de interés, competidores nuevos, inflación, alta volatilidad, entre otros (Oosterhof, 2001; Al-Momani & Gharaibeh, 2008; Kapusuzoglu & Karacaer, 2015). Así mismo, la operación es afectada por diferentes acontecimientos a nivel nacional e internacional, dado esto, Mayorga y Solarte (2012) señala que es necesario entender y gestionar las diferentes clases de riesgos que surjan durante la operación, por tal razón, Bolaño et al (2014) sugieren el desarrollo de una división de gestión de riesgo como parte de los procesos administrativos y organizacionales.

De acuerdo con Xiong et al (2015) la permanencia de las empresas en el mercado depende directa o indirectamente de los precios de activos financieros, por tal razón, establecer diferentes escenarios con el fin de gestionar el riesgo por variaciones en el mismo, se ha vuelto una actividad relevante. Según Jankelova (2017), tal incertidumbre ha aumentado por la liberación de mercados y la alta especialización en los sistemas de producción. Por su parte, Cortina (1995), Knigh (2006), Professionals (2011) y Deloitte (2015) comentan que, el riesgo tiene diferentes formas y características, pero coinciden que el riesgo en precio es uno de los más significativos, al ser el determinante principal de oferta y demanda (Rossi, 2013).

Uno de los sectores donde el riesgo en precio se encuentra presente de una forma significativa es el agrícola (McPhail, Du, & Muhammad, 2012). Dentro de los productos agrícolas más importantes a nivel mundial se encuentra el maíz (FAO, 2019), siendo México el tercer importador nivel global (Trade Map, 2019). De tal modo que, gestionar el riesgo en precio de este producto es relevante para los diversos productores y compradores a nivel nacional. Sin embargo, aun utilizando diferentes tipos de metodologías, los niveles de error de pronóstico continúan siendo elevados dificultando el proceso de toma de decisiones y diseño estratégico (Kenyon, Jones & McGuirk, 1993; Kantanantha, Serban & Griffin, 2010; Adjemian & Smith, 2012).

En escenarios caracterizados por la incertidumbre, señalan Kaufmann & Gil Aluja (1990) y Alfaro-Garcia (2019) que se requieren herramientas que apoyen el proceso de toma de decisiones y metodologías que generen resultados adecuados ante las expectativas de los mercados, desarrollando y definiendo mejores estrategias.

Por lo anteriormente señalado, esta tesis de investigación se centró en determinar la efectividad de los modelos basados en operadores de agregación de información Ordered Weighted Average (OWA) para pronosticar el precio futuro del maíz a comparación de los precios futuros observados en la Chicago Board of Trade (CBOT) mediante la incorporación de las expectativas de los expertos, probabilidades e interrelación de los datos que conforman la información. Esta propuesta es una combinación de las medias Bonferroni (Bonferroni, 1950), operadores *Ordered Weighted Average* (OWA) (Yager, 1988) y operadores probabilísticos (Merigó, 2009). Dentro de los resultados se pudo identificar que este nuevo operador disminuye el error de pronóstico, siendo una herramienta que contribuye a la toma de decisiones en mercados inciertos y volátiles.

1.2. Planteamiento del problema

La gestión del riesgo en precio se centra en la reducción del error en el pronóstico, con la finalidad de disminuir el impacto que la volatilidad pueda tener en los ingresos y costos de los oferentes y demandantes (Ghosh, 1993; Andersen et al., 2003; McPhail, Du, & Muhammad, 2012). Por lo tanto, dentro de las estrategias corporativas se deben encontrar la construcción de escenarios basados en modelos teóricos y herramientas para la simulación de la incertidumbre del mercado (Tufano, 1996; Cárcamo & Franco, 2012, Gaudenzi et al., 2018).

Dentro de los *commodities*[1], se han realizado investigaciones donde se han desarrollado modelos de estimación de precios. Algunas de estas son la Nelson-Siegel (Cortazar, Schwartz, & Naranjo, 2007), modelo Gaussiano (Casassus & Collin-Dufresne, 2005) y

[1] Se denomina *commodities* a aquellas materias primas o bienes de consumo que cuentan con un valor y utilidad. Suelen ser productos genéricos y que se caracterizan además por su uniformidad y por no contar con una clara diferenciación entre sí.

Markov chain Monte Carlo (MCMC) (Chiang, Ethan, Hughen, & Sagi, 2015), sin embargo, ninguna de estas ha presentado errores de pronóstico aceptables.

En el sector agrícola existen diversos *commodities,* uno de los más relevantes a nivel mundial es el maíz. En 2017, alrededor de 375 millones de toneladas de este grano se produjeron en Estados Unidos (EEUU), seguido por China con 259 millones y en quinto lugar México con 28 millones (FAO, 2019). Según datos del portal Trade Map (2019) EEUU en 2018 participó con el 39% del total de las exportaciones a nivel global. Por otra parte, México ha sido uno de los países que observó mayor dependencia a las importaciones de maíz, dado que la producción local no satisface la demanda (González-Rojas, et al, 2011). Según la Organización de las Naciones Unidas para la Alimentación y la Agricultura (FAO por sus siglas en inglés) (2019), México paso del tercer lugar (2009) al segundo lugar (2018) en importación de maíz a nivel global (Trade Map,2019).

Autores como Agnolucci (2009), Chen, Rogoff, & Rossi (2010) y Mensi et al. (2013), han investigado sobre las estimaciones de precio de los *commodities* en base a sus determinantes macroeconómicas, sin obtener resultados estadísticamente aceptables, los cuales presentan una R-cuadrada muy baja (Kalari & Power, 2013). En cambio, García-Ascanio & Maté (2010) y Cheung (2007) se han enfocado en modelos de alta eficiencia como el vector auto-regresivo (VAR) y el de error vectorial de correlación (VECM), que al ser métodos estadísticos tradicionales funcionan eficientemente en estudios lineales y no así en estudios no lineales, como es el caso de los productos del sector agrícola (Xiong, Li, Bao, Hu, & Zhang, 2015).

Por su parte, Giot & Laurent (2004), Martens, Van Dijk, & De Pooter (2009) y Lieberman & Phillips (2008) afirman que los modelos de series de tiempo son más eficientes y permiten detectar estacionalidades en series muy grandes. En contraste, Tian, Yang, & Chen (2017) comentan que el error de pronóstico es amplio debido a su incapacidad de captar el valor de las variables en el tiempo y por lo tanto, cuentan con coeficientes de predicción débiles. Adicionalmente, Clinton y Lewis (2007) comentan que estas técnicas solo toman en cuenta información histórica sin considerar expectativas de los expertos.

Estas condiciones de incertidumbre generan la necesidad de proporcionar herramientas que auxilien en la toma de decisiones y estrategias (Kaufmann & Gil-Aluja, 1990) y siendo la

volatilidad del precio de los productos agrícolas un riesgo que debe ser precisado y racionalizado mediante estrategias que permitan reducirlo a niveles aceptables, es importante generar modelos para conocer el valor futuro con niveles de error reducido.

Ante esta problemática, la presente investigación desarrolla nuevos operadores de agregación de información que permiten incorporar la incertidumbre y subjetividad de los mercados actuales en modelos de pronóstico que tomen en cuenta las expectativas, conocimiento y experiencias de los tomadores de decisiones.

1.3. Objeto estudio de investigación

En el sector agrícola uno de los granos más importantes es el maíz, ocupando el tercer lugar en producción después del trigo y el arroz (Asturias, 2004). Su importancia deriva de los diferentes usos que este asume, donde según la Organización de las Naciones Unidas para la Alimentación y la Agricultura (FAO) (2019), se utiliza principalmente para consumo humano y de alimento para el ganado, el Servicio de Información Agroalimentaria y Pesquera (SIAP) agrega otros usos, tal es la producción de biocombustible, y como ingrediente para la elaboración de más de 4,000 artículos como jarabes, aceites, harinas, féculas, jugos, refrescos, pinturas, pasta para dientes, papel, cosméticos (SIAP, 2019), plásticos y medicamentos, entre otros productos (González & Ávila, 2014). El top 15 de países exportadores e importadores se encuentra en la Figura 1.

Figura 1. Top 15 de exportadores e importadores de maíz
Elaboración propia con datos de Trade Map, 2019

Uno de los problemas que se enfrentan a la par los compradores y productores de maíz es la volatilidad del precio (McKenzie, 2008). Estudios realizados por Petersona & Tomekb (2005) señalan que el valor de los productos agrícolas varía debido a calidad, plazos de entrega, tasas de interés, ubicación, oferta y demanda, a estos factores se les adiciona la variable de estacionalidad, ya que en periodos de cosecha los precios se comportan centrados y se van dispersando los datos a medida que se aleja de la etapa de cosecha, la volatilidad del precio del maíz puede observarse en la Figura 2.

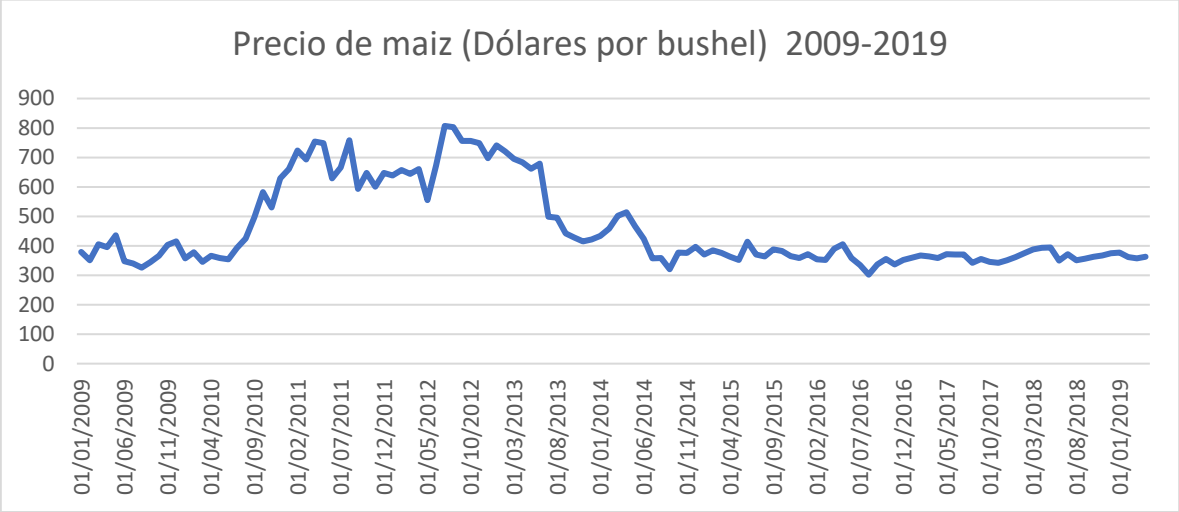

Figura 2. Comportamiento del precio del maíz 2009-2019
Elaboración propia con datos de www. Barchart.com (2019)

1.4. Formulación del problema de investigación

Considerando la importancia de la producción del maíz y las deficiencias de los modelos de pronóstico para estimar su precio, es necesario desarrollar nuevas metodologías que permitan gestionar el riesgo y ayuden en la toma de decisiones de los diferentes actores económicos (Harwood et al, 1999), esto con el fin de lograr administrar la incertidumbre que genera la volatilidad de los precios y que afecta directamente los resultados al final del periodo de cosecha y comercialización (Bjornson & Carter, 1997).

Por tal razón, Riazi (2016) sugiere que los involucrados en esta actividad, tengan conocimiento sobre las causas de la volatilidad de los precios de los productos agrícolas. Dentro de las causales de fluctuación del precio del maíz se encuentra la oferta y la demanda (Chambers & Bailey, 1996; Lanteri, 2012), tipo de cambio (Gilbert, 2010; McPhail, Du, &

Muhammad, 2012; Arteaga, Granados, & Joya, 2013), precio del petróleo (FAO, 2019; Mitchel, 2008), por mencionar algunos.

La ineficiencia de los modelos de pronóstico tradicionales ha generado la necesidad de buscar nuevas técnicas de estimación de precio de maíz (Kawai, 1983; Just & Pope, 2003), los modelos que no toman en cuenta las expectativas de mercado y las opiniones de los tomadores de decisión no presentan un buen ajuste de predicción del precio (Clinton & Lewis, 2007). Por tal razón, la implementación de modelos que tomen en cuenta las expectativas y opiniones de expertos, permiten responder de forma rápida a los cambios del mercado, generando metodologías flexibles y adaptables a las necesidades del entorno caracterizado por su volatilidad (Gil-Lafuente, 2005).

Dentro de las metodologías que permiten la incorporación de las expectativas y conocimiento de los tomadores de decisión dentro de las formulaciones se encuentran las relacionadas con operadores de agregación de información. De acuerdo con Blanco-Mesa, Leon-Castro & Merigo (2019) el operador más importante y citado dentro de esta área es el *Ordered Weigthed Average* (OWA) desarrollado por Yager (1988). Su principal característica es que permite mediante el uso de un vector de peso y un proceso de reordenamiento obtener de forma parametrizada el resultado máximo y mínimo de una problemática. Además, mediante las variaciones que puede tener el vector de peso incorporar las expectativas del experto dentro de la temática a tratar.

Desde la formulación del operador OWA, se han generado una importante cantidad de nuevas formulaciones, las cuales buscan adicionar diferentes herramientas para el tratamiento de la incertidumbre y mejorar los procesos de toma de decisión en escenarios volátiles. Una de estas es la que conjuga las medias Bonferroni (1950), este operador llamado Bonferroni OWA (Yager, 2009) y tiene como principal característica un proceso de interrelación entre las variables, además del vector de peso y reordenamiento característico del operador OWA. Por otro lado, el operador probabilístico OWA (Yager, Engemann & Filev, 1995; Merigo, 2010) es un operador que permite incluir en una sola formulación información objetiva a través del vector de probabilidades e información subjetiva a través del vector de peso.

El operador OWA y las extensiones que se mencionan con anterioridad son metodologías que permiten la inclusión de información subjetiva, expectativas y conocimiento de los

tomadores de decisores en una formulación matemática, permitiendo hacer frente a los escenarios inciertos de los mercados actuales. Ante esto, son herramientas útiles para el pronóstico de los precios futuros de los *commodities*, caracterizados por una alta volatilidad. En base a esto, se plantea la pregunta central de investigación.

¿Cuál es la efectividad de los modelos basados en operadores de agregación de información *Ordered Weighted Average (OWA)* para pronosticar el precio futuro del maíz a comparación de los precios futuros observados en la Chicago Board of Trade (CBOT) mediante la incorporación de las expectativas de los expertos, probabilidades e interrelación de los datos que conforman la información?

Adicionalmente se plantea una pregunta auxiliar

¿Cuál es la formulación matemática de un operador de agregación de información basado en la media Bonferroni, operador *Ordered Weighted Average* y operadores probabilísticos?

1.5. Justificación

El riesgo en precio, generalmente expresado como volatilidad, afecta las decisiones de los diferentes actores. En un contexto de toma de decisiones, la información sobre la volatilidad futura es particularmente importante para los participantes del mercado, ya que les permite evaluar las alternativas y posibles escenarios de sus productos. En los mercados de futuros, por ejemplo, la información sobre la volatilidad puede proporcionar a los participantes del mercado una comprensión de los costos y riesgos relativos de colocar y compensar coberturas durante diferentes períodos de tiempo (Egelkraut, Garcia & Sherrick, 2007).

Dentro de los modelos que se han desarrollado para pronosticar el precio en *commodities*, en específico del maíz, se encuentran aquellos basados en expectativas de diferentes situaciones como son los presentados por Mjelde et al. (1997) y Kogan, Salzar & Roytman (2012), cuyos modelos hacen énfasis en el análisis del clima para pronosticar el precio del maíz. Por otro lado, siguiendo los fundamentos de precios de mercado de Fama (1970) se encuentran aquellos basados en la idea de que el precio refleja la información disponible, por lo que no contar con la cantidad completa de información lleva a volatilidad en los precios y especulación de los participantes (Baur & Orazem, 1994; Karali et al., 2020),

La ineficiencia de los modelos de pronóstico tradicionales y la complejidad del fenómeno estudiado ha generado la necesidad de buscar nuevas técnicas de estimación de precio de maíz (Xu, 2018; Huang, Serra & Garcia, 2020; Jankovic, Kovačević & Jeločnik, 2020). Por tal razón, la implementación de modelos que tomen en cuenta las expectativas y opiniones de expertos, permitirán responder de forma rápida a los cambios del mercado, siendo así una metodología flexible y adaptable a las necesidades del entorno caracterizado por su volatilidad (Gil-Lafuente, 2005).

Una forma de dar respuesta a la complejidad e incertidumbre de los mercados actuales es mediante metodologías que utilicen operadores de agregación de información. Estas representan un conjunto de técnicas modernas que se emplean para una amplia gama de propósitos, incluyendo el análisis y pronostico en temas de economía, administración, finanzas e ingeniería. El propósito de estos operadores es unir diferentes piezas de información proporcionada por varias fuentes para llegar a una conclusión o una decisión (Detyniecki et al., 2000). Además, se asegurarán de que el proceso de toma de decisiones no omita e incluya información cualitativa y cuantitativa (Herrera & Martinez, 2000; Yu, 2015). Por tanto, son fundamentales para la fusión de información y son aplicados para combinar varios valores en un solo resultado (Beliakov, Pradera & Calvo, 2007; Rickard & Aisbett, 2013).

La presente investigación presenta un nuevo operador basado en el operador OWA (Yager, 1988), las medias Bonferroni (Bonferroni, 1950) y los operadores probabilísticos (Merigo, 2010) el cual tiene como característica presentar en una sola formulación diferentes atributos de la subjetividad e incertidumbre de los mercados actuales y mediante este pronosticar el precio futuro del maíz, el cual será contrastado con aquellos publicados por el CBOT y finalmente analizados a través de las técnicas de error de pronóstico error de porcentaje medio (MAPE por sus siglas en inglés), desviación absoluta media (MAD por sus siglas en inglés) y la desviación media cuadrada (MSD por sus siglas en inglés) permitiendo observar si la incorporación de dicha información subjetiva mejora o no el pronóstico en comparación con el precio real.

1.6. Objetivos e hipótesis

Objetivo General

Determinar la efectividad de los modelos basados en operadores de agregación de información Ordered Weighted Average (OWA) para pronosticar el precio futuro del maíz a comparación de los precios futuros observados en la Chicago Board of Trade (CBOT) mediante la incorporación de las expectativas de los expertos, probabilidades e interrelación de los datos que conforman la información

Objetivo específico

- Formular matemáticamente un operador de agregación de información basado en la media Bonferroni, el operador *Ordered Weighted Average* y operadores probabilísticos

Hipótesis

Los precios de los productos agrícolas tienen altos niveles de volatilidad. Tal variación impacta directamente a los ingresos y utilidades de productores, comercializadores y procesadores (McPhail et al, 2012; Jacks et al, 2009). Dado esto, es pertinente buscar nuevas formas de gestión, que permitan mejorar la toma de decisiones de los involucrados en esta actividad (Kawai, 1983; Just & Pope, 2003).

Según Casassus & Collin-Dufresne (2005), Cortazar et al, (2007) y Chiang et al, (2015), los modelos de estimación de precios son herramientas que han sido utilizadas para mitigar el riesgo. Giot & Laurent (2004), Martens et al. (2009) y Lieberman & Phillips (2008) proponen diferentes herramientas estadísticas para la estimación de precios, sin embargo, Tian et al. (2017) comenta que este tipo de metodologías presentan errores de pronóstico amplio debido a que solo toman en cuenta datos históricos y dejan de lado conocimientos del mercado y expectativas del tomador de decisión. Considerar las expectativas del mercado y la experiencia de los tomadores de decisión dentro de diferentes problemas financieros y económicos a través del uso de operadores OWA y sus extensiones ha mostrado ser una metodología efectiva ante la incertidumbre y subjetividad de la información (Merigó & Gil-

Lafuente, 2010; Merigó & Casanovas, 2011; Yang & Chen, 2012; Lan et al., 2013; Espinoza-Audelo et al., 2019, Leon-Castro et al., 2019:2018)

Dado lo anterior se plantea la hipótesis de la investigación:

<< El pronóstico del precio del maíz basado en medias Bonferroni, operadores *Ordered Weighted Average* (OWA) y operador probabilístico OWA (POWA) disminuye el error de pronóstico en comparación del precio futuro publicado en la Chicago Board of Trade (CBOT) mediante la incorporación de las expectativas de los expertos, probabilidades e interrelación de los datos que conforman la información>>

1.7. Estructura y contenido

La presente tesis doctoral se encuentra estructurada en cinco capítulos, los cuales desarrollan de forma complementaria el tema objeto de investigación. A continuación, se describen cada uno de ellos.

En el capítulo 1 denominado introducción, detalla las generalidades de la investigación como son el planteamiento de problema, objeto estudio de investigación, formulación del problema de investigación, justificación, objetivos e hipótesis.

Dentro del capítulo 2 nombrado pronostico del precio del maíz y los operadores Ordered Weighted Average (OWA) se desarrollan las principales postulaciones teórico-metodologicas relacionadas con las principales temáticas de la presente investigación.

Por su parte el Capítulo 3 identifica el esquema metodológico que se ha utilizado para llevar a cabo la investigación y se presenta la nueva propuesta metodológica del operador Bonferroni Probabilístico Ordered Weighted Average (Bon-POWA).

Dentro del Capítulo 4 se presentan los principales resultados de la aplicación del operador Bon-POWA para el pronóstico del precio del maíz y su comparación con el precio de los futuros del Chicago Board of Trade (CBOT) a través del error de porcentaje absoluto, medio (MAPE por sus siglas en inglés), desviación absoluta media (MAD por sus siglas en inglés) y la desviación media cuadrada (MSD por sus siglas en inglés).

En el Capítulo 5 se presentan las principales conclusiones de la investigación, al igual que sus limitaciones y futuras líneas de investigación. Por último, se presentan las referencias utilizadas y dentro del apartado de apéndices los artículos publicados relacionados con el trabajo realizado.

La representación esquemática de la presente investigación es la siguiente (Ver Figura 3).

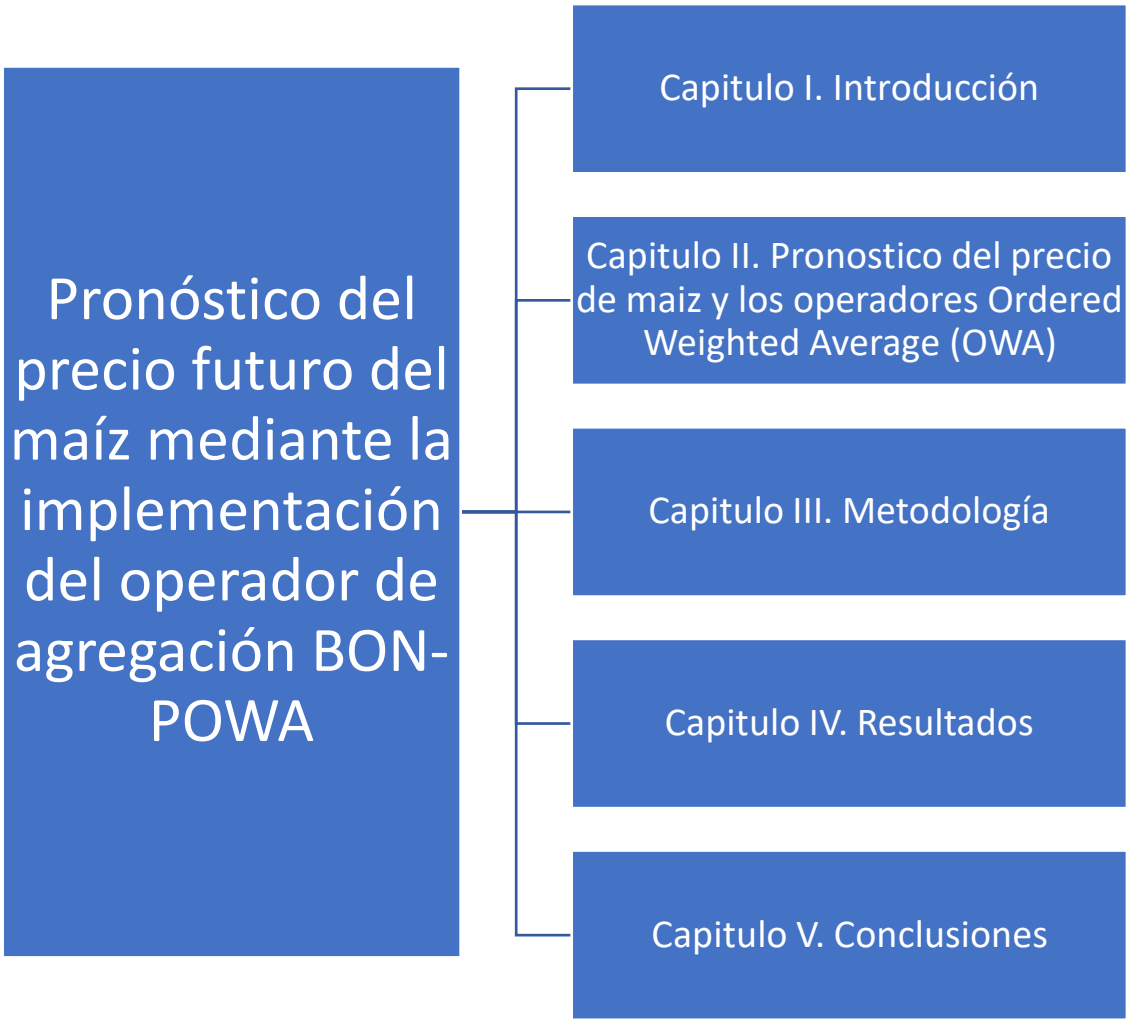

Figura 3. Esquema de estructura de investigación
Elaboración propia

Capítulo II. Pronostico del precio de maíz y los operadores Ordered Weighted Average (OWA)

2.1 Factores determinantes del precio del maíz

Las fluctuaciones de los precios son influenciadas por la oferta y la demanda del bien. En cuestión de productos básicos según Retes et al (2014) argumentan presencia de inelasticidad de la demanda, dado que según Tomek y Robinson (1990) el patrón de consumo no se modifica en igual proporción al alterar la oferta.

Según Chambers y Bailey (1996) un elemento importante en la estimación en una trayectoria temporal de precio del maíz es el conocimiento de la cantidad disponible del producto y el estado actual de la producción o perturbaciones de las cosechas (Chambers & Bailey, 1996).

En este sentido, la investigación realizada por (Lanteri, 2012) se observa que el inventario tiene una correlación inversa al precio del maíz, confirmando la hipótesis de Ye et al (2006) el cual sugiere la incorporación del inventario en ecuaciones mejora el ajuste y la capacidad de pronóstico del precio del grano, tal argumento lo comparten Abbott et al (2019).

Dado que la investigación de Lanteri (2012) analizó el precio del maíz tomando en cuenta diferentes variables tales como:

1. el índice de producción industrial de las economías avanzadas (prodind).
2. la tasa de interés real de corto plazo de los EE UU (irealus).
3. Tipo de cambio (neer).
4. Inventario (stock)
5. Precio real agrícola

La construcción de la ecuación se generó de la siguiente forma

$$Y_t = [\text{prodind}, \text{irealus}, \text{neer}, \text{stock}, \text{precio real agrícola}]$$

Arrojando mayor importancia el tipo de cambio y el nivel de inventario en el pronóstico de precio del maíz (Lanteri, 2012).[2]

[2] Se publicaron nuevas metodologías para el pronóstico del tipo de cambio MXN/USD en el journal Fuzzy Economic Review (Ver apéndice 1).

De igual manera, en estudios realizados en 2012 por Balbi, Kahlenberg y Martínez, comentan que, caídas en el nivel de inventario ocasionan que los precios tengan mayor sensibilidad ante movimientos en la oferta. Así mismo, (Lavarello & Selva, 2008) fortalecen esta teoría argumentando que al obtener el grano en una forma de producción rigida, éste no responde rápidamente al crecimiento de la demanda por su dependencia al ciclo biológico y que el precio está en función de la disponibilidad del producto.

Por otro lado, Ridler y Yandle (1972) demuestran que una depreciación del tipo de cambio debería de incrementar el precio de la materia prima, así mismo, Arteaga et al (2013) comentan que el tipo de cambio es un determinante en el precio dado que impacta directamente en el poder de compra de consumidores y la competitividad de los productores. De igual forma, Gilbert (2010) y McPhail et al (2012) comentan que una depreciación en la moneda estadounidense debería compensarse mediante un incremento en el precio del grano y una caída en los precios en otras monedas (Gilbert, 2010).

Gilbert desarrolla esto en la siguiente ecuación.

$$P^f = E * P$$

Gilbert explica la fórmula de la siguiente manera, donde P representa los precios de las materias primas en dólares, E el tipo de cambio nominal del dólar en términos de una unidad de otra divisa y P^f indica el precio de la materia prima en unidades de la divisa externa. De esta forma, una depreciación del dólar (una caída de E) se correspondería con un precio en dólares más elevado. La condición de arbitraje aseguraría que los precios en dólares se incrementen y que los precios en moneda externa se reduzcan.

Por su parte, Akram (2009) mediante modelos VAR obtiene que el tipo de cambio es altamente significativo para la estimación de los precios de productos agrícolas dado que esta variable explica en un 50% la variación de los precios.

Adicionalmente la literatura afirma que los hidrocarburos especialmente el petróleo tiene una influencia directa en los precios de los productos agrícolas en el corto plazo, debido al empleo de estos últimos en la producción de etanol, (FAO, 2008) así mismo, Mitchel (2008) asevera que existe relación positiva entre los precios de la energía, del maíz y de los fertilizantes y químicos, dado que la producción y transporte de esto últimos requiere utilización de

combustible en gran medida ocasionando un incremento en el costo de la energía y por consecuencia en el de transporte, esto incentiva a la producción de etanol, lo cual implica una mayor demanda de maíz por parte de la industria de biocombustibles.

2.2 Modelos de pronóstico del precio del maíz

Existe diferentes determinantes de la variación del precio en las materias primas agrícolas. Samuelson (1965) señala que los precios del contrato futuro se verán afectados por el tiempo de maduración, ya que al incorporar más información cerca de la expiración, la volatilidad del precio debe aumentar cuando el tiempo de madurez disminuye. Esta teoría la trabajan y amplían Anderson & Danthine (1983) y Streeter & Tomek (1992) al indicar que esto se debe a la relación con la oferta y demanda del producto, debido a que el inicio del contrato las necesidades de producción y demanda son inciertas, por lo que el precio se ajusta cuando la información se va publicando.

Bajo una propuesta similar se encuentra la determinación del precio basados en la razón inventario-uso (Anderson, 1985; Westcott & Hoffman, 1999). Los principales supuestos son que los inventarios se ajustan para responder a la oferta y demanda, por lo que estos disminuirán cuando haya una crisis en la producción, lo que generara mayores precios y aumentarán cuando la producción sea alta, lo que generara precios bajos. El total uso, es la suma del consumo local y las exportaciones, por lo que es generalmente estable y tiende a cambiar gradualmente con el tiempo.

De tal forma, Labys (1973), Westcott & Hoffman (1999) señalan que el modelo de equilibrio para esta teoría es: la oferta (S) se encuentra en función del precio (p) (o mejor dicho en la expectativa del precio futuro) y los desplazadores de la oferta (z) que reflejan crisis en producción, de tal forma que

$$S_t = s(p_t, z_t) \tag{1}$$

La demanda (D) esta en función del precio y otros desplazadores de la demanda (y), por lo que

$$D_t = d(p_t, y_t) \tag{2}$$

Los inventarios (K) son influenciados por el precio y otros posibles factores (v) que reflejan el costo de almacenaje y capacidad, así que

$$K_t = k(p_t, v_t) \tag{3}$$

El equilibrio del mercado requiere que $S_t - D_t - K_t = 0$. Esto reduce la expresión a que el precio se encuentra en función de los inventarios y desplazadores de oferta y demanda, de tal modo que

$$p_t = f(K_t, z_t, y_t) \tag{4}$$

Dentro de los desplazadores de oferta y demanda se encuentran:

a) Según Frenkel (2006) y Calvo (2008) aseguran que la implementación de política monetaria, a través de las tasas de interés y expectativas de inflación provocan cambios importantes en los inventarios de producto, lo que influencia cambios drásticos en la oferta y demanda y por ende en el precio. En este sentido, Frenkel & Andrew (2010) encuentran que posterior al año 2000 y con los recortes en tasas de interés real de la Reserva Federal de Estados Unidos de América el costo de mantener inventario disminuyo, lo que contribuyó al incremento en la demanda de producto para almacenaje, lo que llevo a un incremento en el precio.

b) El precio de la energía, ya que esto incrementa el costo de almacenaje, mantenimiento y otros relacionados con la entrega del producto a destino (Hamilton, 2009). Además, el precio del petróleo ha demostrado tener un impacto en los precios de los commodity agrícolas (Wang *et al.*, 2014; Serra & Zilberman, 2013; Zilberman *et al.*, 2012).

c) La especulación de los mercados financieros (Peck, 1981; Krugman, 2008; Wolf, 2008).

d) El clima y la lluvia (Brown & Kshirsagar, 2015; Mu, 2007; Hennessy & Wahl (1996).

En algunas investigaciones como las realizadas por Robinson y Ethridge (1988) probaron la estimación de precios del maíz con el método de regresión de mínimos cuadrados ordinarios en los cuales se tomaron como variables

$$lnPc' = a - X_1(SBc) + X_2(Uc) - X_3(PDc)$$

Pc'= Precio promedio de comercialización del maíz en EE. UU. dólares por tonelada métrica, (1985-86).

U.S.= GNP deflactor (1985-86 ~ 100).
Uc = Utilización mundial en el año de comercialización de maíz, millones de toneladas métricas.
PDc = Producción mundial de maíz en el año de comercialización, millones de toneladas métricas.
Ln= Logaritmo natural

Esta metodología también fue utilizada para pronósticos de precios de materias primas por (Roy & Ireland, 1975; Paddock, 1985; Johnson, et al, 1986) en procedimientos de dos y tres etapas.

Con la globalización, la incertidumbre incrementó en cuestiones de estimación de precios ya que aparecieron diferentes factores que no se tenían en cuenta y tampoco fácil acceso a estos datos, de esta manera hubo la transición en la utilización de modelos determinísticos a modelos estocásticos donde algunos elementos no se conocen con certeza incorporando la incertidumbre.

Por otro lado, ARIMA o Box Jenkins son modelos para serie de tiempos estadísticos que toman en cuenta la dependencia de los datos utilizados, es decir, cada dato calculado está en función de valores anteriores (Box & Jenkins, 1976), para operar esta herramienta es necesario seguir los siguientes pasos:

- Recolección de datos
- Graficar
- Transformación Logarítmica de la serie
- Eliminar de tendencias
- Identificar el modelo a elegir
- Estimación de coeficientes
- Validación del modelo
- Análisis de errores
- Selección del modelo
- Predicción

la expresión general de un modelo ARIMA (p,d,q) viene dada por:

$$\Delta^d y_t = \phi_1 \Delta^d y_{t-1} + \cdots + \phi_p \Delta^d y_{t-p} + e_t + \theta_1 e_{t-1} + \cdots + \theta_p e_{t-p}$$

donde $\Delta^d y_t$, expresa que sobre la serie original y_t, se han aplicado d diferencias, el valor de la variable y en el momento t es función del valor tomado en el período $t-1$, ϕ_1 es un parámetro que multiplica al valor de la variable y en el período $t-1$.

En 1998 Ntungo & Boyd realizaron estimaciones del precio de materias primas utilizando esta metodología. Analizaron el comportamiento de los rendimientos en marcos germanos, maíz y plata en el periodo 1974-1995, en 2016 Mutavdžić, et al utilizaron esta metodología para realizar pronósticos de precios de maíz y trigo con datos de 1994-2014.

Adicionalmente se han usado modelos de heterocedasticidad condicional autorregresiva generalizada (GARCH). Estos procedimientos estocásticos sirven para estimar la volatilidad, los cuales parten del modelo ARCH creado por Engle (1982), el cual desarrollo la formulación general de la siguiente manera:

$$y_t = \epsilon_t \quad donde\ \epsilon_t \,/\, \Psi_{t-1} \sim N(0, h_t)$$

$$u_t = \frac{\epsilon_t}{\sqrt{h_t}}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$$

Donde u_t es el residual estandarizado, y h_t el componente de volatilidad, en el cual α_1 representa la magnitud en la que se considera el error anterior en el periodo actual.

Bollerslev (1986) generalizó ese primer modelo y propuso el GARCH (Generalized autoregressive conditional heteroscedasticity), GARCH se representa con la formula siguiente:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

Según (Riazi, 2016) los productos agrícolas tienen precios con volatilidad variable en el tiempo, que es bien capturado por modelo GARCH. Sin embargo, hay pocos estudios que se ocupan de la estacionalidad de los productos agrícolas mediante la ejecución de esta metodología. También Yang y Brorsen (1993) utilizan esta metodología para probar la

varianza en los días de la semana los cuales determinan que los lunes son más volátiles y los miércoles son los que presentan menos variabilidad. Adicionalmente Hennessy y Wahl (1996) muestran que la estacionalidad afecta la volatilidad de los precios del maíz, la soja y el trigo.[3]

[3] Se publicaron nuevas metodologías para el pronóstico de la volatilidad dentro del journal Technological and Economic Development of Economy (Ver apéndice 2).

2.3 Operador *Ordered Weighted Average* (OWA)

El operador OWA fue introducido por Yager (1988). Proporciona una familia parametrizada de operadores de agregación que se han utilizada en muchas aplicaciones (Emrouznejad and Marra, 2014; Yager & Kacprzyk, 1997; Yager *et al*., 2011).

Entre las formas que existen para agregar información de la toma de decisiones en la operación matemática es el operador Ordered Weighted Average (OWA) (Yager, 1988). Las principales ventajas de este operador es que es posible modificar el promedio en diferentes criterios y entre un máximo y un mínimo de resultados. La definición es la siguiente.

A continuación, se da la definición del operador OWA introducida por Yager (1988).

Un operador OWA de dimensión *n* es una aplicación $F: R^n \to R$ con un vector de asociación de peso $w = [w_1, w_2, \dots, w_n]^T$ así que $w_j \in [0, 1]$, $1 \leq i \leq n$ y

$$\sum_{i=1}^{n} w_{i=} w_1 + w_2 + \cdots + w_n = 1,$$

donde

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j \, b_j,$$

siendo b_j el *j*th es el elemento más grande de la colección $a_1, a_2, \dots, a_n$ y $a_1, a_2, \dots, a_n$ son los argumentos que integran la formulación.

Obsérvese que se distingue la diferencia entre el operador OWA descendiente (DOWA) y el operador ascendiente OWA (AOWA). Los pesos de estos operadores está relacionado por $w_j = w^*_{n-j+1}$, donde w_j es el *j*th de peso del operador DOWA y w^*_{n-j+1} el *j*th peso del operador AOWA.

Las decisiones dentro de los operadores OWA se pueden dar bajo cuatro criterios

Criterio optimista. Se basa en el supuesto de que se presentara el estado más favorable, de tal modo que se debe seleccionar el resultado más favorable de cada alternativa y de los resultados obtenidos seleccionar el más favorable de todos. De tal modo que este criterio se basa en una máxima que se formula

$$Decisión = Max\{E_i\} = Max[Max\{Max\{a_j\}]$$

Criterio pesimista o de Wald. Sostiene que el decisor debe seleccionar la alternativa que proporcione mayor nivel de seguridad, de tal modo que nuestra decisión debe ser el resultado más favorable de entre los más desfavorables para cada alternativa. A este método se le conoce como max min y su fórmula es

$$Decisión = Max\{E_i\} = Max[Min\{a_j\}]$$

Criterio de Hurwics. Consiste en ponderar con un coeficiente optimista y otro pesimista al mejor y peor caso respectivamente, posteriormente se suman los valores y se escoge aquella alternativa que propone un mayor resultado. La fórmula de este criterio es

$$Decisión = Max\{E_j\} = Max[\alpha Max\{a_j\} + (1-\alpha)Min\{a_j\}]$$

Donde $\alpha + (1-\alpha) = 1$.

4. Criterio de Laplace. Se basa en el principio de razón insuficiente, de tal modo que se asocia un mismo grado de probabilidad a los distintos escenarios, siempre que no se presenten indicios de lo contrario. La fórmula es

$$Decisión = Max\{E_j\} = Max[({}^{1}/_{n})\sum_{j=1}^{n} a_j]$$

Hay otra extensión que añade más información a los resultados finales como probabilidades, este operador OWA probabilístico (POWA) operador. En este operador, hay dos diferentes vectores que se agregan a los resultados finales, uno es el vector de la ponderación tradicional operador OWA y el otro es un operador probabilístico que se basa en la probabilidad de que los acontecimientos sucedieron. Ambos están limitados a 1. La definición es la siguiente (Merigó, 2012).

Un POWA operador de dimensión n es un mapeo $POWA: R^n \to R$ tener asociado un vector de ponderación *P*, donde $p_i \in [0,1]$ y $\sum_{i=1}^{n} p_j = 1$, que se expresa de la siguiente manera

$$POWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \widehat{p_j}\, b_j, \qquad (4)$$

Dónde b_j está el elemento j th, el más grande de la colección $a_1 a_2, \dots, a_n$, donde cada argumento a_i se asocia con una probabilidad p_i donde $\sum_{i=1}^{n} p_i = 1$ y $p_i \in [0,1]$,$\hat{p}_j = \beta w_j + (1-\beta) p_j$ $\beta \in [0,1]$ y p_j es la probabilidad de que la p_i ordenada según b_j, según el *j*th elemento mayor de a_i.

Otra técnica utilizada en la presente investigación es la media Bonferroni (1950) cuya formulación es la siguiente:

$$B(a_1, a_2, \dots, a_n) = \left(\sum_{k=1}^{n} a_i^r \left(\frac{1}{1-n} \sum_{\substack{j=1 \\ j \neq i}}^{n} a_j^q \right) \right)^{\frac{1}{r+q}}, \qquad (1)$$

donde *r* y *q* son parámetros que $r, q \geq 0$ y los argumentos $a \geq 0$.

La media Bonferroni se puede combinar con los operadores OWA (Yager, 2009) lo que permite agregar información tomando en cuenta una serie de argumentos e incorporando esta información con los datos de los expertos, esta se define de la siguiente manera.

$$BON - OWA\,(a_1, a_2, \dots, a_n) = \left(\frac{1}{n} \sum_{i} a_i^r OWA_W\left(V^i\right) \right)^{\frac{1}{r+q}} \qquad (1)$$

donde $OWA_W\left(V^i\right)$ representa la expresión $\left(\frac{1}{\mathrm{n}-1} \sum_{\substack{\mathrm{j}=1 \\ \mathrm{j} \neq \mathrm{i}}}^{\mathrm{n}} \mathrm{a}_j^q \right)$ de la ecuación media Bonferroni:

Capitulo III. Metodología

3.1. Esquema metodológico

A continuación, se presenta un esquema y un breve resumen de los que se realizó en la presente investigación que tiene como objetivo desarrollar una metodología que permita gestionar el riesgo en precio de maíz mediante la reducción del error de pronóstico, apoyado de la metodología BON-POWA (Ver Figura 4).

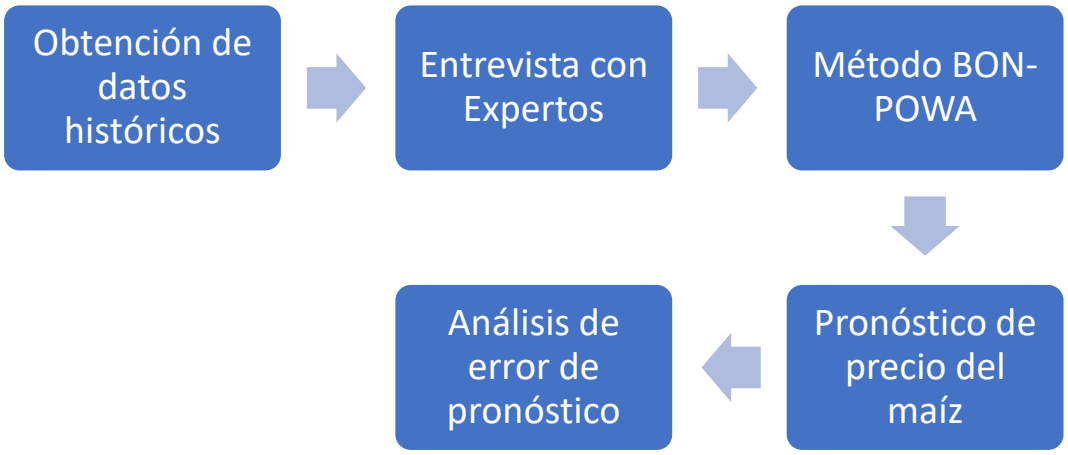

Figura 4. Esquema metodológico
Elaboración propia

Describiendo la figura 4, se indican las principales actividades realizadas en cada paso:

1.- Obtener los datos históricos, se utilizó base de datos de cotizaciones el precio del maíz en la CBOT.

2.- Realizar entrevistas a expertos para identificar sus conocimiento y expectativa del mercado y poderlas agregar al modelo de pronóstico BON-POWA.

4. Desarrollo de la formulación matemática de un operador que combine las medias Bonferroni, el operador OWA y el operador probabilístico OWA.

3.- Estimar precios de maíz con la metodología BON-POWA.

4.- Analizar el error de pronósticos con metodologías MSD, MAPE y MAD

3.2 El operador *Bonferroni Probabilistic Ordered Weighted Average* (BON-POWA)

En la presente investigación se propone un nuevo operador BON-POWA, el cual nos permite ordenar la información dentro de un máximo y un mínimo, unificando mediante vectores de pesos W_i considerados por el grado de importancia para los tomadores de decisión, y la probabilidad p_i en una misma formulación. Esta formulación es como sigue:

$$BON - POWA = \left(\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n-1} a_i^r L_{w_j}\right)^{\frac{1}{r+q}} \quad (2)$$

Donde, $a_i$tiene asociado un elemento w_j de ponderación en el vector W con $\sum_{j=1}^{n} w_j = 1$ y $w_j \in [0,1]$, y una probabilidad p_i del vector P con $\sum_{i=1}^{n} p_i = 1$ y $p_i \in [0,1]$. Sea L_{w_j} un vector de agregación del factor de ponderación y de las probabilidades para $a_i \neq a_j$ tal que, $L_{w_j} = \beta\left(w_j b_j^q\right) + (1-\beta)(p_j b_j^q)$, donde $p_j$es la probabilidad ordenada según b_j, según el j-ésimo más grande de los a_i.

Los valores r y q son los grados de los promedios múltiples que generaliza la media doble ajustando el nivel de confianza de cada uno de los intervalos a_i y a_j.

Adicional a la agregación del OWA y las probabilidades de los argumentos, este nuevo operador permite la comparación múltiple entre las entradas y captura su interrelación. Así pues, cuando $\beta = 0$, se tiene la agregación de las probabilidades de los argumentos $a_i \neq a_j$, y con $\beta = 1$ se tiene el operador OWA_w excluyente el argumento de comparación.

La anterior formulación también puede expresarse así:

$$\begin{aligned} BON - POWA\,(a_1, a_2, \ldots, a_n) = \beta &\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r OWA_W\left(V^i\right)\right)^{\frac{1}{r+q}} \\ &+ (1-\beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r PA_{w_j}\right)^{\frac{1}{r+q}} \end{aligned} \quad (3)$$

Donde (V^i) es la tupla de todos los $a_j \neq a_i$ como $(a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$, y sea W el vector de peso w_j de dimensión n con componentes $w_j \in [0,1]$ cuando $\sum_{j=1}^{n} w_j = 1$. Así podemos definir esta agregación como $OWA_w\ (V^i) = \sum_{j=1}^{n-1} W_j\, a^q_{\pi_i(j)}$, dónde $a^q_{\pi_i(j)}$ es el j-iesimo elemento de la tupla V^i.

En la parte correspondiente a las probabilidades se tiene PA_{w_j}, es la agregación de las probabilidades de los argumentos ordenados según b_j y con $a_j \neq a_i$, siendo $PA_{w_j} = \sum_{j=1}^{n-1} p_j b_j^q$, donde $\sum_{i=1}^{n} p_i = 1\ y\ p_i \in [0,1]$.

Los valores r y q son los grados de los promedios múltiples que generaliza la media doble ajustando el nivel de confianza de cada uno de los intervalos a_i y a_j.

Obsérvese que cuando $\beta = 0$ se obtiene estrictamente a BON-PO y si $\beta = 1$ se obtiene estrictamente el operador BON-OWA. Ahora bien, con respecto al valor de a_i, si todos los $a_i = 0$ se obtiene que $BON - POWA\ (0, \dots, 0) = 0$. Cuando $a_i = 1$ entonces $BON - POWA\ (1, \dots, 1) = \beta \left(\frac{1}{n}\sum_i OWA_W\left(V^i\right)\right)^{\frac{1}{r+q}} + (1-\beta)\left(\frac{1}{n}\sum_i PA_{w_j}\right)^{\frac{1}{r+q}}$. En esto, debe considerarse que V^i es la tupla de los $a_j \neq a_i$ y tendrán por tanto valor de uno, dándose la propiedad de idempotencia para el operador, $BON - POWA\ (1, \dots, 1) = 1$.

Se presenta también monotonicidad del operador OWA. Si se tiene $c_i \geq a_i$, y si se considera $BON - POWA\ (a_1, a_2, \dots, a_n)$ y $BON - POWA\ (c_1, c_2, \dots, c_n)$, entonces dado que, bajo esta propiedad del operador, $OWA_W\left(a_1, a_2, \dots, a_{i-1}, a_{i+1,} \dots, a_n\right) \leq OWA_W(c_1, c_2, \dots, c_{i-1}, c_{i+1}, \dots, c_n)$ y por consiguiente, $BON - POWA\ (a_1, a_2, \dots, a_n) \leq BON - POWA\ (c_1, c_2, \dots, c_n)$.

Además, desde la anterior situación, considerando la delimitación, si $a^* = Max_i[a_i]$, entonces $BON - POWA\ (a_1, a_2, \dots, a_n) \leq BON - POWA\ (a^*, \dots, a^*) = a^*$ y, si $a_* = Min_i[a_i]$ entonces $BON - POWA\ (a_1, a_2, \dots, a_n) \geq BON - POWA\ (a_*, \dots, a_*) = a_*$. Por tanto, $Min_i[a_i] \leq BON - POWA\ (a_1, a_2, \dots, a_n) \leq Max_i[a_i]$. A partir de esto, se tiene que el operador $BON - POWA$ provee una clase de media como operador de agregación.

Se presentan algunos casos especiales de acuerdo a los valores que toma W. Considérese la situación donde $W = W^*$, en esto, para $k = 2$ a $n-1$, $w_1 = 1\ y\ w_k = 0$,así se obtiene un BON-POWA con el máximo de (V^i):

$$BON - POWA_{W^*}(a_1, a_2, \dots, a_n) = \beta \left(\frac{1}{n}\sum_{i=1}^{n} a_i^r Max(V^i)\right)^{\frac{1}{r+q}} + (1-\beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r PA_{w_j}\right)^{\frac{1}{r+q}} \quad (4)$$

Ahora, el $Max(V^i)$ corresponde al elemento más grande de a con $i \neq j$. Además, sea $a_{ind(j)}$ el j-esimo elemento más grande de a_i, entonces, para $i \neq ind(1)$ se tiene que el $Max(V^i) = a_{ind(1)}$, y por tanto, $Max(V^i) = a_{ind(1)} = Max_i[a_i]$; ahora, para $i = ind(1)$ entonces el $Max(V^i) = a_{ind(2)}$. Con valores $r = q = 1$, remplazando en (8),

$$BON - POWA_{W^*}(a_1, a_2, \dots, a_n) = \beta \left(\frac{1}{n}\sum_{i=1}^{n} a_{ind(j)}(a_{ind(1)}) + a_{ind(1)}(a_{ind(2)})\right)^{\frac{1}{2}} + (1-\beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i PA_{w_j}\right)^{\frac{1}{2}} = \beta \left(\frac{1}{n}\left(a_{ind(1)}\sum_{i=1}^{n} a_{ind(j)} + a_{ind(2)}\right)\right)^{\frac{1}{2}} + (1-\beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i PA_{w_j}\right)^{\frac{1}{2}}$$

$$BON - POWA_{W^*}(a_1, a_2, \dots, a_n) = \beta \left(a_{ind(1)}\left(\frac{1}{n}\sum_{i=1}^{n} a_{ind(j)} - (a_{ind(1)} - a_{ind(2)})\right)\right)^{\frac{1}{2}} + (1-\beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i PA_{w_j}\right)^{\frac{1}{2}}$$

$BON - POWA_{W^*}\,(a_1, a_2, \dots, a_n) \approx \beta\,(Max_i[a_i] * promedio(a_1, a_2, \dots, a_n))^{\frac{1}{2}} + (1 - \beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i PA_{w_j}\right)^{\frac{1}{2}}$

Para el caso de $W = W^*$ con $w_{n-1} = 1$ y $w_k = 0$ para $k = 1$ a $n - 2$, se tiene un BON.POWA al mínimo,

$$BON - POWA_{W_*}\,(a_1, a_2, \dots, a_n) = \beta\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r Min(V^i)\right)^{\frac{1}{r+q}} + (1 - \beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r PA_{w_j}\right)^{\frac{1}{r+q}}$$

Siendo, $Min(V^i)$ el elemento más pequeño de a con $i \neq j$. Además, sea $a_{ind(j)}$ el j-ésimo elemento más pequeño de a_i, entonces, para $i \neq ind(n)$ se tiene que el $Min(V^i) = a_{ind(n)}$, y por tanto, $Min(V^i) = a_{ind(n)} = Min_i[a_i]$; ahora, para $i = ind(n)$ entonces el $Min(V^i) = a_{ind(n-1)}$. Siguiendo las operaciones algebraicas del caso anterior, con valores $r = q = 1$,se obtiene:

$$BON - POWA_{W_*}\,(a_1, a_2, \dots, a_n) \approx \beta\,(Min_i[a_i] * promedio(a_1, a_2, \dots, a_n))^{\frac{1}{2}} + (1 - \beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i PA_{w_j}\right)^{\frac{1}{2}}$$

Para determinar el vector de peso existen diferentes enfoques desarrollados en la literatura (Xu, 2005; Merigó & Casanovas, 2011; Blanco-Mesa, Merigó, & Kacprzyk, 2016). De estos, algunas medidas a tener en cuenta son la entropía de dispersión, el operador de balance la divergencia de W y el grado de *orness*. Estos son definidos así:

La entropía es definida como,

$$H(W) = -\sum_{i=1}^{n} w_i \ln w_i$$

Para el BON-POWA la entropía es,

$$H(W) = \beta\left(-\left(\frac{1}{n}\sum_{i=1}^{n}\ln w_i\left(\sum_{\substack{j=1\\ i\neq j}}^{n} w_j \ln w_j\right)\right)^{\frac{1}{r+q}}\right) + (1-\beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r PA_{w_j}\right)^{\frac{1}{r+q}} \quad (5)$$

El grado de *orness* es definido como,

$$orness(W) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i$$

Dicho grado de *orness* para el BON-POWA se obtiene de,

$$orness(W) = \beta\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{n-i}{n-1}\right)\left(\sum_{\substack{j=1\\ i\neq j}}^{n}\left(\frac{n-j}{n-1}\right)w_j\right)\right)^{\frac{1}{r+q}} + (1-\beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r PA_{w_j}\right)^{\frac{1}{r+q}} \quad (6)$$

Así, la divergencia de W se obtiene,

$$Div(W) = \beta \left(\frac{1}{n} \sum_{i=1}^{n} \left(\frac{n-i}{n-1} \right. \right.$$
$$\left. - orness(W) \right)^2 \left(\sum_{\substack{j=1 \\ i \neq j}}^{n} \left(\frac{n-j}{n-1} - orness(W) \right)^2 w_j \right) \Bigg)^{\frac{1}{r+q}} \quad (7)$$
$$+ (1-\beta) \left(\frac{1}{n} \sum_{i=1}^{n} a_i^r PA_{w_j} \right)^{\frac{1}{r+q}}$$

El operador de balance para el BON-POWA resulta de,

$$Bal(W) = \beta \left(\frac{1}{n} \sum_{i=1}^{n} \left(\frac{n+1-2_i}{n-1} \right) \left(\sum_{\substack{j=1 \\ i \neq j}}^{n} \left(\frac{n+1-2_j}{n-1} \right) w_j \right) \right)^{\frac{1}{r+q}} \quad (8)$$
$$+ (1-\beta) \left(\frac{1}{n} \sum_{i=1}^{n} a_i^r PA_{w_j} \right)^{\frac{1}{r+q}}$$

Además, si $q = 0$ entonces BON-POWA sería:

$$
\begin{aligned}
BON - POWA^{r,0}\,(a_1, a_2, \dots, a_n) &= \beta \left(\frac{1}{n}\sum_{i=1}^{n} a_i^r OWA_W\left(V^i\right)\right)^{\frac{1}{r+0}} \\
&+ (1-\beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r PA_{w_j}\right)^{\frac{1}{r+0}} \\
&= \beta\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r\right)^{\frac{1}{r}} + (1-\beta)\left(\frac{1}{n}\sum_{i=1}^{n} a_i^r\right)^{\frac{1}{r}} \\
&= \left(\frac{1}{n}\sum_{i=1}^{n} a_i^r\right)^{\frac{1}{r}} (\beta + 1 - \beta) = \left(\frac{1}{n}\sum_{i=1}^{n} a_i^r\right)^{\frac{1}{r}}
\end{aligned}
\tag{9}
$$

Si $r = 1$ y $q = 0$, remplazando en (13) , BON-POWA se reduce a un promedio:

$$
BON - POWA^{1,0}\,(a_1, a_2, \dots, a_n) = \frac{1}{n}\sum_{i=1}^{n} a_i \tag{10}
$$

Si $r = 2$ y $q = 0$ entonces el BON-POWA se reduce a una media cuadrática, se obtiene al remplazar en (13) así:

$$
BON - POWA^{2,0}\,(a_1, a_2, \dots, a_n) = \left(\frac{1}{n}\sum_{i=1}^{n} a_i^2\right)^{\frac{1}{2}} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} a_i^2} \tag{11}
$$

Si $r \to +\infty$ y $q = 0$, remplazando en (13), el BON-POWA se reduce al operador MAX, es decir, el máximo valor en los argumentos a_i

$$\lim_{r \to +\infty} BON - POWA^{r,0}\,(a_1, a_2, \dots, a_n)$$
$$= \left(\frac{1}{n}\sum_{i=1}^{n} a_i^r\right)^{\frac{1}{r}} = \left(\frac{1}{n}\sum_{i=1}^{n} a_i^{\infty}\right)^{\frac{1}{\infty}} = \frac{1}{n}\sum_{i=1}^{n} a_i^{\frac{\infty}{\infty}} = max\{a_i\} \quad (12)$$

Si $r \rightarrow 0$ y $q = 0$, remplazando en (13), BON-POWA se reduce a la media geométrica:

$$\lim_{r \to 0} BON - POWA^{r,0}\,(a_1, a_2, \dots, a_n) = \left(\frac{1}{n}\sum_{i=1}^{n} a_i^r\right)^{\frac{1}{r}} = \left[\left(\prod_{i=1}^{n} a_i^r\right)^{\frac{1}{n}}\right]^{\frac{1}{r}}$$
$$= \left(\prod_{i=1}^{n} a_i\right)^{\frac{r}{nr}} = \left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}} \quad (13)$$

Si $r = q = 1$, remplazando en (7) se obtiene la expresión reducida del BON-POWA (6)

$$BON - POWA = \frac{1}{n}\left(\beta\left(\sum_{i=1}^{n} a_i OWA_W\left(V^i\right)\right)^{\frac{1}{2}} + (1-\beta)\left(\sum_{i=1}^{n} a_i^r PA_{w_j}\right)^{\frac{1}{2}}\right)$$
$$= \left(\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n-1} a_i L_{w_j}\right)^{\frac{1}{2}} \quad (14)$$

Capitulo IV. Resultados

4.1. Aplicación del operador BON-POWA en la estimación del precio futuro del maíz

Se aplica la metodología propuesta en este documento para estimar el precio del maíz, la cual considera la probabilidad de que un evento suceda, las consideraciones de los expertos y un análisis de influencia de cada atributo. Los datos que se muestran en el paso 1 son los precios de los futuros tomando en cuenta los periodos de los vencimientos (marzo, mayo, julio, septiembre y diciembre) los spots y los datos de comparación para cada contrato de tomaron el publicado el 15 de agosto del 2018.

Paso 1: Se construye la tabla de pagos para los precios de cada periodo. Los datos están expresados en centavos de dólar.

Tabla 1. Tabla de pagos

	S1	**S2**	**S3**	**S4**	**S5**
P1 Sep' 19	336	374	367	369	470
P2 Jul' 19	336	336	374	367	369
P3 May' 19	392	336	336	374	367
P4 Mar' 19	367	392	336	336	374

Paso 2: con base en la valoración dada por expertos, así como información conocida de eventos anteriores, se determina los vectores de peso $W_i = \{0.15,0.05,0.05,0.2,0.55\}$ y de probabilidad $P_i = \{0.1,0.2,0.2,0.05,0.45\}$ para los estados de la naturaleza $S = \{S_1, S_2, S_3, S_4, S_5\}$ respectivamente.

Paso 3: se establecen los vectores V^i que permiten calcular el OWA_W para $a_j \neq a_i$.

$$(Pr_1S_1) = \{374,367,369,470\}$$
$$(Pr_1S_2) = \{336,367,369,470\}$$
$$(Pr_1S_3) = \{336,374,369,470\}$$
$$(Pr_1S_4) = \{336,374,367,470\}$$
$$(Pr_1S_5) = \{336,374,367,369\}$$
$$(Pr_2S_1) = \{336,374,367,369\}$$
$$(Pr_2S_2) = \{336,374,367,369\}$$
$$(Pr_2S_3) = \{336,336,367,369\}$$
$$(Pr_2S_4) = \{336,336,374,369\}$$

$$(Pr_2S_5) = \{336,336,374,367\}$$
$$(Pr_3S_1) = \{336,336,374,367\}$$
$$(Pr_3S_2) = \{392,336,374,367\}$$
$$(Pr_3S_3) = \{392,336,374,367\}$$
$$(Pr_3S_4) = \{392,336,336,367\}$$
$$(Pr_3S_5) = \{392,336,336,374\}$$
$$(Pr_4S_1) = \{392,336,336,374\}$$
$$(Pr_4S_2) = \{367,336,336,374\}$$
$$(Pr_4S_3) = \{367,392,336,374\}$$
$$(Pr_4S_4) = \{367,392,336,374\}$$
$$(Pr_4S_5) = \{367,392,336,336\}$$

Paso 4: con los vectores V^i para todos los procede, se procede a calcular el valor del $OWA_W(V^i)$ a partir de las alternativas excluyentes de a_i multiplicado por el vector de peso *W*, así:

Tabla 2. Resultados de los OWAS

	OWAw(PiSj)				
	S1	**S2**	**S3**	**S4**	**S5**
P1	369	401	401	346	161
P2	312	345	343	289	159
P3	310	352	352	294	167
P4	309	345	348	297	159

Paso 5: los resultados anteriores se multiplican por el pago de cada precio en un estado de la naturaleza (Pr_iS_j) y la sumatoria se divide entre el número de argumentos y se eleva al cociente entre 1 y la suma de *r* y *q*.

Tabla 3 Resultados del BON-OWA

	BON-OWAw(PiSj)
P1	353
P2	320
P3	325
P4	324

Paso 6: para completar el cálculo del BON-POWA es necesario determinar el valor de la probabilidad agregada PA_{W_j}, obtenida a partir de la multiplicación del vector V^i por la probabilidad p_j ordenada según b_j.

Tabla 4 Datos para calcular BON-PO

	Pawj				
	S1	**S2**	**S3**	**S4**	**S5**
P1	378	337	338	393	200
P2	326	293	285	341	194
P3	318	290	290	339	192
P4	331	289	300	351	199

Paso 7: los resultados de la probabilidad agregada se multiplican con el pago de cada precio en un estado de la naturaleza (Pr_iS_j) y la sumatoria se divide entre el número de argumentos, elevado a la razón de 1/(*r*+*q*).

Tabla 5 Resultados BON-PO

	BON-Powj
P1	351
P2	320
P3	322
P4	325

Paso 8: tómese para este ejemplo que el decisor ha establecido un valor de $\beta = 0.6$. Con este parámetro y la información del *BON-OWA* y del *BON-PO*, es posible calcular el *BON-POWA*.

Tabla 6 Resultados BON-POWA

	BON-POWA(PjSj)	**futuro CME**	**Spot**	**CME Vs Spot**	**BON-POWA Vs Spot**
P1	352	399	369	30	16
P2	320	401	336	65	16
P3	324	393	349	44	25
P4	324	392	354	38	30

En el resultado obtenido en la tabla 7 se puede visualizar que con la nueva formulación BON-POWA y teniendo en cuenta diferentes informaciones ofrecidas por el decisor, se ha logrado

reducir el error de pronóstico, tomando en cuenta el precio de contado (spot) comparándolo con el futuro publicado por el CME y el importe calculado con la formulación propuesta en el documento. Se puede apreciar que en todos los periodos se obtuvo una reducción del error. Como se puede observar en el proceso de obtención de resultados, con el BON-POWA es posible generar diferentes escenarios modificando el vector de peso, situación que no es posible generar en modelos de estimación tradicionales, ya que depende solamente de información histórica y no permite construir diferentes escenarios que permitan abrir un abanico de opciones para los tomadores de decisión.

Con la finalidad de evaluar los resultados obtenidos con la metodología BON-POWA en el pronóstico de precio de maíz, se analizaron los errores con las metodologías de error de porcentaje medio (MAPE por sus siglas en inglés), desviación absoluta media (MAD por sus siglas en inglés) y la desviación media cuadrada (MSD por sus siglas en inglés), a continuación, se muestra sus fórmulas.

$$MAPE = \frac{\sum|y_t - \hat{y}_t|/y_t}{n} \text{x}100,$$

$$MAD = \frac{\sum|y_t - \hat{y}_t|}{n},$$

$$MSD = \frac{\sum|y_t - \hat{y}_t|^2}{n},$$

donde y_t es el valor actual, $\hat{y}_t$ es el valor pronosticado y n es el número de observaciones es el valor pronosticado y n es el número de observaciones. En el recuadro siguiente se muestran los resultados obtenidos con estas tres formulaciones.

	CME Vs Spot	BON-POWA Vs Spot
MAPE	12.70	6.18
MAD	44.25	21.75
MSD	2,126.25	509.25

Derivado de los resultados obtenidos en la tabla anterior, se puede observar que la evaluación de error del pronóstico obtenido con el operador de agregación BON-POWA son mejores en las tres formulaciones (MAPE, MAD y MSD). Esto evidencia la eficiencia del operador.

Capitulo V. Conclusiones generales

El precio es una variable de decisión importante para las organizaciones, tener conocimiento o estimación de este en escenarios futuros genera certidumbre y asegura la permanecías de las empresas. Por lo tanto, en la actualidad e importante generar metodologías que mejoren el pronóstico de precios.

Existe una amplia literatura que aborda el pronóstico de precios, para el caso de la presente investigación la cual se aplicó en el caso del maíz, encontramos autores que habla de que su precio tiene periodos estacionarios y cíclicos, por otra parte, algunos aseguran que sus datos históricos tienen un comportamiento lineal, mientras que otros dicen que su comportamiento es no lineal, sin embargo, todos los resultados presentan errores amplios en su pronóstico. En este sentido, no existe una metodología clara y predominante para el pronóstico de precio del maíz, de tal manera que, existe dificultad en elegir un método eficiente para las organizaciones.

Por lo anterior, surge la importancia del objetivo del presente documento, tomando en cuenta la literatura revisada y los resultados obtenidos se considera atendido y atinada la propuesta de implementación de las medias Bonferroni y operadores OWA y probabilísticos para pronosticar el precio futuro del maíz, ya que el error de pronóstico se disminuyó considerablemente en los contratos con vencimiento en marzo, mayo, julio y septiembre de 2019.

Adicionalmente, la formulación de un nuevo operador de agregación de información que permitiera incluir información histórica utilizando las medias Bonferroni y probabilidad tomando en cuenta expectativas, conocimiento y experiencia del tomador de decisiones en el pronóstico. El desarrollo del operador BON-POWA permitió identificar nuevos escenarios, que los modelos tradicionales de pronósticos no permiten ya que solo consideran información histórica. Esto, permitirá a las organizaciones la construcción de estrategias para la toma de decisiones.

Por otro lado, analizando los resultados obtenidos se puede afirmar la hipótesis de esta investigación, así mismo da respuesta a la pregunta central ya que se desarrolla y aplica el operador de agregación de información BON-POWA el cual permitió mejorar el error de

pronóstico en comparación con los futuros del CME, esto permite a tomador de decisión tener mayor certidumbre y diseñar mejores estrategias.

Recomendaciones

La principal recomendación de la presente investigación es que la perspectiva del experto o informante clave cambia en función al lapso de periodo productivo en el que hace su aportación al modelo. Lo anterior, debido a que el empresario agrícola cuenta con información dosificada de acuerdo a cada periodo (Antes de siembra, siembra, cultivo, cosecha y postcosecha).

Limitaciones

La presente investigación tiene como limitaciones es que la información se obtuvo por parte de los expertos y tomadores de decisión, de tal manera que al cambiar de informante y/o periodos de toma de información se puede tener variación en los resultados.

Los resultados y afirmaciones obtenidos en el presente documento son aplicables solo para el precio futuro del maíz, tomando como referencia los contratos de futuros de marzo, mayo, julio y septiembre del 2019 publicados por el CME.

Por último, es importante mencionar que con los puntos desarrollados en la presente investigación se permitió contribuir al conocimiento y con esto abrir la puerta a nuevas investigaciones que permita generar nuevas aplicaciones y extensiones en los operadores de agregación de información que contribuya positivamente a los tomadores de decisión.

Futuras líneas de investigación

Durante el desarrollo del presente documento se pudieron identificar las siguientes líneas de investigación.

- Utilización del operador de agregación de información BON-POWA propuesto en la construcción de banda de precios para el maíz.
- Utilizar el operador de agregación de información BON-POWA para el pronostico de precios de los diferentes cereales (frijol, garbanzo, arroz, entre otros).
- Lograr agrupar la metodología propuesta BON-POWA con otros modelos que permita reducir en mayor medida el error de pronóstico.
- Por último, la generación de extensiones del operador OWA que ayuden a la toma de decisiones y al diseño de estrategias empresariales

Referencias

Abbott, P. C., Hurt, C., & Tyner, W. E. (14 de 03 de 2019). *What's driving food prices?* Obtenido de agecon search: https://ageconsearch.umn.edu/record/37951/

Adjemian, M. K., & Smith, A. (2012). Using USDA forecasts to estimate the price flexibility of demand for agricultural commodities. *American Journal of Agricultural Economics*, *94*(4), 978-995.

Agnolucci, P. (2009). Volatility in crude oil futures: A comparison of the predictive ability of GARCH and implied volatility models. *Energy Economics*, *31*(2), 316-321.

Akram, Q. F. (2009). Commodity Prices, Interest Rates and the Dollar. *Energy Economics*, 838-851.

Alfaro-Garcia, V. (2019). Toma de decisiones en la incertidumbre: técnicas y herramientas ante escenarios altamente desafiantes. *Inquietud Empresarial,* 19(2), I-III.

Al-Momani, R., & Gharaibeh, M. (2008). Foreign exchange risk management practices by Jordanian nonfinancial firms. *Journal of Derivatives & Hedge Funds*, 198-221.

AMIS-FAO. (24 de Abril de 2019). Obtenido de http://statistics.amis-outlook.org/data/index.html

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2003). Modeling and forecasting realized volatility. *Econometrica*, *71*(2), 579-625.

Anderson, R. W. (1985). Some determinants of the volatility of futures prices. *The Journal of Futures Markets (pre-1986)*, *5*(3), 331.

Arbeláez, L. C., & Ceballos, L. E. (2005). El valor en riesgo condicional CVaR como medida coherente de riesgo. *Revista Ingenierías Universidad de Medellín*, 43-54.

Arteaga, C., Granados, J., & Joya, J. O. (2013). Determinantes de los precios internacionales de los bienes básicos. *Ensayos sobre política económica*, 85-107.

Asturias, M. A. (2004). *Maíz, de alimento sagrado a negocio del hambre. Acción Ecológica.* Quito- Ecuador: Acción Ecológica, Red por una América latina libre de transgénicos.

Bailey, W., & Chan, K. (1993). Macroeconomic influences and the variability of the Commodity Futures Basis. *The Journal of Finance*, 555–573.

Baur, R. F., & Orazem, P. F. (1994). The rationality and price effects of US Department of Agriculture forecasts of oranges. *The Journal of Finance*, *49*(2), 681-695.

Beliakov, G., Pradera, A., & Calvo, T. (2007). *Aggregation functions: A guide for practitioners* (Vol. 221). Heidelberg: Springer.

Blanco-Mesa, F., León-Castro, E., & Merigó, J. M. (2019). A bibliometric analysis of aggregation operators. *Applied Soft Computing*, *81*, 105488.

Blanco-Mesa, F., Merigó, J. M., & Kacprzyk, J. (2016). Bonferroni means with distance measures and the adequacy coefficient in entrepreneurial group theory. *Knowledge-Based Systems, 1*, 1-11. doi:10.1016/j.knosys.2016.08.016

Bolaño, R. Y., Robaina, D. A., Pérez, B. A., & Arias, P. M. (2014). Strategic management model based on the risks management. *Ingeniería industrial*, 344-357.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics, 31*, 307-327.

Box, G. E., & Jenkins, C. M. (1976). *Time Series Analysis: Forecasting and Control.* San Francisco: Holden Day.

Cárcamo, U. C., & Franco, L. E. (2012). Una aproximación a la estimación de rendimientos de conveniencia y precios teóricos de futuros para commodities agropecuarios en Colombia. *Cuad. admon.ser.organ, 25*, 141-173.

Carter, C., Rausser, G., & Smith, A. (2011). Commodity Booms and Busts. *Annual Review of Resource Economics*, 87–118.

Casassus, J., & Collin-Dufresne, P. (2005). Stochastic convenience yield implied from commodity futures and interest rates. *Journal of Finance,* 2283-2331.

Chambers, M. J., & Bailey, R. E. (1996). A Theory of Commodity Price Fluctuations. *Journal of Political Economy*, 924-957.

Chen, Y. C., Rogoff, K. S., & Rossi, B. (2010). Can exchange rates forecast commodity prices?. *The Quarterly Journal of Economics*, *125*(3), 1145-1194.

Cheung, Y. (2007). An empirical model of daily highs and lows, Int. *Finance Econ., 12*, 1–20.

Chiang, I., Ethan, H. H., & Sagi, J. S. (2015). Estimating oil risk factors using information from equity and derivatives markets. *Journal of Finance,* 769-804.

Chiang, I., Ethan, H., Hughen, W. K., & Sagi, J. S. (2015). Estimating oil risk factors using information from equity and derivatives markets. *Journal of Finance,* 769-804.

Clinton, J. D., & Lewis, D. E. (2007). Expert Opinion, Agency Characteristics, and Agency Preferences. *Political Analysis*, 3-20.

Comisión nacional del mercado de valores. (2013). *Guía para la elaboración del informe de gestión de las entidades cotizadas.* España: Composiciones Rali, S.A.

Cortazar, G., Schwartz, E., & Naranjo, L. (2007). Term-structure estimation in markets with infrequent trading. *Journal of Financial Economics,* 353-369.

De La Lastra, E. (2007). *Supervisión basada en riesgo en Panamá. .* Panamá: Superintendencia de bancos.

De Lara, H. A. (2002). *Medición y control de riesgos financieros.* México: Limusa Editores.

Del Carpio, C., & Zevallos, M. (2010). Estimación de capital por riesgo de precio: Evaluando metodologías para el caso peruano. *Estudios Económicos,* 47-62.

Detyniecki, M., Bouchon-meunier, D. B., Yager, D. R., & Prade, R. H. (2000). Mathematical aggregation operators and their application to video querying.

Dhanani, A. (2004). The Management of Exchange-Rate Risk: A Case from the Manufacturing Industry. *Thunderbird International Business Review*, 317-338.

Díez de Castro, L., & Mascareñas, J. (1994). *Ingenieria Financiera. La gestion en los mercados financieros internacionales.* España: McGraw Hill.

Drouillard, J. S. (2018). Current situation and future trends for beef production in the United States of America—A review. *Asian-Australasian journal of animal sciences,* 1007-1007.

Egelkraut, T. M., Garcia, P., & Sherrick, B. J. (2007). The term structure of implied forward volatility: Recovery and informational content in the corn options market. *American Journal of Agricultural Economics*, *89*(1), 1-11.

Engle, R. (1982). Autoregresive Conditional Heteroscedasticity whit Estimates of the Variance of United Kingdom Inflation. *Econometrica, 50*(4), 987-1007.

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The journal of Finance*, *25*(2), 383-417.

FAO. (20 de Abril de 2019). Obtenido de http://www.fao.org/faostat/es/#compare

FAO. (24 de Abril de 2019). Obtenido de http://www.fao.org/3/x7650s/x7650s08.htm

FAO. (24 de Abril de 2019). Obtenido de http://www.fao.org/worldfoodsituation/csdb/es/

García-Ascanio, C., & Maté, C. (2010). Electric power demand forecasting using interval time series: a comparison between VAR and iMLP,. *Energy Policy, 38*, 715–725.

Gaudenzi, B., Zsidisin, G. A., Hartley, J. L., & Kaufmann, L. (2018). An exploration of factors influencing the choice of commodity price risk mitigation strategies. *Journal of Purchasing and Supply Management*, *24*(3), 218-237.

Ghosh, A. (1993). Hedging with stock index futures: Estimation and forecasting with error correction model. *The Journal of Futures Markets (1986-1998)*, *13*(7), 743.

Gilbert, C. (2010). How to understand High Food Prices. *Journal of Agricultural Economics*, 398-425.

Giot, P., & Laurent, S. (2004). Modelling daily value-at-risk using realized volatility and ARCH type models. . *Journal of Empirical Finance, 11*, 379–398.

Gómez, D. C., & López, J. M. (2002). *Riesgos financieros y operaciones internacionales.* Madrid: ESIC.

González, M. A., & Ávila, C. J. (2014). El maíz en Estados Unidos y en México: Hegemonía en la producción de un cultivo. *Argumentos*, 215-237.

Harwood, J., Heifner, R., Coble, K., Perry, J., & Somwaru, A. (1999). *Managing Risk in Farming: Concepts, Research, and Analysis.* U.S. Department of Agriculture, Market and trade economics division and resource economics division, Economic Research Service. Washington: USDA.

Hennessy, D. A., & Wahl, T. I. (1996). The Effects of Decision Making on Futures Price Volatility. *American Journal of Agricultural Economics*, 591-603.

Herrera, F., & Martínez, L. (2000). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on fuzzy systems*, *8*(6), 746-752.

Huang, J., Serra, T., & Garcia, P. (2020). Are futures prices good price forecasts? Underestimation of price reversion in the soybean complex. *European Review of Agricultural Economics*, *47*(1), 178-199.

Jankelova, N., Masar, D., & Moricova, S. (2017). Risk factors in the agriculture sector. *Agric. Econ*, 247–258.

Jankovic, I., Kovačević, V., & Jeločnik, M. (2020). Production costs and market price of wheat behavior analysis as a support for hedging strategies. *Economics of Agriculture*, *67*(2), 495-509.

Just, R. E., & Pope, R. D. (2003). Agricultural Risk Analysis: Adequacy of Models, Data, and Issues. *American Journal of Agricultural Economics*, 1249-1256.

Kalari, B., & Power, G. (2013). Short and long-run determinants of commodity price volatility. *American Journal of Agricultural Economics*, 724–738.

Kantanantha, N., Serban, N., & Griffin, P. (2010). Yield and price forecasting for stochastic crop decision planning. *Journal of agricultural, biological, and environmental statistics*, *15*(3), 362-380.

Kapusuzoglu, A., & Karacaer, M. (2015). The interactions between agricultural commodity and oil prices: an empirical analysis . *Agric. Econ, 61*, 410–421.

Karali, B., Irwin, S. H., & Isengildina-Massa, O. (2020). Supply fundamentals and grain futures price movements. *American Journal of Agricultural Economics*, *102*(2), 548-568.

Kaufmann, A., & Aluja, J. G. (1990). *Las matemáticas del azar y de la incertidumbre: elementos básicos para su aplicación en economía*. Centro de Estudios Ramón Areces.

Kawai, M. (1983). Price Volatility of Storable Commodities under Rational Expectations in Spot and Futures Markets. *International Economic Review*, 435-459.

Kenyon, D., Jones, E., & McGuirk, M. A. (1993). Forecasting performance of corn and soybean harvest futures contracts. *American Journal of Agricultural Economics*, *75*(2), 399-407.

Knight, F. (2006). *Risk, Uncertainty and Profit.* Dover Publications.

Kogan, F., Salazar, L., & Roytman, L. (2012). Forecasting crop production using satellite-based vegetation health indices in Kansas, USA. *International journal of remote sensing*, *33*(9), 2798-2814.

Koster, K. (2009). *International Project Management.* London: SAGE publications.

Lan, J., Sun, Q., Chen, Q., & Wang, Z. (2013). Group decision making based on induced uncertain linguistic OWA operators. *Decision Support Systems*, *55*(1), 296-303.

Lanteri, L. (2012). Determinants of real prices of agricultural commodities. The role of inventories and macroeconomic. *Lecturas de Economía*, 189-217.

Lavarello, P., & Selva, R. A. (2008). Mercados internacionales de granos y precios frente a la desaceleración de la economía mundial. *Entrelíneas de la Política Económica*, 14-19.

Leon-Castro, E., Avilés-Ochoa, E. A., & Gil-Lafuente, A. M. (2016). Exchange rate USD/MXN forecast through econometric models, time series and HOWMA operators. *Economic Computation & Economic Cybernetics Studies & Research, 50*(4).

León-Castro, E., Avilés-Ochoa, E., Merigó, J. M., & Gil-Lafuente, A. M. (2018a). Heavy Moving Averages and Their Application in Econometric Forecasting. *Cybernetics and Systems, 49(1),* 26-43.

Lieberman, O., & Phillips, P. C. (2008). Refined inference on long memory in realized volatility. *Econometric, 27*, 254–267.

Martens, M., Van Dijk, D., & De Pooter, M. (2009). Forecasting S&P 500 volatility: Long memory, level shifts, leverage effects, day-of-the week seasonality, and macroeconomic announcements. . *International Journal of Forecasting, 25*, 282–303.

Maylor, H. (2010). *Project Management.* London: Financial Times Prentice Hall.

Mayorga, X. S., & Solarte, J. C. (2012). Medición del riesgo de liquidez.: Una aplicación en el sector cooperativo. *Entramado*, 90-98.

McKenzie, A. M. (2008). Pre-harvest price expectations for corn: The information content of USDA reports and new crop futures. *American Journal of Agricultural Economics*, *90*(2), 351-366.

McMichael, P. (2009). A food regime analysis of the world food crisis. *Agriculture and Human Values* , 281–295.

McPhail, L. L., Du, X., & Muhammad, A. (2012). Disentangling corn price volatility: The role of global demand, speculation, and energy. *Journal of Agricultural and Applied Economics,*, 401-410.

McPhail, L. L., Du, X., & Muhammad, A. (2012). Disentangling Corn Price Volatility: The Role of Global Demand, Speculation, and Energy . *Journal of Agricultural and Applied Economics*, 401-410.

Mensi, W., Beljid, M., Boubaker, A., & Managi, S. (2013). Correlations and volatility spillovers across commodity and stock markets: Linking energies, food, and gold. *Economic Modelling*, *32*, 15-22.

Merigó, J. M. (2009). Probabilistic Decision Making with the OWA Operator and its Application in Investment Management . *IFSA-EUSFLAT*, 1364-1369.

Merigó, J. M. (2010). On the unification between the probability, the weighted average and the OWA operator. In *Intelligent Decision Making Systems* (pp. 375-380): World Scientific.

Merigó, J. M. (2015). Decision-making under risk and uncertainty and its application in strategic management. *Journal of Business Economics and Management, 16(1)*, 93-116.

Merigó, J. M., & Casanovas, M. (2011). Decision-making with distance measures and induced aggregation operators. *Computers & Industrial Engineering, 60*, 66-76. doi:10.1016/j.cie.2010.09.017

Merigó, J. M., & Casanovas, M. (2011). Induced aggregation operators in the Euclidean distance and its application in financial decision making. *Expert Systems with Applications*, *38*(6), 7603-7608.

Merigó, J. M., & Gil-Lafuente, A. M. (2010). New decision-making techniques and their application in the selection of financial products. *Information Sciences*, *180*(11), 2085-2094.

Miguel, R. D., & Marqués de Morante. (1887). *Nuevo Diccionario Latino-Español Etimológico.*

Mitchel, D. (2008). A note on Rising Food Prices, Policy Research Working Paper, Development Prospects Group. *The World Bank*, 1-20.

Mjelde, J. W., Thompson, T. N., Hons, F. M., Cothren, J. T., & Coffman, C. G. (1997). Using southern oscillation information for determining corn and sorghum profit-maximizing input levels in east-central Texas. *Journal of production agriculture*, *10*(1), 168-175.

Nadal, A., & Wise, T. (2005). El comercio de maíz entre México y EE. UU. en el marco del NAFTA. *Globalización y medio ambiente: Lecciones de las Américas*, 49-92.

Oosterhof, C. (2001). *An overview of the literature on corporate risk management.* Netherlands: SOM Research Report Nr. 01E06.

Patel, R. (2008). *Obesos y famélicos. El impacto de la globalización en el sistema alimentario mundial.* Barcelona: Los libros del lince.

Petersona, H. H., & Tomekb, W. G. (2005). How much of commodity price behavior can a rational expectations storage model explain? *Agricultural Economics*, 289–303.

Peyrard, J. (1986). *Risque de Change.* Francia: Vuibert Gestion.

Pindyck, R., & Rotemberg, J. J. (1990). the Excess Co-Movements of Commodity Prices. *The Economic Journal*, 1173–1189.

Retes, M. F., Torres, S. G., & Garrido, S. R. (2014). Un modelo econométrico de la demanda de tortilla de maíz en México, 1996-2008. *Estudios sociales*, 38-59.

Riazi, M. H. (2016). The Economics of Price Volatility in Commodity Futures Markets: A Survey. *Journal of Business Studies*, 45-70.

Riazi, M. H. (2016). The Economics of Price Volatility in Commodity Futures Markets: A Survey . *Journal of Business Studies*, 45-70.

Rickard, J. T., & Aisbett, J. (2013). New classes of threshold aggregation functions based upon the Tsallis q-exponential with applications to perceptual computing. *IEEE Transactions on Fuzzy Systems*, *22*(3), 672-684.

Robinson, J. R., & Don, E. (1988). Simplified short-run price forecasting models For wheat and corn. *Journal Of Agriculture And Natural Resources*.

Robinson, J. R., & Ethridge, D. E. (1988). Simplified short-run price forecasting models. *Texas journal of agriculture and natural resources,*, 37-40.

Rossi, D. G. (2013). La Volatilidad en mercados financieros y de commodities un repaso de sus causas y la evidencia reciente. *Invenio*, 59-74.

Samuelson, K. L., Hubbert, M. E., Galyean, M. L., & Löest, C. A. (2016). Nutritional recommendations of feedlot consulting nutritionists: The 2015 New Mexico State and Texas Tech University survey. *J Anim Sci*, 2648–2663.

SIAP. (24 de Abril de 2019). Obtenido de http://siaprendes.siap.gob.mx/contenidos/2/03-maiz/contexto-5.html

Tian, F., Yang, K., & Chen, L. (2017). Realized volatility forecasting of agricultural commodity futures using the HAR model with time-varying sparsity. . *International Journal of Forecasting , 33*, 132–152.

Tomek, W. G., & Robinson, K. L. (1990). *Agricultural Product Prices.* Ithaca, N. Y. USA: Cornell University Press.

Trade Map. (18 de Marzo de 2019). *Trade Map*. Obtenido de https://www.trademap.org/Country_SelProduct_TS.aspx?nvpm=3%7c%7c%7c%7c%7c1005%7c%7c%7c4%7c1%7c1%7c1%7c2%7c1%7c2%7c1%7c1

Tufano, P. (1996). Who manages risk? An empirical examination of risk management practices in the gold mining industry. *the Journal of Finance*, *51*(4), 1097-1137.

USDA. (12 de Diciembre de 2018). Obtenido de https://www.usda.gov/oce/commodity/wasde/wasde1218.pdf

Venegas, G. M. (2016). Producción y comercialización del maíz en México, cobertura de riesgo con derivados. *AMECIDER –ITM*.

Vivel, B. M. (2010). El riesgo cambiario y su cobertura financiera. *Revista Galega de Economía, 19*(2). Recuperado el 20 de Enero de 2017

Xiong, T., Li, C., Bao, Y., Hu, Z., & Zhang, L. (2015). A combination method for interval forecasting of agricultural commodity futures prices. *Knowledge-Based Systems*.

Xu, X. (2018). Cointegration and price discovery in US corn cash and futures markets. *Empirical Economics*, *55*(4), 1889-1923.

Xu, Z. (2005). An Overview of Methods for Determining OWA Weights. *International Journal of Intelligent Systems, 20*, 843-865. doi:10.1002/int.20097

Yager, R. (1988). On Ordered Weighted Averaging Aggregation Operators in Multi-Criteria Decision Making. *IEEE Transactions on Systems, Man and Cubernetics*, 183-190.

Yager, R. D. (1988). On ordered weighted averaging aggregation operators in multicriteria decisionmaking. *IEEE Transactions on systems, Man, and Cybernetics,*, 183-190.

Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Transactions on Systems, Man and Cybernetics, 18*, 183–190.

Yager, R. R. (2002). Heavy OWA operators. *Fuzzy Optimization and Decision Making,1(4)*, 379-397.

Yager, R. R. (2009). On generalized Bonferroni mean operators for multi-criteria aggregation. *International journal of approximate reasoning*, 1279–1286.

Yager, R. R., Engemann, K. J., & Filev, D. P. (1995). On the concept of immediate probabilities. *International Journal of Intelligent Systems, 10*(4), 373-397.

Yang, S. R., & Brorsen, B. W. (1993). Non - linear Dynamics of Daily Futures Prices: Conditional Heteroske dasticity or Chaos? *The Journal of Futures Markets*, 175 - 191.

Yang, W., & Chen, Z. (2012). The quasi-arithmetic intuitionistic fuzzy OWA operators. *Knowledge-Based Systems*, *27*, 219-233.

Ye, M., Zyren, J., & Shore, J. (2006). Forecasting short-run crude oil price using high-and low-inventory variables. *Energy Policy*, 2736-2743

Yu, D. (2015). A scientometrics review on aggregation operator research. *Scientometrics*, *105*(1), 115-133.

Apéndices

Apéndice 1. Articulo publicado en Fuzzy Economic Review

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FORECASTING PERFORMANCE OF EXCHANGE RATE MODELS WITH HEAVY MOVING AVERAGE OPERATORS

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The objective of the paper is to demonstrate the effectiveness of a model which fuses econometric models and Heavy Ordered Weighted Moving Average (HOWMA) operators. The evaluation of the methodology is validated with a forecast exercise of the monthly USD / MXN parity for 2015-2017. Within the results, it is identified that the use of HOWMA operators decreases the forecast error in contrast to the time series. The limitation lies in obtaining the weights of the operator, since a change generates different results, however, this becomes its main advantage, since it allows to incorporate expectations and knowledge of market experts. The originality of the article is the presentation of a fuzzy econometric model based on information aggregation operators.

Keywords: *exchange rate, exchange rate risk, forecast, OWA operator, time series.*

JEL Classification: *F31, G32, C53, C32, D81.*

1. INTRODUCTION

One of the central questions in the literature on exchange rate risk management is focused on how to reduce the level of exposure to risk by minimizing the forward exchange rate forecast error.

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The objective of this work is to provide evidence of the efficacy of a tool that combines the classical quantitative methodology with uncertainty mathematical techniques.

To fulfill the central objective of this work, the Heavy Ordered Weighted Moving Average (HOWMA) operators are used with regression models. An important aspect of this tool is the inclusion of a new operator for the aggregation of information, which brings together historical behavior, through moving averages, and the expectations of the economic agents, through Heavy Ordered Weighted Average (HOWA) operators, to predict the future value of the exogenous variables of the change in parity.

In the literature, models are compiled such that the traditional explanatory variables are inflation, interest rate and monetary flows of the balance of payments. However, these have not generated satisfactory results, as they were outperformed by models based on random walk in the short term (Cheung *et al.*, 2002; Obstfeld and Rogoff, 2001). On the other hand, we identify the unstructured models, including regression, Markov, support vector regression, artificial neural networks and genetic algorithms (Preminger and Franck, 2007; Nikolsko-Rzhevskyy and Prodan, 2012).

Recently, agent-based models were introduced, integrating the knowledge and interpretation of experts from the financial system and relaxing the hypothesis that postulated the presence of homogeneous information and perfectly informed participants, such that the degree of risk aversion, skills and experience and knowledge of the global environment play an outstanding role in decision-making (Menkhoff and Taylor, 2007; Dick *et al.*, 2015).

This article examines a combination of different models and techniques. As an aggregation information method provided by experts, the Ordered Weighted Average operators (OWA) developed by Yager (1988), have been tested by many authors (Yager and Kacprzyk, 1997; Merigó and Gil-Lafuente, 2010). These contributions to the literature have generated a wide range of extensions to solve decision making problems (Yager, 2009; Blanco-Mesa *et al.*, 2018a).

In order to analyze the effectiveness of the model a USD / MXN exchange rate forecast exercise is carried out with monthly averages for the years 2015-2017. For the case of 2017, the results are contrasted with the future exchange rates determined in the Mexican Derivatives Market (MEXDER). For this purpose, econometric models based on Purchasing Power Parity (PPP), Interest Rate Parity (IRP) and Balance of Payment (BoP) were made. In the forecast of the future value of the exogenous variables, time series and HOWMA operators are used. Next, an analysis of the results obtained is performed.

Section 2 describes the methodological and theoretical framework. Section 3 presents the HOWMA operator and explains the reasons why our approach may serve as a useful forecasting tool. Section 4 presents the state of the art

concerning exchange rate forecasting and the contribution of the paper. Section 5 presents our data, forecast evaluation techniques and empirical results. Section 6 concludes the paper.

Finally, there is an appendix that includes some details of the models.

2. METHODOLOGICAL AND THEORETICAL APPROACH

2.1. EXCHANGE RATE FORECASTING

Among the classical theories of exchange rate forecasting are the Purchasing Power Parity (PPP), Interest Rate Parity (IRP) and Monetary Approach to the Balance of Payments (BoP) (Dornbusch, 1979; Engel *et al.*, 2007).

Definition 1. The PPP theory, in its relative version, postulates that the variation of the exchange rate, in a given period, must be equal to the inflationary differential between two economies. That is, the weakest currency should be depreciated or discounted to compensate for the price gap (Dornbusch, 1985; Rogoff, 1996):

$$\hat{e} = \pi_{MEX} - \pi_{USA} \quad (1)$$

where $\hat{e}$ is the exchange rate variation; π_{MEX} is domestic inflation (Mexico), and π_{USA}, is foreign inflation (USA).

Definition 2. The IRP theory, for its part, argues that there is parity if the difference between domestic and foreign interest rates exactly compensates the premium to the stronger currency futures (Fama, 1984; Flood and Rose, 2002):

$$e^f = e^s \frac{1+r_{MEX}}{1+r_{USA}} \quad (2)$$

where e^f is the forward exchange rate; e^s is the spot exchange rate; r_{MEX} is the domestic interest rate and r_{USA} is the foreign interest rate.

Definition 3. The BoP theory (Dornbusch, 1979) assumes that the variation of the exchange rate is a function of the deficit or surplus of the accounts that compose the Balance of Payments, which alter the supply and demand of foreign exchange and domestic currency:

$$\hat{e} = f(FPI, FDI, BT, R) \quad (3)$$

where FPI is Foreign Portfolio Investment; FDI is Foreign Direct Investment; BT is Balance of Trade and, R is Reserves.

The three previous definitions are the theoretical base of the econometric models used to forecast the future exchange rate. In each model, the following variables are added: exchange rate with a lag and exchange rate volatility (Taylor, 2001).

To generate the values of the exogenous variables for the period 2015-2017, two forecasting techniques are used: multiplicative time series decomposition and heavy ordered weighted moving average (HOWMA) operators.

Definition 4. To solve the problem of serial correlation of error, in each of the models, a first order autoregressive variable is used, which expresses the dependent variable and the forecast as a function of previous values of the time series (Messe and Rogoff, 1983):

$$\widehat{e_t} = \beta_0 + \beta_1 e_{t-1} \tag{4}$$

where e_{t-1} is the exchange rate with a lag.

Definition 5. To calculate the volatility, the coefficient of variation is used and is defined (Everitt, 1998) as:

$$v = \frac{\sigma}{\mu} \tag{5}$$

where v is volatility, σ is standard deviation and μ is the average.

Definition 6. The model of the decomposition of multiplicative time series, as indicated by Fischer and Planas (2000), is formulated as follows:

$$e_t = T_t * S_t * I_t * C_t \tag{6}$$

where e_t is the observed value, T_t is the trend, S_t is the seasonality, C_t is the cycle and I_t is the irregularity.

2.2. AGGREGATION INFORMATION OPERATORS

To understand better the formulation of the HOWMA operator that is presented in this paper, some of the basic concepts that utilizes are presented.

First, it is important to understand the basic concept of the OWA operator that was introduced by Yager (1988). This is the following.

Definition 7. An OWA operator of dimension n is an application $F: R^n \to R$ with an associated weight vector $\boldsymbol{w} = [w_1, w_2, \ldots, w_n]^T$, with $w_j \in [0, 1]$, $1 \leq j \leq n$, and

$$\sum_{j=1}^{n} w_{j=} w_1 + w_2 + \cdots + w_n = 1 \tag{7}$$

where

$$F(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j \, b_j \tag{8}$$

with b_j denoting the j-th largest element of the collection $a_1, a_2, \ldots, a_n$.

Second, when the OWA operator is not bounded to a sum of weights equal to 1, then the operator becomes the heavy ordered weighted average (HOWA) operator (Yager, 2002). The definition is as follows.

Definition 8. A heavy aggregation operator is an extension of the OWA operator that allows the sum of weights to be up to n. Thus, a HOWA operator is an application $R^n \to R$ which is associated with a weight vector $\boldsymbol{w}$ with $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, so that:

$$\text{HOWA}(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j \, b_j \tag{9}$$

where b_j is the j-th largest element of the collection $a_1, a_2, \ldots, a_n$.

One of the main characteristics of the HOWA operator is that the weight vector can range from $-\infty$ to ∞. Thus, the sum of the weights in the weighting vector $\boldsymbol{w}$ is unbounded: $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$. In this case, we can drastically under- or over-estimate the results according to the available information.

3. HEAVY ORDERED WEIGHTED MOVING AVERAGE OPERATOR

The HOWMA operator is an extension of the OWA operator and it combines in the same operator the characteristics of the HOWA operator with the characteristics of the moving average. In general, this operator uses a moving average with an associated weighting vector that goes from $-\infty$ to ∞. This is useful when we know that the result of the moving average will be lower or higher based on the expectations of the future. The definition is as follows (León *et al.*, 2016; 2018a).

Definition 9. A HOWMA operator is a mapping $R^{N-n+1} \to R$ defined as a given sequence $\{a_i\}_{i=1}^{N}$, where one obtains a new sequence $\{s_i\}_{i=1}^{N-n+1}$ that is multiplied by a heavy weighting vector, such that:

$$\text{HOWMA}(s_1, s_2, \ldots, s_{N-n+1}) = \sum_{j=1}^{N-n+1} w_j b_j \tag{10}$$

where b_j is the j-th largest element of the collection $s_1, s_2, \ldots, s_{N-n+1}$, and $\boldsymbol{w}$ is an associated weighting vector of dimension m with: $1 \leq \sum_{i=1+t}^{N-n+1+t} w_i \leq n$ and $w_i \in [0,1]$. Observe that here we can also expand the weighting vector from $-\infty$ to ∞. Thus, the weighting vector $\boldsymbol{w}$ becomes $-\infty \leq \sum_{j=1}^{N-n+1} w_j \leq \infty$.

With the use of the HOWMA operator to forecast the value of the variables that are used in an econometric model, we can use not only past information but also the expectations and future scenarios. With this operator it is easier to generate different scenarios, including the knowledge of the decision maker about the future of the fundamentals and the exchange rate. Our empirical work combines traditional econometric models and the HOWMA operator, as discussed in section 5.

4. EXCHANGE RATE FORECASTING. THE DEBATE IN THE LITERATURE

In emerging economies such as Mexico, companies must manage exchange rate risk constantly. The effect of an unanticipated depreciation or appreciation of the local currency generate an important impact on their sales, costs, profits, cash flows and investments, putting at risk their existence (Schiozer and Saito, 2009).

In free market economy, the behavior of exchange rate parity has a special resonance in decision making. The expectations of its trajectory influence the direction and intensity of capital flows. In addition, they affect the performance of financial instrument portfolios and the cost of risk hedging.

As result of the exchange rate crisis, which began in December 1994, Carstens and Werner (2000) state that the Bank of Mexico abandoned the controlled sliding regime and adopted a free floating regime. Since then, parity oscillations are in function of market conditions, which generates an unexpected, indeterminate and volatile exchange rate that jeopardizes the future cash flows of the company (Adler and Dumas, 1984; Bartram, 2008).

One of the main problems with predictions of floating exchange is that a random walk usually forecasts better than another model (Engel *et al.*, 2015). This is usually seen as a weakness of international macroeconomics (Bacchetta and van Wincoop, 2006) that is why many researchers have used different techniques to find success in forecasting exchange rate, for example, the use of panel data (Groen, 2005), Neural Networks (Sermpinis *et al.*, 2013), vector autoregressive (VAR) models (Born *et al.*, 2013), generalized autoregressive conditional heteroscedasticity (GARCH) model (Pilbeam and Langeland, 2014) and many more.

The contribution of this article is the use of an extension of OWA operators, the operator heavy ordered weighted moving average (HOWMA) (León-Castro *et al.*, 2016) as predictive technique of the future value of exogenous variables of econometric models based on fundamentals economical.

The use of this technique allows to include the expectations and knowledge of the decision maker; likewise, it makes possible the identification of new future scenarios which, with the use of traditional time series techniques, cannot be delineated.

5. DATA AND FORECASTING EVALUATION

In this section, the econometric models for the three theories are presented (PPP, IRP and BoP) and are generated with the following information: a) 2015: January 1994-December 2014, b) 2016: January 1994-December 2015 and c) 2017: January 1994-December 2016.

Each model uses the variables defined in the theoretical section. In addition, two variables are added to all the models. The first one, e_{t-1} which refers to the dependent variable with a lag[1] and the second, v_t representing the volatility of the period (Table 1-3).

Table 1. Variables in the PPP model

Variable	Name	Type of data[2]
$\boldsymbol{e_t}$	Exchange rate	Logarithm of the nominal value
$\boldsymbol{v_t}$	Volatility	Logarithm of the nominal value
$\boldsymbol{e_{t-1}}$	Exchange rate with a lag	Logarithm of the nominal value
$\boldsymbol{pci_t^{MEX}}$	Consumer price index in MEX	Logarithm of the nominal value
$\boldsymbol{pci_t^{USA}}$	Consumer price index in USA	Logarithm of the nominal value

Table 2. Variables in the IRP model

Variable	Name	Type of data
$\boldsymbol{e_t}$	Exchange rate	Logarithm of the nominal value
$\boldsymbol{v_t}$	Volatility	Logarithm of the nominal value
$\boldsymbol{e_{t-1}}$	Exchange rate with a lag	Logarithm of the nominal value
$\boldsymbol{r_t^{MEX}}$	Interest rate in MEX	Logarithm of the nominal value
$\boldsymbol{r_t^{USA}}$	Interest rate in USA	Logarithm of the nominal value

Table 3. Variables in the BoP model

Variable	Name	Type of data[3]
$\boldsymbol{e_t}$	Exchange rate	Nominal value
$\boldsymbol{v_t}$	Volatility	Nominal value
$\boldsymbol{e_{t-1}}$	Exchange rate with a lag	Nominal value
$\boldsymbol{FPI_t^{MEX}}$	Foreign portfolio investment in MEX	Nominal value
$\boldsymbol{FDI_t^{MEX}}$	Foreign direct investment in MEX	Nominal value
$\boldsymbol{R_t^{MEX}}$	International reserves in MEX	Nominal value
$\boldsymbol{BT_t^{MEX}}$	Balance of trade in MEX	Nominal value

[1] The dependent variable with a lag is used because the value of the exchange rate is significantly influenced by the value at the end of the previous period.

[2] The logarithm of the nominal value is used in PPP model and IRP model to linearize the relationship

[3] The logarithm transformation cannot be used in BoP model because some data is negative.

Table 4. PPP model from 2015 to 2017[4]

Variable	2015	2016	2017
v_t	$\beta_1 = 0.0234641$	$\beta_1 = 0.0230913$	$\beta_1 = 0.0237885$
e_{t-1}	$\beta_2 = 0.9141334$	$\beta_2 = 0.9302370$	$\beta_2 = 0.9428252$
pci_t^{MEX}	$\beta_3 = -0.0194534$	$\beta_3 = 0.0101966$	$\beta_3 = 0.0260894$
pci_t^{USA}	$\beta_4 = 0.0407992$	$\beta_4 = 0.0242960$	$\beta_4 = 0.0127441$
Constant	$\beta_0 = 0.1066104$	$\beta_0 = 0.0527191$	$\beta_0 = 0.0264180$
r^2	0.9899	0.9906	0.9915
adjusted r^2	0.9897	0.0991	0.9914

Analyzing the level of individual significance[5] in every year, for the case of the constant, pci_t^{MEX} and pci_t^{USA} the null hypothesis is not rejected, being rejected for e_{t-1} and v_t (Appendix: Table A1). On the other hand, for the complete model (Appendix: Table A4) the null hypothesis is rejected.[6]

In terms of multicollinearity[7] there are high levels for e_{t-1}, pci_t^{MEX} and pci_t^{USA} and low for v_t[8] (Appendix: Table A5). The values of the Durbin-Watson (D-W) statistic indicate that there is no autocorrelation at a significance level of 0.05 and 0.01 (Appendix: Table A8).

Table 5. IRP model from 2015 to 2017[9]

Variable	2015	2016	2017
v_t	$\alpha_1 = 0.2254560$	$\alpha_1 = 0.2243830$	$\alpha_1 = 0.0237093$
e_{t-1}	$\alpha_2 = 0.9604932$	$\alpha_2 = 0.9618081$	$\alpha_2 = 0.9644360$
r_t^{MEX}	$\alpha_3 = -0.0022782$	$\alpha_3 = -0.0022228$	$\alpha_3 = -0.0031719$
r_t^{USA}	$\alpha_4 = -0.0001871$	$\alpha_4 = -0.0007320$	$\alpha_4 = -0.0001735$
Constant	$\alpha_0 = 0.0983360$	$\alpha_0 = 0.8955890$	$\alpha_0 = 0.9064130$
r^2	0.9896	0.9905	0.9914
adjusted r^2	0.9895	0.9903	0.9913

[4] The formula for the PPP model is: $e_t = \beta_0 + \beta_1 v_t + \beta_2 e_{t-1} + \beta_3 pci_t^{MEX} - \beta_4 pci_t^{USA}$.

[5] The significance test is performed by analyzing the P statistic (Appendix: Table A1-A3) where the null hypothesis is rejected: $P < 0.05$ and not rejected when $P > 0.05$.

[6] An important aspect to consider in the study is that variables with non-significant levels are studied because it is desired to keep the original theoretical formulation of each model, in this sense, even if the variable does not contribute relevantly to the equation is taken into account.

[7] Multicollinearity in a regression model occurs when some predictors of the model are correlated with other predictors. Variance Inflation Factor (VIF) analysis is used to identify its presence.

[8] It is important to note that the presence of multicollinearity is not relevant in the present study, because the objective is to predict and not simplify the model. Although it is necessary to use theoretically robust explanatory variables, it is not intended to examine their individual significance or to eliminate some of them that diminishes the explanatory capacity of the model.

[9] The formula for the IRP model is: $e_t = \alpha_0 + \alpha_1 v_t + \alpha_2 e_{t-1} + \alpha_3 r_t^{MEX} - \alpha_4 r_t^{USA}$

Examining the level of significance, it is detected, for all years, and in the case of r_t^{MEX} and r_t^{USA} the null hypothesis is not rejected, being rejected for the constant, e_{t-1} and v_t (Appendix: Table A2). On the other hand, for the complete model (Appendix: Table A4), the null hypothesis is rejected.

The presence of multicollinearity is low for r_t^{MEX}, r_t^{USA}, e_{t-1} and v_t (Appendix: Table A6). Finally, D-W values indicate that there is no autocorrelation at a significance level of 0.05 and 0.01 (Appendix: Table A8).

Table 6. BoP model from 2015 to 2017[10]

Variable	2015	2016	2017
$\boldsymbol{v_t}$	$\theta_1 = 4.5873750$	$\theta_1 = 4.6158630$	$\theta_1 = 4.9582940$
$\boldsymbol{e_{t-1}}$	$\theta_2 = 0.9780775$	$\theta_2 = 0.9778967$	$\theta_2 = 0.9791574$
$\boldsymbol{FPI_t^{MEX}}$	$\theta_3 = -0.0000227$	$\theta_3 = -0.0000228$	$\theta_3 = -0.0000170$
$\boldsymbol{FDI_t^{MEX}}$	$\theta_4 = -0.0000162$	$\theta_4 = -0.0000157$	$\theta_4 = -0.0000132$
$\boldsymbol{R_t^{MEX}}$	$\theta_5 = 0.0000023$	$\theta_5 = 0.0000024$	$\theta_5 = 0.0000021$
$\boldsymbol{BT_t^{MEX}}$	$\theta_6 = -0.0000411$	$\theta_6 = -0.0000434$	$\theta_6 = -0.0000540$
Constant	$\theta_0 = 0.1738603$	$\theta_0 = 0.1629673$	$\theta_0 = 0.1406526$
$\boldsymbol{r^2}$	0.9876	0.9896	0.9902
adjusted $\boldsymbol{r^2}$	0.9873	0.9894	0.9900

Analyzing the level of individual significance, for the year 2015 the null hypothesis is not rejected for the constant and BT_t^{MEX}, in contrast, it is rejected for the variables: v_t, e_{t-1}, FPI_t^{MEX} FDI_t^{MEX} and R_t^{MEX}. In 2016, it is accepted for the constant and FPI_t^{MEX}, refusing to v_t, e_{t-1}, FDI_t^{MEX} R_t^{MEX} and BT_t^{MEX}. Finally, for 2017 it is accepted for the constant and, it is rejected for v_t, e_{t-1}, FPI_t^{MEX} R_t^{MEX}, FDI_t^{MEX} and BT_t^{MEX} (Appendix: Table A3). On the other hand, it is identified that for the complete model (Appendix: Table A4), the null hypothesis is rejected.

In terms of multicollinearity, we have low levels for v_t, e_{t-1}, FPI_t^{MEX} R_t^{MEX}, FDI_t^{MEX} and BT_t^{MEX} (Appendix: Table A7). According to D-W values, there is no autocorrelation at a significance level of 0.05 and 0.01 (Appendix: Table A8).

5.1. FORECAST OF THE EXOGENOUS VARIABLES

To forecast, we first used a multiplicative decomposition time series method in which we determined the type of trend, season, cycle and irregularity (Appendix: Table A9). Additionally, HOWMA operators were used to forecast the future of the variables in which a sequence $n = 6$ was used.

[10] The formula for the BoP model is: $e_t = \theta_0 + \theta_1 \boldsymbol{v_t} + \theta_2 \boldsymbol{e_{t-1}} - \theta_3 FPI_t^{MEX} - \theta_4 FDI_t^{MEX} - \theta_5 R_t^{MEX} - \theta_6 BT_t^{MEX}$.

Table 8. Forecast exchange rate USD/MXN in 2015 using HOWMA*

Date	Spot Exchange rate e_t	PPP Model $\widehat{e_t}$	Error	IRP Model $\widehat{e_t}$	Error	BoP Model $\widehat{e_t}$	Error
01-15	14.6808	14.4197	-0.2611	14.4255	-0.2553	14.5561	-0.1360
02-15	14.9230	14.4971	-0.4259	14.4952	-0.4278	14.5631	-0.3629
03-15	15.2136	14.8056	-0.4080	14.8092	-0.4044	14.8342	-0.3734
04-15	15.2208	15.1007	-0.1201	15.1098	-0.1110	15.1095	-0.0940
05-15	15.2475	15.4455	0.1980	15.4622	0.2147	15.4608	0.2407
06-15	15.4692	15.7565	0.2873	15.7792	0.3100	15.7469	0.3221
07-15	15.9225	16.0647	0.1422	16.0965	0.1740	16.0656	0.2047
08-15	16.5032	16.3817	-0.1215	16.4208	-0.0824	16.3778	-0.0523
09-15	16.8519	16.7113	-0.1406	16.7585	-0.0934	16.7060	-0.0603
10-15	16.5813	17.0465	0.4652	17.1021	0.5208	17.0394	0.5572
11-15	16.6325	17.3888	0.7563	17.4533	0.8208	17.3816	0.8627
12-15	17.0365	17.7368	0.7003	17.8103	0.7738	17.7273	0.8198
Average	15.8569	15.9462	0.0893	15.9769	0.1200	15.9640	0.1071

* The results are obtained by the Tables 4-6, year 2015

Table 9. Forecast exchange rate USD/MXN in 2016 using time series*

Date	Spot Exchange rate e_t	PPP Model $\widehat{e_t}$	Error	IRP Model $\widehat{e_t}$	Error	BoP Model $\widehat{e_t}$	Error
01-16	18.0255	17.0488	-0.9767	17.1546	-0.8709	17.1498	-0.8758
02-16	18.4777	16.8809	-1.5968	16.9827	-1.4949	17.0689	-1.4088
03-16	17.6923	17.0252	-0.6671	17.1248	-0.5675	17.1437	-0.5486
04-16	17.4905	17.1666	-0.3239	17.2738	-0.2167	17.3333	-0.1572
05-16	18.0980	17.4955	-0.6025	17.6169	-0.4811	17.6715	-0.4266
06-16	18.6506	17.6676	-0.9830	17.7936	-0.8570	17.9268	-0.7237
07-16	18.5862	17.3921	-1.1941	17.5030	-1.0832	17.7274	-0.8588
08-16	18.4715	17.5371	-0.9344	17.6466	-0.8249	17.8680	-0.6035
09-16	19.1955	17.8172	-1.3783	17.9344	-1.2611	18.0945	-1.1010
10-16	18.9157	17.8131	-1.1026	17.9220	-0.9937	17.9934	-0.9223
11-16	20.0371	18.0194	-2.0177	18.1422	-1.8949	18.2600	-1.7771
12-16	20.5156	17.9193	-2.5963	18.0382	-2.4774	18.1306	-2.3850
Average	18.6797	17.4819	-1.1978	17.5944	-1.0852	17.6973	-0.9824

*The results are obtained by the Table A9 (Appendix), year 2016.

Table 10. Forecast exchange rate USD/MXN in 2016 using HOWMA*

Date	Spot Exchange rate e_t	PPP Model $\widehat{e_t}$	Error	IRP Model $\widehat{e_t}$	Error	BoP Model $\widehat{e_t}$	Error
01-16	18.0255	16.9908	-1.0348	17.1103	-0.9152	17.0096	-1.0160
02-16	18.4777	17.6152	-0.8625	17.7435	-0.7342	17.6267	-0.8510
03-16	17.6923	17.9259	0.2336	18.0531	0.3608	17.9444	0.2521
04-16	17.4905	18.2969	0.8063	18.4220	0.9314	18.2786	0.7880
05-16	18.0980	18.6428	0.5447	18.7637	0.6657	18.5983	0.5003
06-16	18.6506	19.0387	0.3881	19.1588	0.5082	18.9734	0.3229
07-16	18.5862	19.4318	0.8456	19.5539	0.9677	19.3622	0.7759
08-16	18.4715	19.8395	1.3680	19.9613	1.4898	19.7537	1.2822
09-16	19.1955	20.2452	1.0497	20.3665	1.1710	20.1448	0.9493
10-16	18.9157	20.6634	1.7477	20.7843	1.8686	20.5480	1.6323
11-16	20.0371	21.0890	1.0519	21.2094	1.1723	20.9599	0.9228
12-16	20.5156	21.5250	1.0094	21.6451	1.1294	21.3819	0.8662
Average	18.6797	19.2753	0.5956	19.3977	0.7180	19.2151	0.5354

*The results are obtained by the Tables 4-6, year 2016

Table 11. Forecast exchange rate USD/MXN in 2017 using time series*

Date	Exchange rate MEXDER	PPP Model $\widehat{e_t}$	Error	IRP Model $\widehat{e_t}$	Error	BoP Model $\widehat{e_t}$	Error
01-17	20.7818	18.4543	-2.3275	18.6534	-2.1284	17.1498	-3.6320
02-17	20.8672	17.9187	-2.9485	18.0962	-2.7710	17.0689	-3.7983
03-17	20.9365	17.9072	-3.0293	18.0782	-2.8583	17.1437	-3.7928
04-17	21.0369	18.0168	-3.0201	18.1912	-2.8457	17.3333	-3.7036
05-17	21.1246	18.1620	-2.9626	18.3392	-2.7854	17.6715	-3.4531
06-17	21.2382	18.5056	-2.7326	18.6927	-2.5455	17.9268	-3.3114
07-17	21.3293	18.7231	-2.6062	18.9137	-2.4156	17.7274	-3.6019
08-17	21.4207	18.4370	-2.9837	18.6133	-2.8074	17.8680	-3.5527
09-17	21.5369	18.4719	-3.0650	18.6435	-2.8934	18.0945	-3.4424
10-17	21.6337	18.8572	-2.7765	19.0374	-2.5963	17.9934	-3.6403
11-17	21.7345	18.7124	-3.0221	18.8909	-2.8436	18.2600	-3.4745
12-17	21.8622	19.0072	-2.8550	19.1883	-2.6739	18.1306	-3.7316
Average	21.2919	18.4311	-2.8608	18.6115	-2.6804	17.6973	-3.5945

*The results are obtained by the Table A9 (Appendix), year 2017.

Table 12. Forecast exchange rate USD/MXN in 2017 using HOWMA*

Date	Exchange rate MEXDER	PPP Model $\widehat{e_t}$	Error	IRP Model $\widehat{e_t}$	Error	BoP Model $\widehat{e_t}$	Error
01-17	20.7818	20.0285	-0.7533	20.1849	-0.5969	19.8278	-0.9540
02-17	20.8672	20.2859	-0.5813	20.4473	-0.4199	20.0535	-0.8137
03-17	20.9365	20.6899	-0.2466	20.8290	-0.1075	20.4116	-0.5249
04-17	21.0369	20.9185	-0.1184	21.0437	0.0068	20.6017	-0.4352
05-17	21.1246	21.4049	0.2803	21.5240	0.3994	21.0536	-0.0710
06-17	21.2382	21.4909	0.2527	21.5936	0.3554	21.1299	-0.1083
07-17	21.3293	21.5661	0.2368	21.6522	0.3229	21.1584	-0.1709
08-17	21.4207	21.8677	0.4470	21.9423	0.5216	21.4325	0.0118
09-17	21.5369	22.6596	1.1227	22.7345	1.1976	22.2041	0.6672
10-17	21.6337	22.4716	0.8379	22.5211	0.8874	21.9762	0.3425
11-17	21.7345	22.7753	1.0408	22.8120	1.0775	22.2494	0.5149
12-17	21.8622	23.0084	1.1462	23.0300	1.1678	22.4500	0.5878
Average	21.2919	21.5973	0.3054	21.6929	0.4010	21.2124	-0.0795

*The results are obtained by the Tables 4-6, year 2017

To evaluate the errors obtained by each of the forecasting models and techniques, the results are analyzed by the mean absolute percentage error ($MAPE$), mean absolute deviation (MAD) and mean squared deviation (MSD), whose formulas are defined below.

$$MAPE = \frac{\sum|y_t - \hat{y}_t|/y_t}{n} \text{x}100$$

$$MAD = \frac{\sum|y_t - \hat{y}_t|}{n}$$

$$MSD = \frac{\sum|y_t - \hat{y}_t|^2}{n}$$

where y_t is the actual value, $\hat{y}_t$ is the forecasted value and n is the number of observations. With these formulas, the following results are obtained (Tables 13-15):

Table 13. Evaluation of the overall errors in 2015

	Time series			HOWMA		
	PPP	IRP	BoP	PPP	IRP	BoP
MAD	0.5796	0.5630	0.6194	0.3355	0.3490	0.3405
MSD	0.5180	0.4848	0.4949	0.1571	0.1796	0.1863
MAPE	3.9477	3.8347	4.2189	2.0988	2.3775	2.3194

Table 14. Evaluation of the overall errors in 2016

	Time series			HOWMA		
	PPP	IRP	BoP	PPP	IRP	BoP
MAD	1.1978	1.0853	0.9824	0.9118	0.9929	0.8466
MSD	1.8008	1.5407	1.3162	0.9862	1.1454	0.8508
MAPE	6.6449	6.0208	5.4499	5.0586	5.5081	4.6966

Table 15. Evaluation of the overall errors in 2017

	Time series			HOWMA		
	PPP	IRP	BoP	PPP	IRP	BoP
MAD	2.8608	2.6804	3.5945	0.5887	0.9929	0.8466
MSD	8.2279	7.2317	12.9420	0.4782	1.1454	0.8508
MAPE	13.7657	12.8977	17.2966	2.8326	5.5081	4.6966

With the information obtained by the three methods of error evaluation, it is observed that the results derived from the use of HOWMA operators are better in any of the three forecast models. Also, considering all the evaluations the best ones for each year are: a) 2014: PPP with HOWMA operator, b) 2015: BoP with HOWMA and c) 2016: PPP with HOWMA.

It is important to note that in 2016, a significantly greater error occurs than in 2015 when the economic scenario was more stable. Finally, by the end of 2017, the MEXDER predicts a parity of 21.8622. In contrast, with the use of HOWMA operators, it is estimated that it oscillates between 22.45 and 23.00 USD / MXN. This proves the importance of the operators and how the results can be improved by adding information of the economic agents through the weighting vector that compose the HOWMA operator.

Additionally, as seen in the calculation process, it is possible to generate different scenarios by modifying the values of the weighting vector according to the knowledge and expectations of the decision maker. This is not possible when time series are used, where the future value of the explanatory variables depends on their tendency and, therefore, the generation of scenarios depending on information about the future of each variable is not possible.

6. DISCUSSION

This article proposes the use of mathematical techniques of uncertainty for the forecast of the exchange rate, specifically the use of HOWMA operators. The main advantage of this operator is that, when combined with traditional

econometric models, it is possible to incorporate the expectations and knowledge of financial market experts and the information and past behavior of the variables. The results are compared using this new operator versus those resulting from the classical technique of a multiplicative decomposition time series.

The HOWMA operator facilitates the generation of different scenarios depending on the expectations of the decision maker by the reordering step of the weights, by changing to an optimistic or pessimistic reorder, this will be important when the decision makers wants to have a conservative (pessimistic) or risky (optimistic) posture and make a forecasting based on that kind of quality measures.

In scenarios of increasing uncertainty, such as those observed during 2016 and 2017, the forecast error is higher and, consequently, the exchange rate risk is increasing. In periods of relative stability, the models with operators generate predictions that allow us to minimize the risk.

The limitations of the study are in the subjectivity of the results because are based on weights defined by the decision maker, in this sense, if the weighting vector values change or the reordering step moves from optimistic to pessimistic, the results can change drastically. These limitations are also one of the advantages of the aggregation operators because by doing these not all the results are the same, like is the case by using time series, but is also true that an consensus of the results and weighting vector is hard to obtain and that is why the results reflect the reality or point of view of the specific decision makers.

For future research, it is proposed that new forecasting tools under uncertainty scenarios, such as new extensions of the OWA operator, including characteristics of induced or probabilistic, as well as other techniques such as expertons or methodology of the forgotten effects be considered for development (Merigó and Gil-Lafuente, 2010; León *et al.*, 2018b; Perez-Arellano *et al.*, 2019; Blanco-Mesa *et al.*, 2018b).

REFERENCES

Adler, M., & Dumas, B. (1984). Exposure to currency risk: definition and measurement. *Financial Management*, *22*(2), 41-50.

Bacchetta, P., & Van Wincoop, E. (2006). Can information heterogeneity explain the exchange rate determination puzzle? *The American Economic Review*, *96*(3), 552-576.

Bartram, S. M. (2008). What lies beneath: Foreign exchange rate exposure, hedging and cash flows. *Journal of Banking & Finance*, *32*(8), 1508-1521.

Blanco-Mesa, F., León-Castro, E., & Merigó, J. M. (2018a). Bonferroni induced heavy operators in ERM decision-making: A case on large companies in Colombia. *Applied Soft Computing*, *72*, 371-391.

Blanco-Mesa, F., León-Castro, E., & Merigó, J. M. (2018b). Bonferroni means with induced ordered weighted average operators. *International Journal of Intelligent Systems*, *34*(1), 3-23.

Born, B., Juessen, F., & Müller, G. J. (2013). Exchange rate regimes and fiscal multipliers. *Journal of Economic Dynamics and Control*, *37*(2), 446-465.

Carstens, A. G., & Werner, A. M. (2000). Mexico's monetary policy framework under a floating exchange rate regime. *Inflation Targeting in Practice: Strategic and Operational Issues and Application to Emerging Market Economies*. Seminar on inflation held at Rio de Janeiro, Brazil (pp.80-86). International Monetary Fund. Retrieved from: https://www.imf.org/external/pubs/ft/seminar/2000/targets/strach10.pdf

Cheung, Y., Chinn, M., & Pascual, A. (2002). Empirical Exchange Rate Models of the Nineties: Are Any Fit to Survive? *Journal of International Money and finance*, *24*(7), 1150-1175.

Dick, C. D., MacDonald, R., & Menkhoff, L. (2015). Exchange rate forecasts and expected fundamentals. *Journal of International Money and Finance*, *53*, 235-256.

Dornbusch, R. (1979). Monetary policy under exchange rate flexibility. *National Bureau of Economic Research*. Working paper No. 311.

Dornbusch, R. (1985). Purchasing Power Parity. *National Bureau of Economic Research*. Working paper No. 1591.

Engel, C., Mark, N. C., & West, K. D. (2015). Factor model forecasts of exchange rates. *Econometric Reviews*, *34*(1-2), 32-55.

Engel, C., Mark, N., & West, K. (2007). Exchange Rate Models Are Not as Bad as You Think. *National Bureau of Economic Research*. Working paper No. w13318.

Everitt, B. (1998). The Cambridge Dictionary of Statistics. UK: *Cambridge University Press*.

Fama, E. (1984). Forward and spot exchange rates. *Journal of Monetary Economics*, *14*(3), 319-338.

Fischer, B., & Planas, C. (2000). Large scale fitting of regression models with ARIMA errors. *Journal of Official Statistics*, *16*(2), 173-184.

Flood, R. P., & Rose, A. K. (2002). Uncovered interest parity in crisis. *IMF staff papers*, *49*(2), 252-266.

Groen, J. J. (2005). Exchange rate predictability and monetary fundamentals in a small multi-country panel. *Journal of Money, Credit, and Banking*, *37*(3), 495-516.

León-Castro, E., Avilés-Ochoa, A., Gil-Lafuente, A. M. (2016). Exchange Rate USD/MXN Forecast Through Econometric Models, Time Series and HOWMA Operators. *Economic Computation and Economic Cybernetics Studies and Research*, *50*(4), 135-150.

León-Castro, E., Avilés-Ochoa, E., & Merigó, J. M. (2018a). Induced heavy moving averages. *International Journal of Intelligent Systems*, *33*(9), 1823-1839.

León-Castro, E., Avilés-Ochoa, E., Merigó, J. M., & Gil-Lafuente, A. M. (2018b). Heavy Moving Averages and Their Application in Econometric Forecasting. *Cybernetics and Systems, 49*(1), 26-43.

Meese, R. A., & Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of international economics, 14*(1-2), 3-24.

Menkhoff, L., & Taylor, M. P. (2007). The obstinate passion of foreign exchange professionals: technical analysis. *Journal of Economic Literature, 45*(4), 936-972.

Merigó, J. M., & Gil-Lafuente, A. M. (2010). New decision-making techniques and their application in the selection of financial products. *Information Sciences, 180*(11), 2085-2094.

Nikolsko-Rzhevskyy, A., & Prodan, R. (2012). Markov switching and exchange rate predictability. *International Journal of Forecasting, 28*(2), 353-365.

Obstfeld, M., & Rogoff, K. (2001). The six major puzzles in international macroeconomics: is there a common cause? *NBER Macroeconomics Annual,15*, 339-412.

Pérez-Arellano, L. A., León-Castro, E., Avilés-Ochoa, E., & Merigó, J. M. (2019). Prioritized induced probabilistic operator and its application in group decision making. *International Journal of Machine Learning and Cybernetics*, *10*(3), 451-462.

Pilbeam, K., & Langeland, K. N. (2015). Forecasting exchange rate volatility: GARCH models versus implied volatility forecasts. *International Economics and Economic Policy, 12*(1), 127-142.

Preminger, A., & Franck, R. (2007). Forecasting exchange rates: A robust regression approach. *International Journal of Forecasting, 23*(1), 71-84.

Rogoff, K. (1996). The purchasing power parity puzzle. *Journal of Economic literature, 34*(2), 647-668.

Schiozer, R., & Saito, R. (2009). The determinants of currency risk management in latin american nonfinancial firms. *Emerging Markets Finance & Trade, 45*(1) 49-71.

Sermpinis, G., Theofilatos, K., Karathanasopoulos, A., Georgopoulos, E. F., & Dunis, C. (2013). Forecasting foreign exchange rates with adaptive neural networks using radial-basis functions and particle swarm optimization. *European Journal of Operational Research, 225*(3), 528-540.

Taylor, J. B. (2001). The role of the exchange rate in monetary-policy rules. *The American Economic Review, 91*(2), 263-267.

Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multicriteria decision making. *Systems, Man and Cybernetics, IEEE Transactions on, 18*(1), 183-190.

Yager, R. R. (2002). Heavy OWA operators. *Fuzzy optimization and decision making, 1*(4), 379-397.

Yager, R. R. (2009). Weighted maximum entropy OWA aggregation with applications to decision making under risk. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on, 39*(3), 555-564.

Yager, R. R., & Kacprzyk, J. (1997). *The ordered weighted averaging operators: Theory*. Norwell: Kluwer Academic Publishers.

DATABASES

USD/MXN Exchange rate retrieved from
http://www.sat.gob.mx/informacion_fiscal/tablas_indicadores/paginas/tipo_cambio.aspx

Consumer price index in Mexico retrieved from
http://www.sat.gob.mx/informacion_fiscal/tablas_indicadores/Paginas/inpc_2017.aspx

Consumer price index in USA retrieved from
https://www.bls.gov/cpi/detailed-report.htm

Interest rate in Mexico retrieved from
http://www.banxico.org.mx/SieInternet/consultarDirectorioInternetAction.do?accion=consultarCuadro&idCuadro=CF107&sector=22&locale=es

Interest rate in USA retrieved from
https://www.treasury.gov/resource-center/data-chart-center/interest- rates/Pages/TextView.aspx?data=billrates

Balance of trade in Mexico retrieved from
http://www.inegi.org.mx/est/contenidos/proyectos/registros/economicas/comercio/default_t.aspx

International reserves in Mexico retrieved from
http://www.banxico.org.mx/SieInternet/consultarDirectorioInternetAction.do?accion=consultarCuadro&idCuadro=CF106

Foreign investment in Mexico retrieved from
http://www.banxico.org.mx/politica-monetariaeinflacion/estadisticas/graficasdecoyuntura/comercio-exterior-y-balanza-de-pagos/inversion-extranjera-directa.html

APPENDIX

Table A1. Significance test for the PPP model

Variable	2015	2016	2017
Constant	0.131	0.421	0.668
e_{t-1}	0.000	0.000	0.000
v	0.000	0.000	0.000
pci^{USA}	0.625	0.784	0.458
pci^{MEX}	0.066	0.238	0.491

Table A2. Significance test for the IRP model

Variable	2015	2016	2017
Constant	0.000	0.000	0.000
e_{t-1}	0.000	0.000	0.000
v	0.000	0.000	0.000
r^{USA}	0.917	0.484	0.628
r^{MEX}	0.627	0.917	0.669

Table A3. Significance test for the BoP model

Variable	2015	2016	2017
Constant	0.066	0.119	0.074
e_{t-1}	0.000	0.000	0.000
v	0.000	0.000	0.000
BT	0.052	0.009	0.031
FDI	0.000	0.003	0.000
FPI	0.020	0.058	0.021
R	0.001	0.001	0.000

Table A4. Significance test for the entire model

Model	2015	2016	2017
PPP	0.000	0.000	0.000
IRP	0.000	0.000	0.000
BoP	0.000	0.000	0.000

Table A5. VIF for the PPP model

Variable	2015	2016	2017
e_{t-1}	12.42	12.07	12.42
v	1.07	1.08	11.45
pci^{USA}	8.97	8.82	5.49
pci^{MEX}	28.28	26.59	5.45

Table A6. VIF for the IRP model

Variable	2015	2016	2017
e_{t-1}	1.85	2.01	1.96
v	1.15	1.14	2.70
r^{USA}	3.22	3.26	4.78
r^{MEX}	3.01	3.26	3.29

Table A7. VIF for the BoP model

Variable	2015	2016	2017
e_{t-1}	2.68	3.12	3.28
v	1.03	1.03	1.03
BT	1.02	1.05	1.04
FDI	2.17	2.10	1.93
FPI	1.39	1.42	1.36
R	3.64	4.14	4.38

Table A8. Durbin-Watson test for all models

Model	2015	2016	2017
PPP	1.9071	1.8910	1.9062
IRP	1.9356	1.9159	1.9288
BoP	1.9137	1.9214	1.8895

Table A9. Trend type and formula for the variables[11]

PPP	2015	2016	2017
e_{t-1}	$6.3461 * 1.00352^t$	$6.3450 * 1.00352^t$	$6.3138 * 1.00358^t$
v	$0.007 * 1.00125^t$	$0.0074 * 1.00110^t$	$0.0073 * 1.00135^t$
pci^{USA}	$147.085 * 1.00201^t$	$147.937 * 1.00197^t$	$148.453 * 1.00193^t$
pci^{MEX}	$31.0370 * 1.00601^t$	$32.0835 * 1.00574^t$	$32.6300 * 1.00555^t$
IRP	2015	2016	2017
e_{t-1}	$6.3461 * 1.00352^t$	$6.3450 * 1.00352^t$	$6.3138 * 1.00358^t$
v	$0.007 * 1.00125^t$	$0.0074 * 1.00110^t$	$0.0073 * 1.00135^t$
r^{USA}	$16.359 * 0.978471^t$	$17.216 * 0.977784^t$	$14.810 * 0.979421^t$
r^{MEX}	$29.3549 * 0.990829^t$	$29.3015 * 0.990834^t$	$28.2408 * 0.991239^t$
BoP	2015	2016	2017
e_{t-1}	$6.3461 * 1.00352^t$	$6.3450 * 1.00352^t$	$6.3138 * 1.00358^t$
v	$0.007 * 1.00125^t$	$0.0074 * 1.00110^t$	$0.0073 * 1.00135^t$
BT	$-17 - 10.13t + 0.0402t^2$	$-126 - 6.22t + 0.0204t^2$	$-192 - 4.26t + 0.01122t^2$
FPI	$-157 - 17.7t + 0.2128t^2$	$-854 + 0.8t + 0.1305t^2$	$-1159 + 9.8t + 0.0883t^2$
FDI	$2840.4 * 1.00396^t$	$2841.02 * 1.00399^t$	$2881.36 * 1.00383^t$
R	$13.6034 * 1.01074^t$	$13781.2 * 1.01066^t$	$13974.2 * 1.01050^t$

[11] In this annex, the trend formulas of each of the variables are presented for the different years. It is identified that most of the tendencies are formulas of the exponential type and some of quadratic type. The above because when a variable has positive and negative data and exponential formula cannot be made.

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A NEW MEASURE OF VOLATILITY USING INDUCED HEAVY MOVING AVERAGES

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Abstract. The volatility is a dispersion technique widely used in statistics and economics. This paper presents a new way to calculate volatility by using different extensions of the ordered weighted average (OWA) operator. This approach is called the induced heavy ordered weighted moving average (IHOWMA) volatility. The main advantage of this operator is that the classical volatility formula only takes into account the standard deviation and the average, while with this formulation it is possible to aggregate information according to the decision maker knowledge, expectations and attitude about the future. Some particular cases are also presented when the aggregation information process is applied only on the standard deviation or on the average. An example in three different exchange rates for 2016 are presented, these are for: USD/MXN, EUR/MXN and EUR/USD.

Keywords: volatility, IHOWMA operator, exchange rate, moving average.

JEL Classification: C13, D81, F31, G17.

Introduction

Volatility is a basic concept in economics for measuring the variance of some variables like exchange rate, stock prices and some other (Garman & Klass, 1980; Rabbani, Grable, Heo, Nobre, & Kuzniak, 2017). To calculate the volatility, the coefficient of variation is used in historical data (Minton & Schrand, 1999), but some researchers have identified other variables that can make the results vary from forecast results, for example: a) in stock markets, Officer

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(1973) indicates that volatility is related to macroeconomic variables, b) in electoral voting, van Biezen, Mair, and Poguntke (2012) and Hooghe and Kern (2015) found that volatility is related to a decline in party memberships and c) in exchange rate, Aristotelous (2011) says that volatility in exchange rate can be related to exchange-rate regimes.

It is important to note, that volatility is not only influenced by the historical data, but can also be influenced by some macroeconomics variables like GDP, interest rate, foreign reserves and others Grossmann, Love, and Orlov (2014), but also there are some other information that can be added to the results, such as, knowledge of the decision maker about the future scenarios that it is important to add when there is important uncertainty in the problem (Yager, 1996, 2006).

In the context of planning of regional development, Kacprzyk and Straszak (1984) introduced a concept of stability of a regional development strategy to express a natural human bias, both of the regional authorities and inhabitants, that the variability of crucial development indicators (e.g. values of life quality indicators) should be limited, to yield some feeling of "stability". The changes of values of these indicators at consecutive planning stages are then subjected to an objective (against targets set by the authorities) and objective (against expectations of inhabitants) limitations, and then their degrees of satisfaction are aggregated, using both simple tools like the t/s-norms or some averages or more sophisticated ones like the various ordered weighted average (OWA) operators (Yager, 1988; Yager, Kacprzyk, & Beliakov, 2011). The approach was then extended by Kacprzyk (2015) and Kacprzyk, Romero, and Gomide (1999), and used for the modeling of regional development planning in many regions around the world. The OWA operators proposed in this paper can yield a new quality in a human consistent aggregation of such an assessment of stability which is clearly opposite to the volatility, and their application to the planning of regional development will be shown in next papers.

One way to add information to the volatility formula, is changing the usual average by adding weights and other tools. In this sense, we can use the ordered weighted average (OWA) operator, developed by Yager (1988) to generate new scenarios between the minimum and the maximum operator. Also, more complex operators using the OWA as a base have been studied by many authors (Yager et al., 2011; Emrouznejad & Marra, 2014; Blanco-Mesa, Merigó, & Gil-Lafuente, 2017). For example, the heavy OWA (HOWA) operator (Yager, 2002), where the weights are not bounded to the sum equal to 1, and the induced OWA (IOWA) operator (Yager & Filev, 1999), where the weights are induced according to the characteristics of the decision makers.

The aim of this paper is to analyze the use of the OWA operator and some of its extensions in the volatility formula. The main advantage of doing this is that it is possible to generate new scenarios identifying different elements that the traditional formula can't include, such as an optimistic and pessimistic results. In this sense, we introduce new concepts of volatility like the OWMA-volatility, IOWMA-volatility, HOWMA-volatility and IHOWMA-volatility.

An application of these new formulas in foreign exchange market is also developed. We use exchange rate USD/MXN, EUR/MXN and EUR/USD information from 2015–2016 to forecast the volatility for all the months for the year 2016. The information provided by the different formulations is presented in tables and graphs for the analysis.

The idea of these new operators and its application is in order to see that in economic markets that uncertainty is a common things and specifically in FOREX market that many of the signals that the market give to the investors and decision makers can be interpreted in many ways, the use of a formulation that will help them in order to include they expectations, knowledge and aptitude toward the market and obtain a result that include that characteristics give them a new approach and will help them to generate new strategies in order to minimize the risk or have a new vision where the market will led and make decisions based on that and not only in the traditional techniques that are usually based only in historical data and leave aside the qualitative information that we can have at the time when the decision is made.

The remainder of the paper is organized as follows. In Section 1, we review the volatility and some aggregation operators. Section 2 introduced different approaches of volatility using OWA operator as a base. Section 3 explains the steps for using the new volatility formulas, and Section 4 presents an application in USD/MXN exchange rate forecast. Last Section summarizes the main conclusions of the paper.

1. Preliminaries

In this section some basic concepts and definitions, such as volatility, aggregation operators, moving average operators and heavy moving average operators are presented.

1.1. Volatility

Since the collapse of the Bretton Woods agreement, exchange rate has presented an increasing volatility growing the risk that the companies have (Ethier, 1973; Héricourt & Poncet, 2015). One way to calculate volatility is based on the coefficient of variation, this can be defined as follows.

Definition 1. Volatility formula is:

$$v = \frac{\sigma}{\mu}, \tag{1}$$

where v is volatility, σ is standard deviation and μ is the average.

As it can be seen, the volatility formula integrates an average, in this sense we can introduce a more complex way of analysis by adding weights and other aggregation operators to the classical formulation.

1.2. Aggregation operators

The weighted average is one of the most common aggregation operators. The formulation is similar to the normal average but also includes a weight that multiplies each of the arguments. The definition is as follows (Merigó, Guillén, & Sarabia, 2015).

Definition 2. A WA operator of dimension n is a mapping $\mathrm{WA}: R^n \to R$ that has an associated weighting vector V, with $v_i \in [0,1]$ and $\sum_{i=1}^{n} v_i = 1$, such that

$$\mathrm{WA}(a_1,\ldots,a_n)=\sum_{i=1}^{n} v_j a_j, \tag{2}$$

where a_j represent the arguments and v_j is the weight related to that argument.

The OWA operator, is an extension of the WA operator, in the way that the weights can be rearranged according to different criteria, such as minimum, maximum, Laplace and Hurwicz. The definition is as follows (Yager, 1988).

Definition 3. An OWA operator of dimension n is a mapping $\mathrm{OWA}: R^n \to R$ with an associated weight vector W of dimension n such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, according to the following formula:

$$\mathrm{OWA}(a_1,a_2,\ldots,a_n)=\sum_{j=1}^{n} w_j b_j, \tag{3}$$

where b_j is the jth largest element of the collection a_i.

The heavy OWA (HOWA) operator (Yager, 2002) is an extension of the traditional OWA, where the main characteristic of this new operator is in the weight vector, which is not bounded by the sum of 1. This operator can be defined as follows.

Definition 4. A HOWA operator is a mapping $\mathrm{HOWA}: R^n \to R$ associated to a weighting vector W where $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, such that

$$\mathrm{HOWA}(a_1,a_2,\ldots,a_n)=\sum_{j=1}^{n} w_j b_j, \tag{4}$$

where b_j is the jth largest element of the collection a_i.

The characteristics of the HOWA operator are that it is monotonic and commutative, but it is not bounded by the minimum and the maximum operator. With the possibility to expand the weighting vector from $-\infty$ to ∞, we can drastically under- or overestimate the results of the HOWA operator, considering new scenarios according to information of the decision maker and some expectations of the future of the case in study.

The induced OWA (IOWA) operator was introduced by Yager and Filev (1999) and the main difference is that the reordering step is developed with order inducing variables according to the information, knowledge and preference of the decision maker. The IOWA operator can be defined as follows.

Definition 5. An IOWA operator of dimension n is an application $\mathrm{IOWA}: R^n \times R^n \to R$ that has a weighting vector associated, W of dimension n where the sum of the weights is 1 and $w_j \in [0,1]$, where an induced set of ordering variables are included (u_i) such that the formula is

$$\mathrm{IOWA}(u_1,a_1,u_2,a_2,\ldots,u_n,a_n)=\sum_{j=1}^{n} w_j b_j, \tag{5}$$

where b_j is the a_i value of the OWA pair $\langle u_i,a_i \rangle$ having the jth largest u_i. u_i is the order inducing variable and a_i is the argument variable.

1.3. Moving average operators

A moving average is an averaging technique that has been used in economics and statistics. This technique is an average that moves toward some part of the sample. The moving average, according to Kenney and Keeping (1962) can be defined as follows.

Definition 6. A moving average is defined as a sequence given $\{a_i\}_{i=1}^{N}$, where a moving average n is a new sequence $\{s_i\}_{i=1}^{N-n+1}$ defined from a_i taking the arithmetic mean of the sequence of n terms, such that

$$s_i = \frac{1}{n}\sum_{j=i}^{i+n-1} a_j, \tag{6}$$

The usual moving average can be extended by using weighted averages, obtaining the weighted moving average (WMA). Merigó and Yager (2013) defined it as follows.

Definition 7. A weighted moving average (WMA) of dimension m is a mapping $\underset{m+t}{\mathrm{WMA}}: R^m \to R$ that has an associated weighting vector W of dimension m with $W = \sum_{i=1+t} w_i = 1$ and $w_i \in [0,1]$, such that

$$\mathrm{WMA}(a_{1+t}, a_{2+t}, \ldots, a_{m+t}) = \sum_{i=1+t}^{m+t} w_i a_i, \tag{7}$$

where a_i is the ith argument, m is the total number of arguments considered from the whole sample and t indicates the movement performed in the average from the initial analysis. Note that if $w_i = 1/m$ for all i, the WMA becomes the MA aggregation.

If weights are added to the WMA we obtained the ordered weighted moving average (OWMA). Merigó and Yager (2013) defined it as follows.

Definition 8. An ordered weighted moving average (OWMA) of dimension m is a mapping $\underset{m+t}{\mathrm{OWMA}}: R^m \to R$ that has an associated weighting vector W of dimension m with $W = \sum_{j=1+t} w_j = 1$ and $w_j \in [0,1]$, such that

$$\mathrm{OWMA}(a_{1+t}, a_{2+t}, \ldots, a_{m+t}) = \sum_{j=1+t}^{m+t} w_j b_j, \tag{8}$$

where b_j is the jth largest argument of the a_i, m is the total number of arguments considered from the whole sample and t indicates the movement in the average from the initial analysis.

Also, note it is possible to add an induced reordering step the induced weighted vector resulting in the IOWMA operator is generated. Merigó and Yager (2013) defined it as follows.

Definition 9. An IOWMA of dimension m is a mapping $\underset{m+t}{\mathrm{IOWMA}}: R^M \times R^M \to R$ that has an associated weighting vector W of dimension m with $W = \sum_{j=1+t} w_j = 1$ and $w_j \in [0,1]$, such that

$$\mathrm{IOWMA}(u_{1+t}, a_{1+t}, u_{2+t}, a_{2+t}, \ldots, u_{m+t}, a_{m+t}) = \sum_{j=1+t}^{m+t} w_j b_j, \tag{9}$$

where b_j is the a_i value of the IOWMA pair u_i, a_i having the jth largest u_i, u_i is the order inducing variable, a_i is the argument variable, m is the total number of arguments considered from the whole sample and t indicates movement in the average from the initial analysis.

If heavy weights are considered in the OWMA operator, the heavy ordered weighted moving average (HOWMA) operator is obtained. The definition is as follows (León-Castro, Avilés-Ochoa, & Gil-Lafuente, 2016).

Definition 10. A HOWMA is defined as a sequence given $\{a_i\}_{i=1}^{N}$, where you get a new sequence $\{s_i\}_{i=1}^{N-n+1}$ which is multiplied by the heavy weight vector, so that

$$\text{HOWMA}(s_i) = \sum_{j=1}^{n} w_j b_j, \tag{10}$$

where b_j is the jth largest element of the collection $a_1 a_2, \ldots, a_n$, and W is an associated weighting vector of dimension m that satisfies $1 \leq \sum_{i=1+t}^{m+t} w_j \leq n$ and $w_j \in [0,1]$. Observe that here we can also expand the weighting vector to the range $-\infty$ to ∞. Thus, the weighting vector w becomes unbounded: $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

Considering the HOWMA operator and adding an induced reordering step, the induced heavy ordered weighted moving average (IHOWMA) operator is generated. León-Castro, Avilés-Ochoa, and Merigó (2018) defined it as follows.

Definition 11. A IHOWMA operator is defined as a given sequence $\{a_i\}_{i=1}^{N}$, where a new sequence $\{s_i\}_{i=1}^{N-m+1}$ is multiplied by a heavy weighting vector, such that

$$\text{IHOWMA}(u_{1+t}, a_{1+t}, u_{2+t}, a_{2+t}, \ldots, u_{n+t}, a_{m+t}) = \sum_{j=1+t}^{m+t} w_j b_j, \tag{11}$$

where b_j is jth element that has the largest value of u_i, u_i is the order inducing variable, and W is an associated weighting vector of dimension m with $W: 1 \leq \sum_{i=1+t}^{m+t} w_j \leq n$ and $w_j \in [0,1]$. Observe that we can also expand the weighting vector like in the case of the HOWMA operator.

2. Heavy moving average operators in the volatility

In order to include knowledge and experience of the decision maker in the traditional volatility formula, we consider different aggregation operators that will change the part of the usual average of the formula. First, we consider the OWMA operator with volatility, the definition is as follows.

Definition 12. An OWMA – Volatility operator of dimension m is a mapping $\text{OWMA} - \text{Volatility}: R^m \to R$ that has an associated weighting vector W of dimension m with $W = \sum_{j=1+t}^{m+t} w_j = 1$ and $w_j \in [0,1]$, such that

$$\text{OWMA} - \text{Volatility}(a_{1+t}, a_{2+t}, \ldots, a_{m+t}) = \frac{\sigma - \text{OWMA}}{\mu - \text{OWMA}}, \tag{12}$$

where $\sigma - \text{OWMA}$ is the OWMA standard deviation, $\mu - \text{OWMA}$ is the OWMA average.

Considering an induced reordering step in the OWMA – Volatility, the IOWMA – Volatility is proposed. The definition is.

Definition 13. An IOWMA – Volatility of dimension m is a mapping IOWMA – Volatility: $R^M \times R^M \to R$ that has an associated weighting vector W of dimension m with $W = \sum_{j=1+t}^{m+t} w_j = 1$ and $w_j \in [0,1]$, such that

$$\text{IOWMA} - \text{Volatility}\left(u_{1+t}, a_{1+t}, \ldots, u_{m+t}, a_{m+t}\right) = \frac{\sigma - \text{IOWMA}}{\mu - \text{IOWMA}}, \tag{13}$$

where $\sigma - \text{IOWA}$ is IOWMA standard deviation and $\mu - \text{IOWMA}$ is IOWMA average.

Also, if the weights are not bounded by 1, the HOWMA-Volatility is generated and the definition is.

Definition 14. A HOWMA – Volatility is defined as a sequence given $\{a_i\}_{i=1}^{N}$, where you get a new sequence $\{s_i\}_{i=1}^{N-n+1}$ which is associated with a weight vector w with $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, so that

$$\text{HOWMA} - \text{Volatility}\left(s_i\right) = \frac{\sigma - \text{HOWMA}}{\mu - \text{HOWMA}}, \tag{14}$$

where $\sigma - \text{HOWMA}$ is HOWMA standard deviation and $\mu - \text{HOWMA}$ is HOWMA average.

Finally, if a reordering step in the weights is used, the IHOWMA-Volatility is presented. The definition is as follows.

Definition 15. A IHOWMA – Volatility operator is defined as a given sequence $\{a_i\}_{i=1}^{N}$, where a new sequence $\{s_i\}_{i=1}^{N-m+1}$ which is associated with a weight vector w with $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, so that

$$\text{IHOWMA} - \text{Volatility}\left(u_{1+t}, a_{1+t}, \ldots, u_{n+t}, a_{m+t}\right) = \frac{\sigma - \text{IHOWMA}}{\mu - \text{IHOWMA}}, \tag{15}$$

where $\sigma - \text{IHOWMA}$ is IHOWMA standard deviation and $\mu - \text{IHOWMA}$ is IHOWMA average.

The properties for the OWMA – Volatility and IOWMA – Volatility are the followings (The proofs are trivial and thus omitted): a) Commutative: Assume f is the OWMA – Volatility or IOWMA – Volatility operator, the $f\left(u_i, a_i, \ldots, u_n, a_n\right) = f\left(u_i, b_i, \ldots, u_n, b_n\right)$. Monotonicity: Assume f is the OWMA – Volatility or IOWMA-Volatility operator; if $\left|u_i, a_i\right| \ge \left|u_i, b_i\right|$ for all i, then $f\left(u_i, a_i, \ldots, u_n, a_n\right) \ge f\left(u_i, b_i, \ldots, u_n, b_n\right)$. Bounded: Assume f is the OWMA – Volatility or IOWMA – Volatility operator, then $\min\{a_i\} \le f\left(u_i, a_i, \ldots, u_n, a_n\right) \le \max\{a_i\}$. Idempotency: Assume f is the OWMA – Volatility or IOWMA – Volatility operator; if $\left|u_i, a_i\right| = a$ for all i, then $f\left(u_i, a_i, \ldots, u_n, a_n\right) = a$. Also, note that HOWMA – Volatility and IHOWMA – Volatility operators have as properties commutative, monotonicity and idempotency, they are not bounded because the weighting vector can range $1 \le \sum_{j=1}^{n} w_j \le n$.

Also note that all formulas have 2 particular cases. One considering an attitude in the standard deviation and not in the average and another one considering an attitude in the average and not in the standard deviation. These cases are presented in Table 1.

Table 1. Volatility operators and their particular cases

Operator	General	Case 1	Case 2
OWMA – Volatility	$\frac{\sigma-\text{OWMA}}{\mu-\text{OWMA}}$	$\frac{\sigma}{\mu-\text{OWMA}}$	$\frac{\sigma-\text{OWMA}}{\mu}$
IOWMA – Volatility	$\frac{\sigma-\text{IOWMA}}{\mu-\text{IOWMA}}$	$\frac{\sigma}{\mu-\text{IOWMA}}$	$\frac{\sigma-\text{IOWMA}}{\mu}$
HOWMA – Volatility	$\frac{\sigma-\text{HOWMA}}{\mu-\text{HOWMA}}$	$\frac{\sigma}{\mu-\text{HOWMA}}$	$\frac{\sigma-\text{HOWMA}}{\mu}$
IHOWMA – Volatility	$\frac{\sigma-\text{IHOWMA}}{\mu-\text{IHOWMA}}$	$\frac{\sigma}{\mu-\text{IHOWMA}}$	$\frac{\sigma-\text{IHOWMA}}{\mu}$

To understand better each of the cases that are explained in Table 1, the new formulations of standard deviation are defined.

Definition 16. The $\sigma-\text{OWMA}$ is defined as follows:

$$\sigma-\text{OWMA}\left(a_{1+t},a_{2+t},\ldots,a_{i+t},\ldots,a_{m+t}\right)=\sum_{j=1+t}^{m+t}\sqrt{w_j\left[D_j^2\right]}, \tag{16}$$

where D_j is the jth largest argument of the $\left(a_{i+t}-\bar{x}\right)$, $\bar{x}$ is the median (or average) of the sample, m is the total number of arguments considered from the whole sample and t indicates the movement in the average from the initial analysis.

Definition 17. The $\sigma-\text{IOWMA}$ formulation is:

$$\sigma-\text{IOWMA}\left(u_{1+t},a_{1+t},\ldots,u_{m+t},a_{m+t}\right)=\sum_{j=1+t}^{m+t}\sqrt{w_j\left[D_j^2\right]}, \tag{17}$$

where D_j is the $\left(a_{i+t}-\bar{x}\right)$ value of the IOWMA pair u_i, a_i having the jth largest u_i, u_i is the order inducing variable, a_i is the argument variable, $\bar{x}$ is the median of the sample, m is the total number of arguments considered from the whole sample and t indicates movement in the average from the initial analysis.

Definition 18. The $\sigma-\text{HOWMA}$ is defined as:

$$\sigma-\text{HOWMA}\left(a_{1+t},a_{2+t},\ldots,a_{m+t}\right)=\sum_{j=1+t}^{m+t}\sqrt{w_j\left[D_j^2\right]}, \tag{18}$$

where D_j is the jth largest argument of the $\left(a_{i+t}-\bar{x}\right)$, $\bar{x}$ is the median of the sample, m is the total number of arguments considered from the whole sample, t indicates the movement in the average from the initial analysis, w is an associated weighting vector of dimension m that satisfies $1\le\sum_{i=1+t}^{m+t}w_j\le n$ and $w_j\in\left[0,1\right]$. Observe that here we can also expand the weighting vector to the range $-\infty$ to ∞. Thus, the weighting vector w becomes unbounded: $-\infty\le\sum_{j=1}^{n}w_j\le\infty$.

Definition 19. The $\sigma - IHOWMA$ is formulated as:

$$\sigma - IHOWMA\left(u_{1+t}, a_{1+t}, \ldots, u_{m+t}, a_{m+t}\right) = \sum_{j=1+t}^{m+t} \sqrt{w_j \left[D_j^2\right]} \tag{19}$$

where D_j is the $\left(a_{i+t} - \overline{x}\right)$ value of the IHOWMA pair u_i, a_i having the jth largest u_i, u_i is the order inducing variable, a_i is the argument variable, $\overline{x}$ is the median of the sample, m is the total number of arguments considered from the whole sample, t indicates the movement in the average from the initial analysis, w is an associated weighting vector of dimension m that satisfies $1 \leq \sum_{i=1+t}^{m+t} w_j \leq n$ and $w_j \in [0,1]$. Observe that here we can also expand the weighting vector to the range $-\infty$ to ∞. Thus, the weighting vector w becomes unbounded: $-\infty \leq \sum_{j=1}^{n} w_j \leq \infty$.

In order to explain these new approaches better, we present a simple numerical example.

Example 1. The exchange rate USD/MXN for 2015 and 2016 is as follow (see Table 2).

Table 2. Exchange rate USD/MXN for 2015–2016

Month	2015	2016
January	14.6808	18.0255
February	14.9231	18.4777
March	15.2136	17.6923
April	15.2208	17.4905
May	15.2475	18.0980
June	15.4692	18.6506
July	15.9225	18.5862
August	16.5032	18.4715
September	16.8519	19.1955
October	16.5813	18.9157
November	16.6325	20.0371
December	16.9437	20.5156

With this information, we calculate volatility for 2016 using the following information:

1. A weighted vector $W = (0.05, 0.05, 0.05, 0.07, 0.07, 0.07, 0.10, 0.10, 0.10, 0.10, 0.12, 0.12)$;
2. An induced vector $U = (12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1)$;
3. A heavy weighted vector $W = (0.05, 0.05, 0.05, 0.07, 0.07, 0.10, 0.10, 0.10, 0.10, 0.15, 0.15)$.

The results are as follows (see Tables 3–5).

Table 3. Volatility for USD/MXN 2016 using the general formulation

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
January	0.0514	0.0502	0.0486	0.0500	0.0477
February	0.0584	0.0552	0.0568	0.0545	0.0572
March	0.0656	0.0612	0.0645	0.0604	0.0656
April	0.0638	0.0609	0.0607	0.0610	0.0606
May	0.0587	0.0577	0.0536	0.0584	0.0516
June	0.0536	0.0539	0.0493	0.0545	0.0477
July	0.0504	0.0494	0.0479	0.0492	0.0473
August	0.0468	0.0453	0.0444	0.0444	0.0440
September	0.0447	0.0445	0.0403	0.0442	0.0394
October	0.0473	0.0476	0.0429	0.0482	0.0423
November	0.0432	0.0442	0.0392	0.0452	0.0389
December	0.0445	0.0423	0.0433	0.0422	0.0435

Table 4. Volatility for USD/MXN 2016 using the case 1 formulation

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
January	0.0514	0.0495	0.0492	0.0521	0.0514
February	0.0584	0.0544	0.0578	0.0567	0.0619
March	0.0656	0.0601	0.0657	0.0628	0.0712
April	0.0638	0.0599	0.0617	0.0633	0.0655
May	0.0587	0.0569	0.0544	0.0607	0.0556
June	0.0536	0.0532	0.0500	0.0568	0.0513
July	0.0504	0.0487	0.0486	0.0513	0.0509
August	0.0468	0.0447	0.0449	0.0464	0.0474
September	0.0447	0.0441	0.0407	0.0462	0.0422
October	0.0473	0.0471	0.0434	0.0503	0.0455
November	0.0432	0.0437	0.0396	0.0472	0.0417
December	0.0445	0.0419	0.0437	0.0441	0.0467

Table 5. Volatility for USD/MXN 2016 using the case 2 formulation

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
January	0.0514	0.0522	0.0507	0.0494	0.0477
February	0.0584	0.0593	0.0575	0.0561	0.0540
March	0.0656	0.0667	0.0644	0.0632	0.0605
April	0.0638	0.0649	0.0628	0.0615	0.0590
May	0.0587	0.0596	0.0579	0.0565	0.0545
June	0.0536	0.0544	0.0529	0.0515	0.0499
July	0.0504	0.0511	0.0497	0.0483	0.0468
August	0.0468	0.0474	0.0463	0.0448	0.0435
September	0.0447	0.0451	0.0442	0.0427	0.0416
October	0.0473	0.0479	0.0468	0.0453	0.0441
November	0.0432	0.0437	0.0427	0.0413	0.0402
December	0.0445	0.0450	0.0441	0.0426	0.0415

3. Generalizations of the IHOWMA – Volatility

In this section, the use of quasi-arithmetic is used in order to generalize the formulation and present some interesting particular cases (Yager, 2004; Merigó & Gil-Lafuente, 2009; Zhou & Chen, 2011). To understand better these new formulations the whole formula will be explained (instead of using $\sigma-$IHOWMA and $\mu-$IHOWMA as simplification for example). The Quasi-IHOWMA-Volatility formulation is as follows.

Definition 20. A Quasi – IHOWMA – Volatility operator is defined as a given sequence $\{a_i\}_{i=1}^{N}$, where a new sequence $\{s_i\}_{i=1}^{N-m+1}$ which is associated with a weight vector w with $w_j \in [0,1]$ and $1 \leq \sum_{j=1}^{n} w_j \leq n$, so that

$$\text{Quasi} - \text{IHOWMA} - \text{Volatility}\left(u_{1+t}, a_{1+t}, \ldots, u_{n+t}, a_{m+t}\right) = g^{-1}\left(\frac{\sum_{j=1+t}^{m+t} \sqrt{w_j\left[\left(g\left(D_j\right)\right)^2\right]}}{\sum_{j=1+t}^{m+t} w_j g\left(b_j\right)}\right), \tag{20}$$

where b_j is the a_{i+t} and D_j is the $\left(a_{i+t} - \overline{x}\right)$ value of the IHOWMA pair u_{i+t}, a_{i+t} having the jth largest u_i, u_i is the order inducing variable, a_i is the argument variable, $\overline{x}$ is the median of the sample, m is the total number of arguments considered from the whole sample, t indicates the movement in the average from the initial analysis, g and h are strictly continuous monotone functions, w is an associated weighting vector of dimension m that satisfies

$1 \le \sum_{i=1+t}^{m+t} w_j \le n$ and $w_j \in [0,1]$. Observe that here we can also expand the weighting vector to the range $-\infty$ to ∞. Thus, the weighting vector w becomes unbounded: $-\infty \le \sum_{j=1}^{n} w_j \le \infty$.

Also, is possible to generalize the OWMA-volatility, IOWMA-volatility and HOWMA-volatility operators by using quasi-arithmetic means. These formulations are as follows.

Definition 21. A Quasi-HOWMA-Volatility operator is defined as a given sequence $\{a_i\}_{i=1}^{N}$, where a new sequence $\{s_i\}_{i=1}^{N-m+1}$ which is associated with a weight vector w with $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, so that

$$\text{Quasi} - \text{HOWMA} - \text{Volatility}(a_{1+t},\ldots,a_{m+t}) = g^{-1}\left(\frac{\sum_{j=1+t}^{m+t}\sqrt{w_j\left[\left(g\left(D_j\right)^2\right]\right.}}{\sum_{j=1+t}^{m+t} w_j g\left(b_j\right)}\right), \quad (21)$$

where b_j is the jth largest argument of the a_{i+t} and D_j is the jth largest argument of the $\left(a_{i+t} - \bar{x}\right)$, $\bar{x}$ is the median of the sample, m is the total number of arguments considered from the whole sample, t indicates the movement in the average from the initial analysis, g and h are strictly continuous monotone functions, w is an associated weighting vector of dimension m that satisfies $1 \le \sum_{i=1+t}^{m+t} w_j \le n$ and $w_j \in [0,1]$. Observe that here we can also expand the weighting vector to the range $-\infty$ to ∞. Thus, the weighting vector w becomes unbounded: $-\infty \le \sum_{j=1}^{n} w_j \le \infty$.

Definition 22. A Quasi-IOWMA-Volatility operator is defined as a given sequence $\{a_i\}_{i=1}^{N}$, where a new sequence $\{s_i\}_{i=1}^{N-m+1}$ which is associated with a weight vector w with $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, so that

$$\text{Quasi} - \text{IOWMA} - \text{Volatility}(u_{1+t},a_{1+t},\ldots,u_{n+t},a_{m+t}) = g^{-1}\left(\frac{\sum_{j=1+t}^{m+t}\sqrt{w_j\left[\left(g\left(D_j\right)^2\right]\right.}}{\sum_{j=1+t}^{m+t} w_j g\left(b_j\right)}\right), \quad (22)$$

where b_j is the a_{i+t} and D_j is the $\left(a_{i+t} - \bar{x}\right)$ value of the IOWMA pair u_i, a_i having the jth largest u_i, u_i is the order inducing variable, a_i is the argument variable, $\bar{x}$ is the median of the sample, m is the total number of arguments considered from the whole sample, t indicates the movement in the average from the initial analysis and g and h are strictly continuous monotone functions.

Definition 23. A Quasi-OWMA-Volatility operator is defined as a given sequence $\{a_i\}_{i=1}^{N}$, where a new sequence $\{s_i\}_{i=1}^{N-m+1}$ which is associated with a weight vector w with $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, so that

$$\text{Quasi}-\text{OWMA}-\text{Volatility}\left(a_{1+t},\ldots,a_{m+t}\right)=g^{-1}\left(\frac{\sum_{j=1+t}^{m+t}\sqrt{w_j\left[\left(g\left(D_j\right)\right)^2\right]}}{\sum_{j=1+t}^{m+t}w_j g\left(b_j\right)}\right), \quad (23)$$

where b_j is the jth largest argument of the a_{i+t} and D_j is the jth largest argument of the $\left(a_{i+t}-\bar{x}\right)$, $\bar{x}$ is the median of the sample, m is the total number of arguments considered from the whole sample, t indicates the movement in the average from the initial analysis and g and h are strictly continuous monotone functions.

In Table 6, there are present some of the main particular cases of the Quasi – IHOWMA – volatility.

Table 6. Families of generalized IHOWMA volatility

Particular case	Quasi – IHOWMA – volatility
$p_i=\frac{1}{n}$, for all i	Quasi-arithmetic induced ordered weighted moving average volatility (Quasi – HOWMA – volatility)
$w_i=\frac{1}{n}$, for all i	Quasi-arithmetic induced heavy moving average volatility (Quasi – IHMA – volatility)
$h\left(D\right)=D^{\lambda}$, $g\left(b\right)=b^{\lambda}$	Generalized IHOWMA – volatility
$h\left(D\right)=D$, $g\left(b\right)=b$	IHOWMA – volatility
$h\left(D\right)=D^2$, $g\left(b\right)=b^2$	Induced heavy ordered weighted quadratic moving average volatility (IHOWQMA – volatility)
$h\left(D\right)\to D^{\lambda}$, $g\left(b\right)\to b^{\lambda}$, for $\lambda\to 0$	Induced heavy ordered weighted geometric moving average volatility (IHOWGMA – volatility)
$h\left(D\right)=D^{-1}$, $g\left(b\right)=b^{-1}$	Induced heavy ordered weighted harmonic moving average volatility (IHOWHMA – volatility)
$h\left(D\right)=D^3$, $g\left(b\right)=b^3$	Induced heavy ordered weighted cubic moving average volatility (IHOWCMA – volatility)
$h\left(D\right)\to D^{\lambda}$, $g\left(b\right)\to b^{\lambda}$, for $\lambda\to\infty$	Maximum operator
$h\left(D\right)\to D^{\lambda}$, $g\left(b\right)\to b^{\lambda}$, for $\lambda\to-\infty$	Minimum operator

4. IHOWMA-Volatility with EUR/MXN and EUR/USD exchange rate

4.1. Theoretical approach

The study of the volatility has been applied in different assets clases such as foreign exchange markets, equity, bond and many more (Carr & Wu, 2008; Mueller, Vedolin, & Yen, 2012; Della Corte, Ramadorai, & Sarno, 2016). Among the largest and important financial market in the world is the foreign exchange market, that is why forecasting volatility is relevant to

organizations, financial institutions and traders wanting to do risk management strategies (Pilbeam & Langeland, 2015).

Different techniques have been used in order to forecast volatility, in this paper we present a new approach using as base the classical formulation of volatility (standard deviation and average) and different OWA operator extensions, specifically the IHOWMA operator and some of its particular cases such as the HOWMA, IOWMA and OWMA operators. Also, three different volatility formulations have been presented adding the information in the standard deviation (case 1) or in the average (case 2). The steps to use these new OWA volatility operators in foreign exchange rate are as follow.

Step 1. It is necessary to identified the large of the moving average that will be used to forecast volatility, it can be twelve months (1 year), six months (Half year) or three months (A quarter).

Step 2. The information of the exchange rate currencies that want to be evaluated.

Step 3. The weighting vector that will be applied to the arguments based in the importance of the information and the decision maker knowledge. It is important to note, for the Heavy weights it can range from $-\infty$ to ∞, this is done when the decision maker wants to underestimate or overestimate the results.

Step 4. The induced vector that will be used to order the weights according to the expectations of the decision maker has to be identified.

Step 5. With the information provided by Step 1 to 5, now it is important to define which of the different OWA-volatility formulations will be made (General, case 1 or case 2).

Step 6. An analysis has to be done about the information obtained by the different operators and volatility formulations.

4.2. Numerical example

Step 1. The large of the moving average will be twelve months

Step 2. The exchange rate that will be analyzed will be EUR/MXN and EUR/USD for 2016 (see Tables 7–8).

Step 3. A weighted vector

$W = (0.05, 0.05, 0.05, 0.07, 0.07, 0.07, 0.10, 0.10, 0.10, 0.10, 0.12, 0.12)$.

And a heavy weighted vector

$W = (0.05, 0.05, 0.05, 0.07, 0.07, 0.10, 0.10, 0.10, 0.10, 0.15, 0.15)$.

Step 4. An induced vector $U = (12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1)$.

Step 5. For both cases (EUR/MXN and EUR/USD) the three OWA-Volatility formulation (General, case 1 and case 2) and the OWMA, IOWMA, HOWMA and IHOWMA operators will be applied (see Tables 9–14).

Table 7. Exchange rate EUR/MXN for 2015–2016

Month	2015	2016
January	16.8343	20.4321
February	16.8269	20.0299
March	16.3233	19.7873
April	16.6941	19.8733
May	17.0320	20.6288
June	17.4147	20.9361
July	17.5338	20.5435
August	18.3213	20.7234
September	18.9434	21.2936
October	18.8437	21.3345
November	18.1610	21.0676
December	18.0803	21.7698

Table 8. Exchange rate EUR/USD for 2015–2016

Month	2015	2016
January	1.1387	1.1026
February	1.1138	1.1041
March	1.0781	1.1274
April	1.0900	1.1355
May	1.1150	1.1270
June	1.1241	1.1240
July	1.1006	1.1063
August	1.1110	1.1200
September	1.1254	1.1201
October	1.1277	1.1110
November	1.0925	1.0895
December	1.0767	1.0599

Table 9. Volatility for EUR/MXN 2016 using the general formulation

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
January	0.0497	0.0476	0.0464	0.0461	0.0443
February	0.0650	0.0612	0.0636	0.0605	0.0641
March	0.0695	0.0661	0.0676	0.0662	0.0685
April	0.0651	0.0619	0.0617	0.0620	0.0608

End of Table 9

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
May	0.0604	0.0585	0.0567	0.0585	0.0551
June	0.0590	0.0575	0.0562	0.0576	0.0551
July	0.0585	0.0572	0.0557	0.0569	0.0552
August	0.0528	0.0516	0.0497	0.0501	0.0488
September	0.0509	0.0503	0.0460	0.0486	0.0445
October	0.0532	0.0543	0.0465	0.0542	0.0456
November	0.0531	0.0561	0.0462	0.0582	0.0454
December	0.0437	0.0460	0.0378	0.0474	0.0368

Table 10. Volatility for EUR/MXN 2016 using the case 1 formulation

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
January	0.0497	0.0470	0.0470	0.0482	0.0476
February	0.0650	0.0603	0.0647	0.0630	0.0693
March	0.0695	0.0650	0.0689	0.0687	0.0744
April	0.0651	0.0608	0.0627	0.0644	0.0657
May	0.0604	0.0577	0.0574	0.0610	0.0593
June	0.0590	0.0567	0.0570	0.0600	0.0594
July	0.0585	0.0563	0.0566	0.0592	0.0597
August	0.0528	0.0510	0.0502	0.0524	0.0525
September	0.0509	0.0498	0.0465	0.0509	0.0478
October	0.0532	0.0536	0.0472	0.0566	0.0491
November	0.0531	0.0553	0.0468	0.0606	0.0489
December	0.0437	0.0455	0.0382	0.0496	0.0394

Table 11. Volatility for EUR/MXN 2016 using the case 2 formulation

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
January	0.0497	0.0503	0.0491	0.0476	0.0463
February	0.0650	0.0661	0.0640	0.0625	0.0602
March	0.0695	0.0707	0.0682	0.0670	0.0640
April	0.0651	0.0662	0.0641	0.0627	0.0603
May	0.0604	0.0612	0.0596	0.0579	0.0560
June	0.0590	0.0599	0.0582	0.0567	0.0548
July	0.0585	0.0594	0.0576	0.0563	0.0542
August	0.0528	0.0534	0.0522	0.0505	0.0491
September	0.0509	0.0514	0.0503	0.0486	0.0474
October	0.0532	0.0538	0.0525	0.0510	0.0494
November	0.0531	0.0538	0.0524	0.0510	0.0493
December	0.0437	0.0442	0.0433	0.0418	0.0408

Table 12. Volatility for EUR/USD 2016 using the general formulation

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
January	0.0182	0.0175	0.0171	0.0171	0.0170
February	0.0160	0.0152	0.0155	0.0150	0.0153
March	0.0158	0.0152	0.0149	0.0150	0.0141
April	0.0150	0.0140	0.0148	0.0135	0.0142
May	0.0155	0.0138	0.0160	0.0132	0.0156
June	0.0160	0.0146	0.0162	0.0140	0.0158
July	0.0160	0.0155	0.0150	0.0148	0.0144
August	0.0157	0.0156	0.0146	0.0148	0.0139
September	0.0158	0.0159	0.0143	0.0152	0.0136
October	0.0156	0.0159	0.0133	0.0155	0.0126
November	0.0151	0.0163	0.0126	0.0165	0.0120
December	0.0154	0.0159	0.0137	0.0160	0.0133

Table 13. Volatility for EUR/USD 2016 using the case 1 formulation

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
January	0.0182	0.0175	0.0171	0.0182	0.0179
February	0.0160	0.0152	0.0155	0.0159	0.0162
March	0.0158	0.0152	0.0149	0.0159	0.0149
April	0.0150	0.0140	0.0148	0.0143	0.0151
May	0.0155	0.0139	0.0160	0.0140	0.0166
June	0.0160	0.0146	0.0162	0.0148	0.0167
July	0.0160	0.0155	0.0151	0.0156	0.0153
August	0.0157	0.0156	0.0146	0.0157	0.0148
September	0.0158	0.0159	0.0143	0.0161	0.0144
October	0.0156	0.0159	0.0133	0.0164	0.0134
November	0.0151	0.0162	0.0127	0.0174	0.0127
December	0.0154	0.0159	0.0137	0.0169	0.0141

Table 14. Volatility for EUR/USD 2016 using the case 2 formulation

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
January	0.0182	0.0182	0.0182	0.0171	0.0172
February	0.0160	0.0160	0.0160	0.0151	0.0151
March	0.0158	0.0158	0.0158	0.0149	0.0149
April	0.0150	0.0150	0.0150	0.0141	0.0141
May	0.0155	0.0155	0.0155	0.0146	0.0146

End of Table 14

Month	Volatility	OWMA – Volatility	IOWMA – Volatility	HOWMA – Volatility	IHOWMA – Volatility
June	0.0160	0.0160	0.0160	0.0151	0.0151
July	0.0160	0.0160	0.0160	0.0151	0.0151
August	0.0157	0.0158	0.0157	0.0149	0.0148
September	0.0158	0.0158	0.0158	0.0149	0.0149
October	0.0156	0.0156	0.0155	0.0147	0.0147
November	0.0151	0.0151	0.0151	0.0143	0.0142
December	0.0154	0.0154	0.0154	0.0146	0.0145

Step 6. Taking into account example 1 (USD/MXN) and the information presented in section 6.2 of this paper (EUR/MXN and EUR/USD) it is possible to note that with the use of different aggregation operators different perspectives for the volatility according if the attitude is given to both standard deviation and average or to one of them can be done. With this it is possible to generate new strategies for risk management according to this new data. For simplify the analysis a graph for the three cases for the general formulation of the OWA-Volatility is presented (see Figures 1–3).

If we analyze the data, it is possible to see that for example the month with more volatility for USD/MXN is march and can range from 0.0601 (OWMA with case 1) to 0.0878 (IHOWMA with case 2), in the case of the EUR/MXN is also march and can range from 0.0640 (IHOWMA with case 2) to 0.0744 (IHOWMA with case 1) and for EUR/USD is January and range from 0.0170 (IHOWMA with general case) to 0.0182 (Volatility). These information is relevant because can be used to different things like speculation, risk management strategy, take decision about buying or selling your products in an specific month of the year and many more (as can be seen there are different levels of volatility among different currencies so it is possible to make approaches where you can buy USD/MXN and then get them back from EUR/USD and change it back to EUR/MXN). Finally, it is important to take in account that with all these new scenarios it is possible to increase the knowledge of the financial market and explain in a better way how they will be in the future.

Additionally, a matrix with scatterplot graphs is presented (see Figure 4) to make a comparison between each of the methods, in a common way, it is found that each of these has a positive relationship, which is obvious derived from using the same data series as base. The interesting thing is in how there are relations that show a strength of relationship not as strong as is the case of IOWMA volatility and the HOWMA volatility and on the other hand we find strong relationships such as those presented with the IOWMA volatility and the IHOWMA volatility. This analysis is important because it allows us to find how the use of different operators can generate results with different interpretations and that they can differ from each other even when the same data is used to analyze.

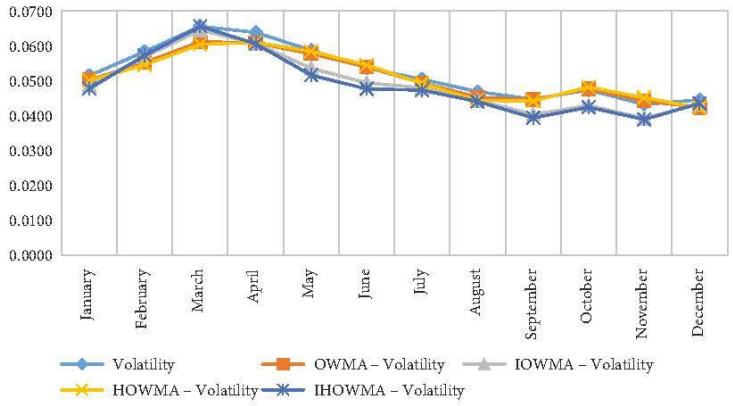

Figure 1. USD/MXN 2016 volatility by different operators for general formulation

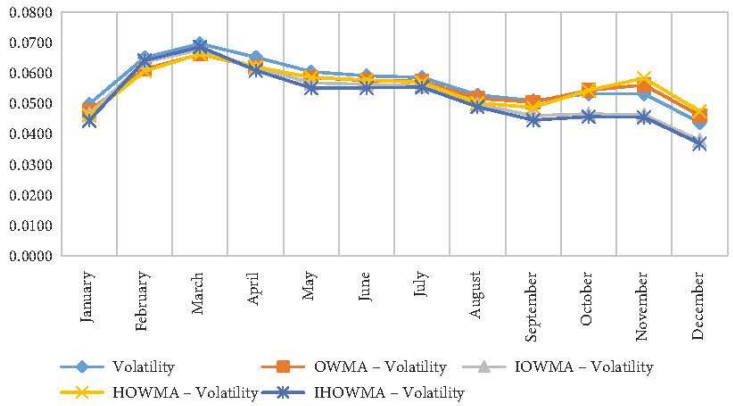

Figure 2. EUR/MXN 2016 volatility by different operators for general formulation

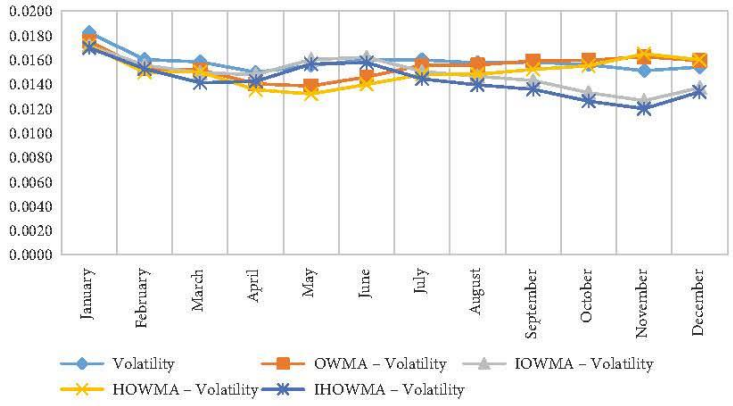

Figure 3. EUR/USD 2016 volatility by different operators for general formulation

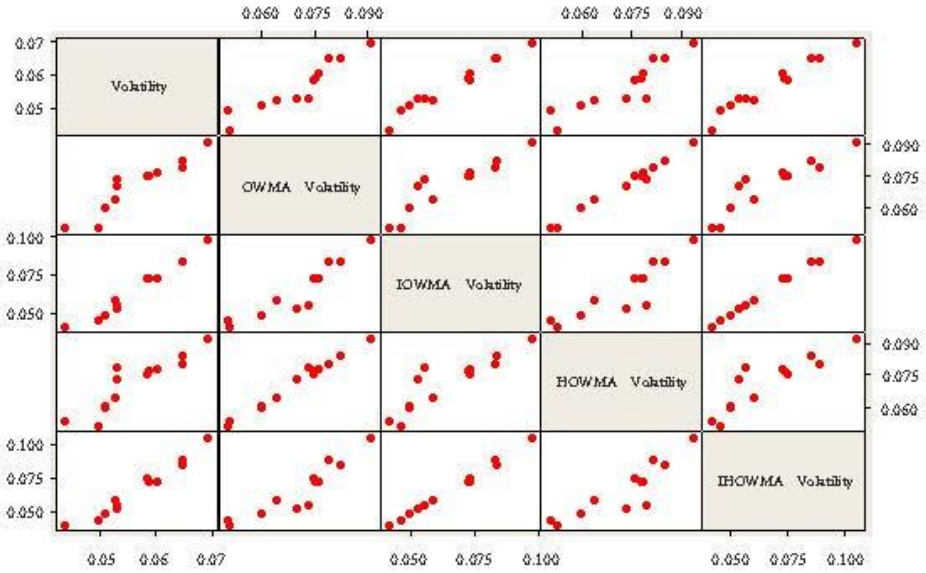

Figure 4. Matrix with scatterplot graphs for each method

Conclusions

The paper introduced an extension of the classical formulation of the volatility called the induced heavy ordered weighted moving average (IHOWMA) volatility. This extension includes the characteristics of the IHOWMA operator in the standard deviation and/or average that are the components of the classical volatility formula, in this sense, it is possible to aggregate the knowledge, experience and expectative of the decision maker by the use of an induced-order variable and a heavy weighting vector.

The definition and some of the families of the IHOWMA-volatility operator are presented. Those ranges of families are the OWMA-volatility, IOWMA-volatility and HOWMA-volatility. Additionally, three different formulations are presented: 1) General formulation where the aggregation process is used in both the standard deviation and the average, 2) Case 1 formulation where the process is only applied to the standard deviation and 3) Case 2 formulation where the aggregation is done in the average only.

An application of this extension is used in foreign exchange rate market. The forecast of the volatility for 2016 for the USD/MXN, EUR/MXN and EUR/USD currencies are presented. As can be seen with the use of the different operators the volatility between the currencies can change, these kind of information is very important because when we analyze the FOREX market it is important to note that not every agent interpreted the information in the same way. In these sense, the use of aggregation operators can help us to have a better idea of how can things can change if we include different aspects of the decision maker or the market into the formulation and in this way generate a better risk management strategy and these is the most important aspect of these new formulations, have a new approach of

the future and know that the market can have different results and interpretations is very important in the economics markets that are characterized for its uncertainty.

In future research, we plan to make more extensions of aggregation operators using prioritized operators (Pérez-Arellano, León-Castro, Avilés-Ochoa, & Merigó 2017; Avilés-Ochoa, Perez-Arellano, León-Castro, & Merigó, 2017), Bonferroni means (Blanco-Mesa, Merigó, & Kacprzyk, 2016; Blanco-Mesa, León-Castro, & Merigó, 2018; Y. He, Z. He, & Chen, 2015), Choquet integrals (Belles-Sampera, Merigó, Guillén, & Santolino, 2014), distance operators (Merigó & Casanovas, 2011; Liu & Chen, 2017), Hesitant fuzzy sets (Ren, Xu, & Hao, 2017; Zeng & Yao, 2018), intuitionistic fuzzy sets (Liu, J. X. You, X. Y. You, & Su, 2016), multicriteria decision making (Zeng, Chen, & Li, 2016) and econometric techniques (León-Castro, Avilés-Ochoa, Merigó, & Gil-Lafuente, 2018; Avilés-Ochoa, León-Castro, Gil-Lafuente, & Merigó, 2018). Additionally, we will also use other operators in different kinds of problems like group decision making (Xu, 2006). Also, the development of the formulation for the OWA volatility and some of its extensions and families have been planned for future research, the main difference is that instead of using the moving average in the aggregation process, the information will be evaluated individually.

We plan to use the powerful OWA type aggregation operators to further extend our models of the broadly perceived decision making in socio-economic systems to obtain a deeper insight of the stability of regional development strategies as already mentioned (Kacprzyk & Straszak, 1984; Yager et al., 2011; Kacprzyk, 2015). Moreover, we plan to further extend the use of our operators for a human consistent aggregation of preferences, testimonies and judgments under a fuzzy majority proposed in Kacprzyk (1986). In particular, we will deal with various classes of group decisions (Zeng, Llopis-Albert, & Zhang, 2018; Merigó, Xu, & Zeng, 2013) and voting (Kacprzyk, Fedrizzi, & Nurmi, 1992), and consensus reaching (Herrera-Viedma, Cabrerizo, Kacprzyk, & Pedrycz, 2014; Kacprzyk & Fedrizzi, 1988, 1989; Kacprzyk, Zadrożny, & Raś, 2010).

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References

Aristotelous, K. (2001). Exchange-rate volatility, exchange-rate regime, and trade volume: evidence from the UK–US export function (1889–1999). *Economics Letters*, *72*(1), 87-94. https://doi.org/10.1016/S0165-1765(01)00414-1

Avilés-Ochoa, E., León-Castro, E., Gil-Lafuente, A. M., & Merigó, J. M. (2018). Forgotten effects in exchange rate forecasting models. In *Complex systems: solutions and challenges in economics, management and engineering* (pp. 423-437). Cham: Springer.

Avilés-Ochoa, E., Perez-Arellano, L. A., León-Castro, E., & Merigó, J. M. (2017). Prioritized induced probabilistic distances in transparency and access to information laws. *Fuzzy Economic Review*, *22*(1). https://doi.org/10.25102/fer.2017.01.04

Belles-Sampera, J., Merigó, J. M., Guillén, M., & Santolino, M. (2014). Indicators for the characterization of discrete Choquet integrals. *Information Sciences*, *267*, 201-216. https://doi.org/10.1016/j.ins.2014.01.047

Blanco-Mesa, F., León-Castro, E., & Merigó, J. M. (2018). Bonferroni induced heavy operators in ERM decision-making: A case on large companies in Colombia. *Applied Soft Computing*, *72*, 371-391. https://doi.org/10.1016/j.asoc.2018.08.001

Blanco-Mesa, F., Merigó, J. M., & Gil-Lafuente, A. M. (2017). Fuzzy decision making: a bibliometric-based review. *Journal of Intelligent & Fuzzy Systems*, *32*(3), 2033-2050. https://doi.org/10.3233/JIFS-161640

Blanco-Mesa, F., Merigó, J. M., & Kacprzyk, J. (2016). Bonferroni means with distance measures and the adequacy coefficient in entrepreneurial group theory. *Knowledge-Based Systems*, *111*, 217-227. https://doi.org/10.1016/j.knosys.2016.08.016

Carr, P., & Wu, L. (2008). Variance risk premiums. *The Review of Financial Studies*, *22*(3), 1311-1341. https://doi.org/10.1093/rfs/hhn038

Della Corte, P., Ramadorai, T., & Sarno, L. (2016). Volatility risk premia and exchange rate predictability. *Journal of Financial Economics*, *120*(1), 21-40. https://doi.org/10.1016/j.jfineco.2016.02.015

Emrouznejad, A., & Marra, M. (2014). Ordered weighted averaging operators 1988–2014: A citation-based literature survey. *International Journal of Intelligent Systems*, *29*(11), 994-1014. https://doi.org/10.1002/int.21673

Ethier, W. (1973). International trade and the forward exchange market. *The American Economic Review*, *63*(3), 494-503.

Garman, M. B., & Klass, M. J. (1980). On the estimation of security price volatilities from historical data. *Journal of business*, 67-78. https://doi.org/10.1086/296072

Grossmann, A., Love, I., & Orlov, A. G. (2014). The dynamics of exchange rate volatility: A panel VAR approach. *Journal of International Financial Markets, Institutions and Money*, *33*, 1-27. https://doi.org/10.1016/j.intfin.2014.07.008

He, Y., He, Z., & Chen, H. (2015). Intuitionistic fuzzy interaction Bonferroni means and its application to multiple attribute decision making. *IEEE transactions on cybernetics*, *45*(1), 116-128. https://doi.org/10.1109/TCYB.2014.2320910

Héricourt, J., & Poncet, S. (2015). Exchange rate volatility, financial constraints, and trade: empirical evidence from Chinese firms. *The World Bank Economic Review*, *29*(3), 550-578. https://doi.org/10.1093/wber/lht035

Herrera-Viedma, E., Cabrerizo, F. J., Kacprzyk, J., & Pedrycz, W. (2014). A review of soft consensus models in a fuzzy environment. *Information Fusion*, *17*, 4-13. https://doi.org/10.1016/j.inffus.2013.04.002

Hooghe, M., & Kern, A. (2015). Party membership and closeness and the development of trust in political institutions: An analysis of the European Social Survey, 2002–2010. *Party Politics*, *21*(6), 944-956. https://doi.org/10.1177/1354068813509519

Kacprzyk, J. (1986). Group decision making with a fuzzy linguistic majority. *Fuzzy sets and systems*, *18*(2), 105-118. https://doi.org/10.1016/0165-0114(86)90014-X

Kacprzyk, J. (2015). Fuzzy dynamic programming for the modeling of sustainable regional development (Survey). *Applied and Computational Mathematics*, *14*(2), 107-124.

Kacprzyk, J., & Fedrizzi, M. (1988). A 'soft'measure of consensus in the setting of partial (fuzzy) preferences. *European Journal of Operational Research*, *34*(3), 316-325. https://doi.org/10.1016/0377-2217(88)90152-X

Kacprzyk, J., & Fedrizzi, M. (1989). A 'human-consistent'degree of consensus based on fuzzy login with linguistic quantifiers. *Mathematical Social Sciences*, *18*(3), 275-290. https://doi.org/10.1016/0165-4896(89)90035-8

Kacprzyk, J., & Straszak, A. (1984). Determination of "stable" trajectories of integrated regional development using fuzzy decision models. *IEEE Transactions on Systems, Man, and Cybernetics*, *14*(2), 310-313. https://doi.org/10.1109/TSMC.1984.6313215

Kacprzyk, J., Fedrizzi, M., & Nurmi, H. (1992). Group decision making and consensus under fuzzy preferences and fuzzy majority. *Fuzzy Sets and Systems*, *49*(1), 21-31. https://doi.org/10.1016/0165-0114(92)90107-F

Kacprzyk, J., Romero, R. A., & Gomide, F. A. (1999). Involving objective and subjective aspects in multistage decision making and control under fuzziness: dynamic programming and neural networks. *International Journal of Intelligent Systems*, *14*(1), 79-104. https://doi.org/10.1002/(SICI)1098-111X(199901)14:1<79::AID-INT6>3.0.CO;2-6

Kacprzyk, J., Zadrożny, S., & Raś, Z. W. (2010). How to support consensus reaching using action rules: a novel approach. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, *18*(04), 451-470. https://doi.org/10.1142/S0218488510006647

Kenney, J., & Keeping, E. (1962). *Moving Averages*. Princeton: Van Nostrand.

León-Castro, E., Avilés-Ochoa, E. A., & Gil-Lafuente, A. M. (2016). Exchange rate USD/MXN forecast through econometric models, time series and HOWMA operators. *Economic Computation and Economic Cybernetics Studies and Research*, *50*(4), 135-150.

León-Castro, E., Avilés-Ochoa, E., & Merigó, J. M. (2018). Induced heavy moving averages. *International Journal of Intelligent Systems*, *33*(9), 1823-1839. https://doi.org/10.1002/int.21916

León-Castro, E., Avilés-Ochoa, E., Merigó, J. M., & Gil-Lafuente, A. M. (2018). Heavy moving averages and their application in econometric forecasting. *Cybernetics and Systems*, *49*(1), 26-43. https://doi.org/10.1080/01969722.2017.1412883

Liu, H. C., You, J. X., You, X. Y., & Su, Q. (2016). Fuzzy Petri nets using intuitionistic fuzzy sets and ordered weighted averaging operators. *IEEE Transactions on Cybernetics*, *46*(8), 1839-1850. https://doi.org/10.1109/TCYB.2015.2455343

Liu, P., & Chen, S. M. (2017). Group decision making based on Heronian aggregation operators of intuitionistic fuzzy numbers. *IEEE Transactions on Cybernetics*, *47*(9), 2514-2530. https://doi.org/10.1109/TCYB.2016.2634599

Merigó, J. M., & Casanovas, M. (2011). Decision-making with distance measures and induced aggregation operators. *Computers & Industrial Engineering*, *60*(1), 66-76. https://doi.org/10.1016/j.cie.2010.09.017

Merigó, J. M., & Gil-Lafuente, A. M. (2009). The induced generalized OWA operator. *Information Sciences*, *179*(6), 729-741. https://doi.org/10.1016/j.ins.2008.11.013

Merigó, J. M., & Yager, R. R. (2013). Generalized moving averages, distance measures and OWA operators. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, *21*(04), 533-559. https://doi.org/10.1142/S0218488513500268

Merigó, J. M., Guillén, M., & Sarabia, J. M. (2015). The ordered weighted average in the variance and the covariance. *International Journal of Intelligent Systems*, *30*(9), 985-1005. https://doi.org/10.1002/int.21716

Merigó, J. M., Xu, Y., & Zeng, S. (2013). Group decision making with distance measures and probabilistic information. *Knowledge-Based Systems*, *40*, 81-87. https://doi.org/10.1016/j.knosys.2012.11.014

Minton, B. A., & Schrand, C. (1999). The impact of cash flow volatility on discretionary investment and the costs of debt and equity financing. *Journal of Financial Economics*, *54*(3), 423-460. https://doi.org/10.1016/S0304-405X(99)00042-2

Mueller, P., Vedolin, A., & Yen, Y. M. (2012). *Bond variance risk premia*. Financial Markets Group, London School of Economics and Political Science.

Officer, R. R. (1973). The variability of the market factor of the New York Stock Exchange. *The Journal of Business*, *46*(3), 434-453. https://doi.org/10.1086/295551

Pérez-Arellano, L. A., León-Castro, E., Avilés-Ochoa, E., & Merigó, J. M. (2017). Prioritized induced probabilistic operator and its application in group decision making. *International Journal of Machine Learning and Cybernetics*, 1-12.

Pilbeam, K., & Langeland, K. N. (2015). Forecasting exchange rate volatility: GARCH models versus implied volatility forecasts. *International Economics and Economic Policy*, *12*(1), 127-142. https://doi.org/10.1007/s10368-014-0289-4

Rabbani, A. G., Grable, J. E., Heo, W., Nobre, L., & Kuzniak, S. (2017). Stock market volatility and changes in financial risk tolerance during the great recession. *Journal of Financial Counseling and Planning*, *28*(1), 140. https://doi.org/10.1891/1052-3073.28.1.140

Ren, P., Xu, Z., & Hao, Z. (2017). Hesitant fuzzy thermodynamic method for emergency decision making based on prospect theory. *IEEE Transactions on Cybernetics*, *47*(9), 2531-2543. https://doi.org/10.1109/TCYB.2016.2638498

Zeng, S., & Xiao, Y. (2018). A method based on TOPSIS and distance measures for hesitant fuzzy multiple attribute decision making. *Technological and Economic Development of Economy*, *24*(3), 969-983. https://doi.org/10.3846/20294913.2016.1216472

Van Biezen, I., Mair, P., & Poguntke, T. (2012). Going, going,... gone? The decline of party membership in contemporary Europe. *European Journal of Political Research*, *51*(1), 24-56. https://doi.org/10.1111/j.1475-6765.2011.01995.x

Xu, Z. (2006). Induced uncertain linguistic OWA operators applied to group decision making. *Information Fusion*, *7*(2), 231-238. https://doi.org/10.1016/j.inffus.2004.06.005

Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multicriteria decisionmaking. *IEEE Transactions on Systems, Man, and Cybernetics*, *18*(1), 183-190. https://doi.org/10.1109/21.87068

Yager, R. R. (1996). On the inclusion of variance in decision making under uncertainty. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, *4*(05), 401-419. https://doi.org/10.1142/S0218488596000238

Yager, R. R. (2002). Heavy OWA operators. *Fuzzy Optimization and Decision Making*, *1*(4), 379-397. https://doi.org/10.1023/A:1020959313432

Yager, R. R. (2004). Generalized OWA aggregation operators. *Fuzzy Optimization and Decision Making*, *3*(1), 93-107. https://doi.org/10.1023/B:FODM.0000013074.68765.97

Yager, R. R. (2006). Generalizing variance to allow the inclusion of decision attitude in decision making under uncertainty. *International Journal of Approximate Reasoning*, *42*(3), 137-158. https://doi.org/10.1016/j.ijar.2005.09.001

Yager, R. R., & Filev, D. P. (1999). Induced ordered weighted averaging operators. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, *29*(2), 141-150. https://doi.org/10.1109/3477.752789

Yager, R. R., Kacprzyk, J., & Beliakov, G. (Eds.). (2011). *Recent developments in the ordered weighted averaging operators: theory and practice* (Vol. 265). Berlin, Heidelberg: Springer. https://doi.org/10.1007/978-3-642-17910-5

Zeng, S., Chen, J., & Li, X. (2016). A hybrid method for pythagorean fuzzy multiple-criteria decision making. *International Journal of Information Technology & Decision Making*, *15*(02), 403-422. https://doi.org/10.1142/S0219622016500012

Zeng, S., Llopis-Albert, C., & Zhang, Y. (2018). A novel induced aggregation method for intuitionistic fuzzy set and its application in multiple attribute group decision making. *International Journal of Intelligent Systems*, *33*(11), 2175-2188. https://doi.org/10.1002/int.22009

Zhou, L. G., & Chen, H. Y. (2011). Continuous generalized OWA operator and its application to decision making. *Fuzzy Sets and Systems*, *168*(1), 18-34. https://doi.org/10.1016/j.fss.2010.05.009